UNIVERSITÀ
DI PARMA

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

BOLOGNA AND PARMA APPROACH AND PROPOSALS

Laura Branchetti*, Paola Fantini, Olivia Levrini****
Michael Lodi, Eleonora Barelli, Giovanni Ravaioli, Giulia Tasquier

*Department of Mathematical, Physical and Computer Sciences, University of Parma (Italy)

** Department of Physics and Astronomy, UNIBO

DISCIPLINES AND SOCIETAL CHALLENGES: IS THERE A TENSION?

- curricula are still organized in **disciplines**
- background of **science and mathematics** teachers and curriculum developers' are, in most cases, disciplinary

VS

- recommendations for an educational switch from knowledge to skills and/or for teaching in a **STEM perspective** have been coming from **outside the schools** (policy makers, entrepreneurial world, labour market)

DISCIPLINES AND SOCIETAL CHALLENGES

What is the role of **traditional disciplines** to prepare students to face **societal challenges**? What space should we reserve for their teaching? Are they becoming unnecessary or do they still play a relevant role?

Neither the “**traditional**” **disciplinary approach** to knowledge nor an **a-disciplinary approach**, based on transversal skills, is productive to address societal challenges and their authentic problems.

What is a **discipline**?

What does it mean the term “**interdisciplinarity**” in relationship with STEM integration and new emerging fields?

DISCIPLINE

Latin root "discere", whose meaning is "to learn"

Forms of knowledge organization

Mathematics?
Physics, Chemistry, ...?

Computer science?
Engineering?

DISCIPLINES AND SCHOOL SUBJECT MATTERS

subject matters usually do not reflect both the nature of contemporary scientific endeavor and the history of science

disciplinary authenticity should be pursued **developing epistemic skills** "by emphasizing the practices of **doing science** and **generating scientific knowledge**, while other, more historical-philosophical–oriented settings may emphasize critical reflection on the **epistemological and historical processes of the development of scientific knowledge**."

(Kapon et al., 2018)

DISCIPLINES AND AUTHENTIC RESEARCH PRACTICES

National Academies Committee on Facilitating Interdisciplinary Research (Kates, 2005): “Interdisciplinary research is a mode of research by teams or individuals that integrates information, data, techniques, tools, perspectives, concepts, and/or theories from two or more **disciplines or bodies of specialised knowledge** to **advance** fundamental understanding or to solve problems whose solutions are **beyond the scope of a single discipline or field of research practice**” (p. 2).

Interdisciplinarity characterize authentic contemporary research practices BUT still needs disciplines!

RESEARCH QUESTIONS

RQ1: How can **disciplinary knowledge and epistemic skills** be exploited or developed in teaching, **whilst coping with authentic (interdisciplinary) STEM issues**?

RQ2: If so, **what** authentic (interdisciplinary) STEM issues can be used to guide students to exploit or develop their disciplinary knowledge and disciplinary epistemic skills?
What teaching approaches can be applied?

TWO CASES FOR TWO DIRECTIONS

Interdisciplinarity ⟷ **Disciplinary authenticity**

1. an **interdisciplinary approach** could help in **understanding better a discipline** (ex. blackbody radiation)

2. **disciplinary knowledge** could help in learning new disciplines or in **dealing with new problems that are not yet organized in a discipline** (ex. artificial intelligence)

BLACKBODY RADIATION

Branchetti, L., Cattabriga, A., Levrini, O. (accepted). *Interplay between mathematics and physics to catch the nature of a scientific breakthrough: The case of the blackbody,* Phys. Rev. Phys. Educ. Res.

MATHEMATICS AND PHYSICS: A HIDDEN RELATIONSHIP

Karam R. (Ed.) (2015). *Introduction to the thematic issue on the interplay of physics and mathematics.* Science & Education.

Physics education → Mathematics as a mere tool to describe and calculate

Mathematics education → Physics as a possible context for the application of abstract concepts

While Mathematics for Physics…

- is an instigator of scientific revolutions (Brush, 2015)
- provides formal structures (e.g. creative power of formal analogies in physics) (Kragh, 2015)

1st CASE: BLACKBODY RADIATION

One of the most interesting **historical case studies:**
the breakthrough that led to Quantum Physics

What contribution can this **historical case** provide to the debate on the **interplay of physics and mathematics**? What are the specific roles of mathematics in this case?

How can the case be **reconstructed for an educational purpose**?

PLANCK, M. (1900): PRIMARY SOURCES

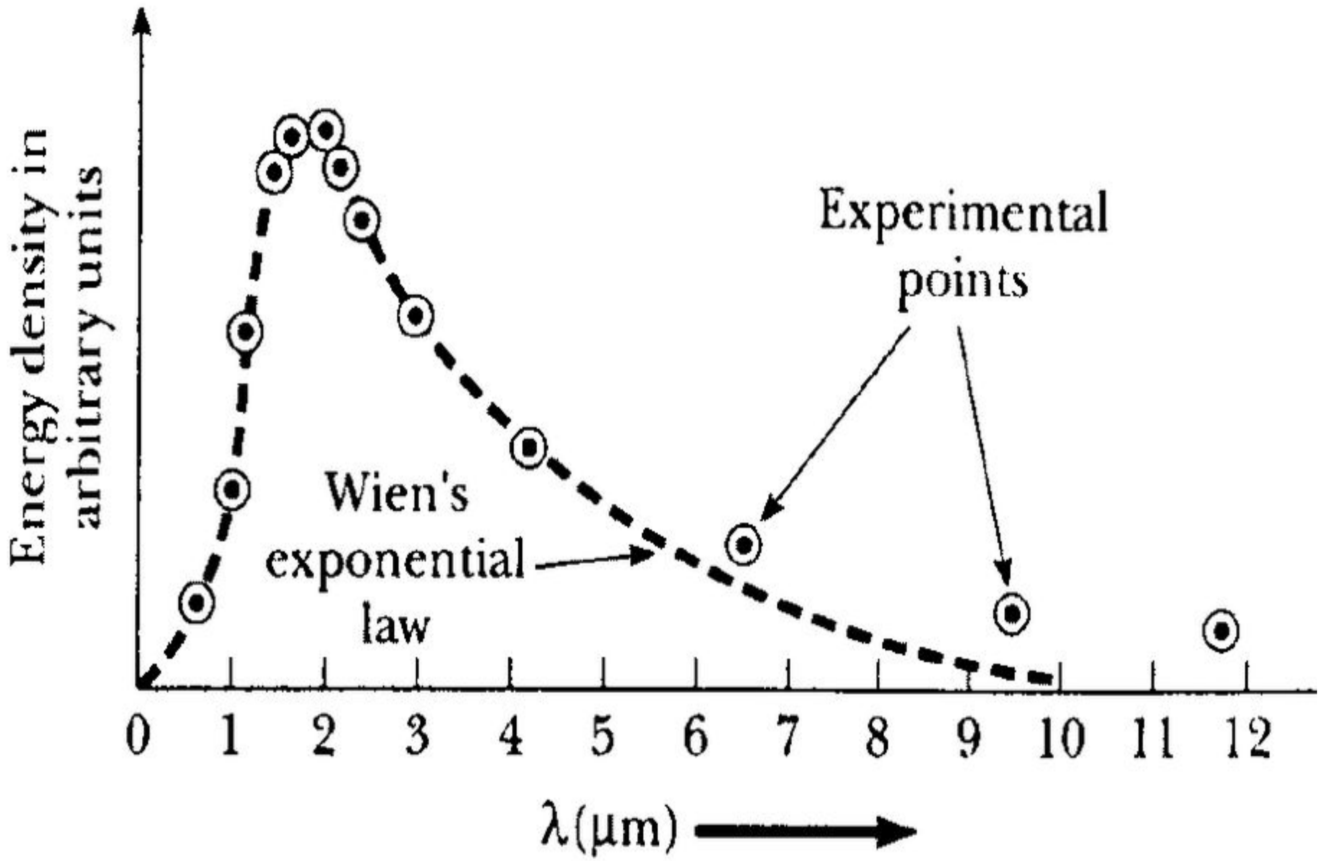
a. *On an Improvement of Wien's Equation for the Spectrum*

b. *On the Theory of the Energy Distribution Law of the Normal Spectrum*

Three main phases (Tronconi, 2016):

1. **mathematical (formal) improvement** of Wien's law with a better-fitting expression of the spectral density;
2. **construction of a model** in analogy with Boltzmann's approach to thermodynamics and **derivation** of the law within a physical theory
3. **Physical interpretation** of the mathematical model

WIEN'S EXPONENTIAL LAW FOR BLACKBODY RADIATION



PLANCK (1900A): FORMULATION OF A CONJECTURE

"**Wien's energy distribution law is not as generally valid**, as many supposed up to now […] my view that Wien's law would be of general validity, was brought about rather by **special considerations**, namely by the **evaluation of an infinitesimal increase of the entropy of a system of *n* identical resonators** in a stationary radiation field […]

From this equation Wien's law follows in $\frac{d^2S}{dU^2} = \frac{\text{const}}{U}$

Following this suggestion **I have finally started to construct completely arbitrary expressions for the entropy** which although they are more complicated than Wien's expression still seem to **satisfy just as completely all requirements of the thermodynamic and electromagnetic theory**."

PLANCK (1900B)

“ Since the entropy of a resonator is thus determined by the way in which the energy is distributed at one time over many resonators, **I suspected** that one should evaluate this quantity in the electromagnetic radiation theory by introducing **probability considerations**, the importance of which for the second law of thermodynamics was first of all discovered by **Mr. Boltzmann.**

This suspicion has been confirmed; I have been able to **derive deductively an expression for the entropy** of a monochromatically vibrating resonator **and thus for the energy distribution in a stationary radiation state,** that is, in the normal spectrum. “

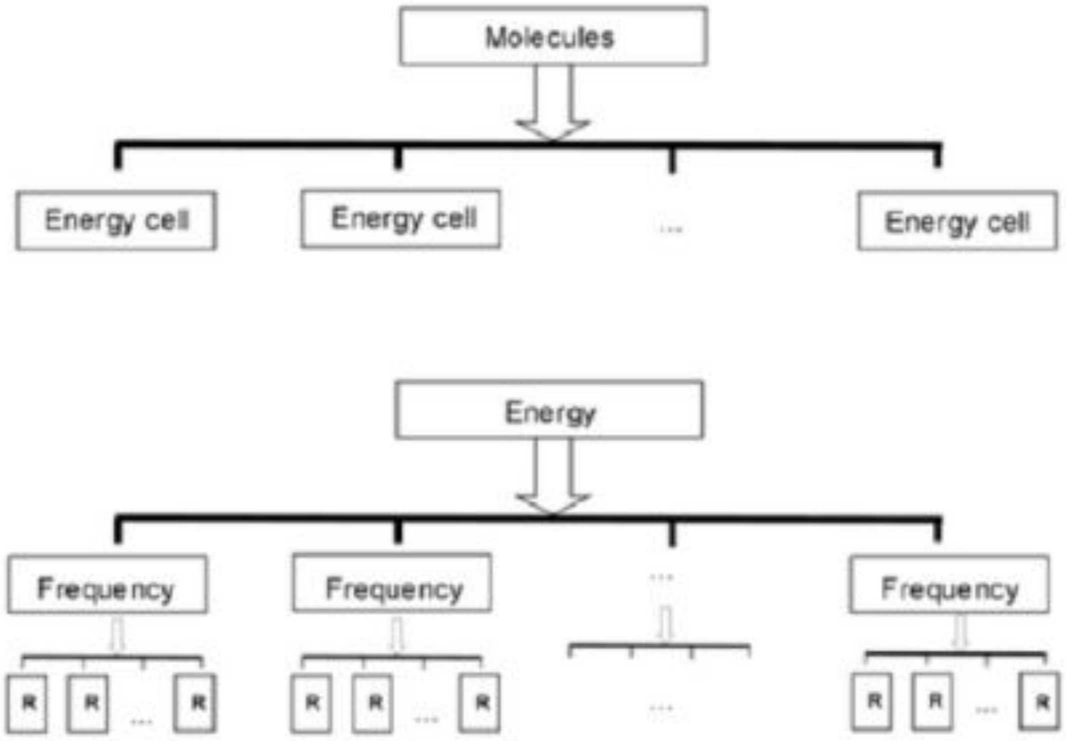
PLANCK (1900B): HYPOTHESIS OF DISCRETIZATION

“We must now **give the distribution** of the energy over the separate resonators of each group, first of all the distribution of the energy E over the N resonators of frequency ν.

If E considered to be continuously divisible quantity, this distribution is possible in infinitely many ways.

We consider, however – this is the most essential point of the whole calculation – **E to be composed of a very definite number of equal parts** and use thereto the constant of nature h= 6.55×10^(−27) erg·sec.”

PLANCK'S HYPOTHESIS



An expression for the **entropy**:
Boltzmann (**S = k log W**)

W = **Number of ways** in which energy exchanged in the interaction radiation-matter in the cavity could be distributed over the resonators with a given frequency

Classical hypothesis: energy is a **continuous variable**

Infinitely many possibilities of distributing energy

TEACHERS' REACTIONS…..

*"**I can follow the whole argument** and I understand the **mathematical problem** that we are asked to solve. Yet, **when I start with mathematics the symbols loose any meaning**. Here there is an "S" but I cannot recognize that it is entropy. For me it a generic variable and **I get lost**. I make my calculation but **I don't understand what they mean physically**".*

"Zooming in" on details and "zooming out" on the whole process needed to shape the interaction between mathematics and physics: to "use the little eye" and to "use the big eye".

If one of the two "eyes" is missing, the process of understanding the interrelation between mathematics and physics gets stuck.

an interdisciplinary approach help

in understanding better a discipline

Interdisciplinarity → **Disciplinary authenticity**

2nd CASE: ARTIFICIAL INTELLIGENCE

I SEE PROJECT (https://iseeproject.eu/)

Inclusive **STEM Education** to Enhance the capacity to aspire and imagine future careers

Structure of the I SEE modules

https://iseeproject.eu/wp-content/uploads/2019/08/O3_DEF.pdf

ARTIFICIAL INTELLIGENCE

A **future-relevant topic,** at the basis of **utopias and dystopias**

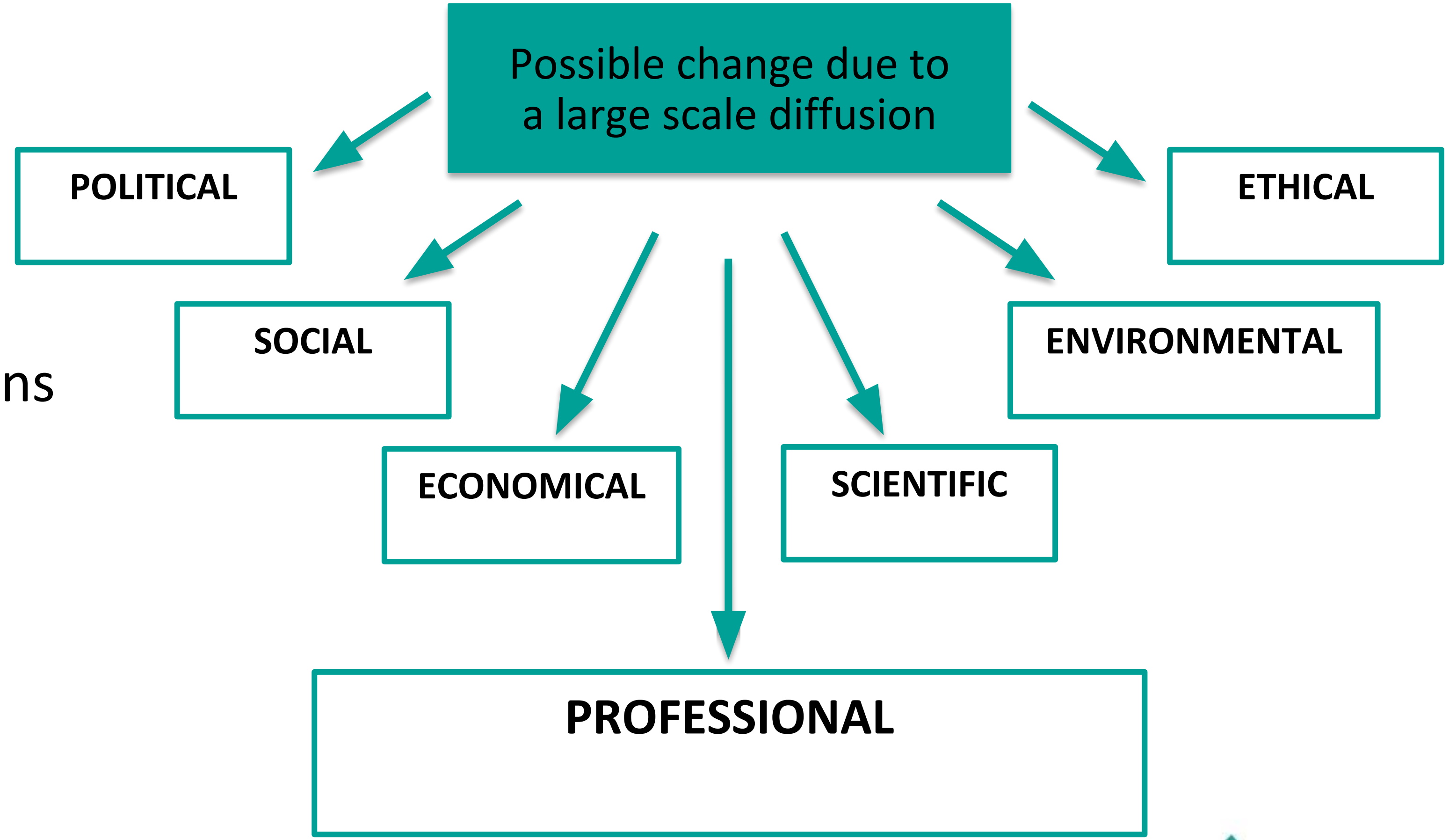
A **new research field** and **a labour market "obsession"**

Is Artificial Intelligence a new "STEM discipline"?

What is the role of Mathematics, Sciences, Technology, Engineering?

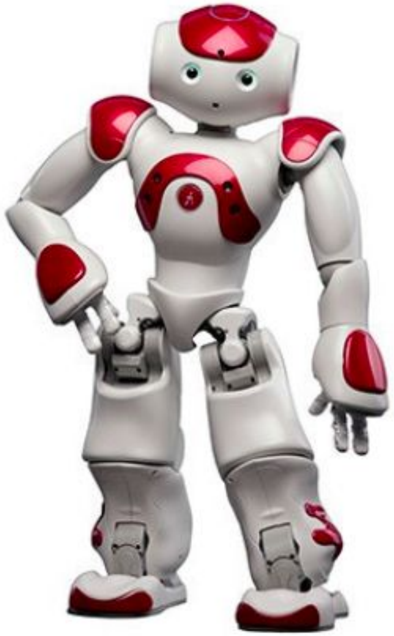
I. GROUP ACTIVITY - AI APPLICATIONS

- Autonomous vehicles
- Archeology
- Arts
- Services
- Scientific research
- Astronomical observations

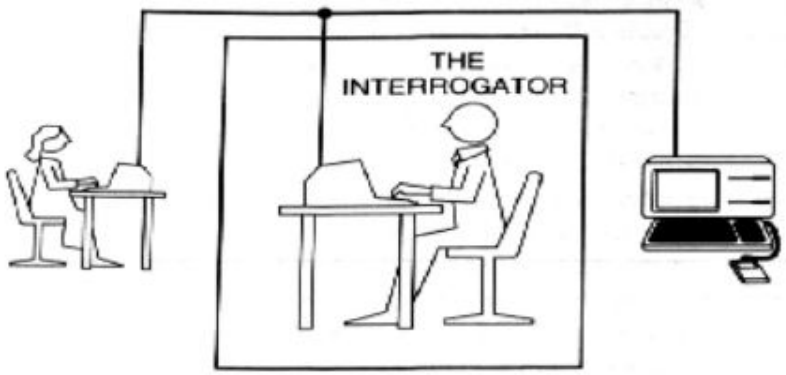


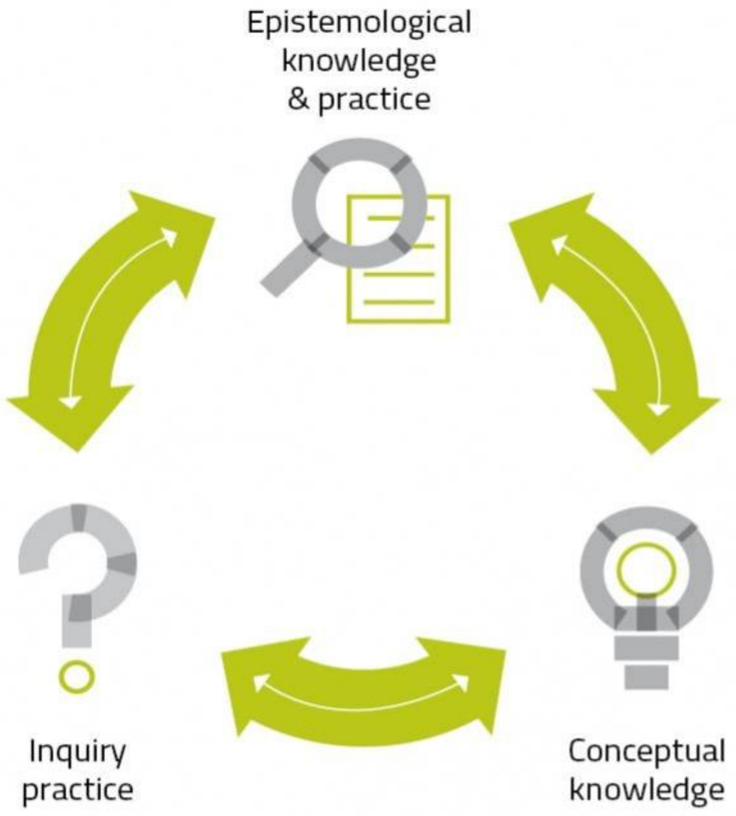
II. LECTURES

- **AI, complexity and culture**
  Prof. Gianni Zanarini, *Physicist*

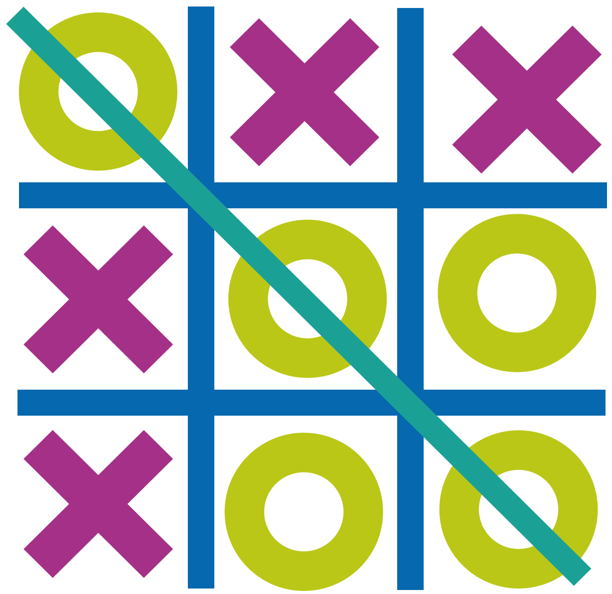


- Overview on **AI in the history**
  Prof. Paola Mello, *Computer Engineer*




Epistemological knowledge & practice
Inquiry practice
Conceptual knowledge

TIC-TAC-TOE



Three possible approaches for teaching a machine to solve a problem

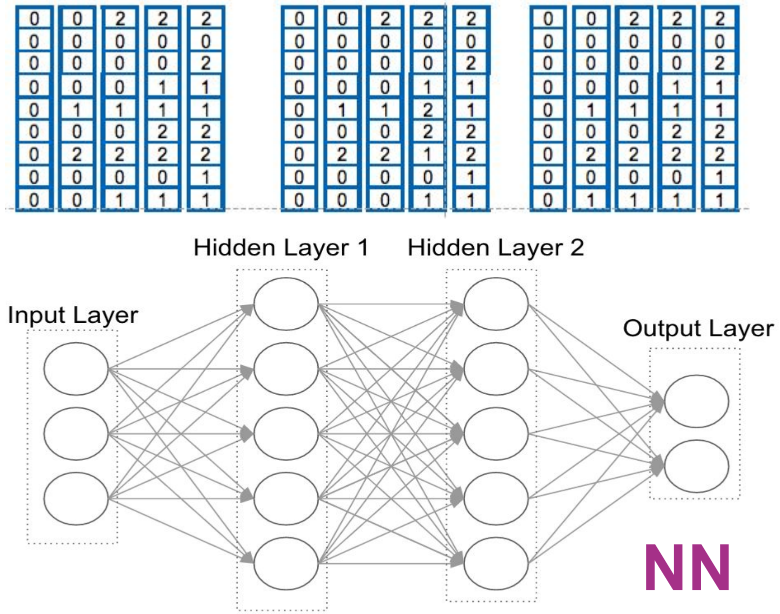
	IMPERATIVE APPROACH	LOGICAL APPROACH	MACHINE LEARNING
the programmer has to	explain to the machine exactly what to do for every possible situation through an ***algorithm***	make some logical statements and the machine will infer the output through an inference engine	collect examples of winning moves and train a neural network (NN) through a learning algorithm

PYTHON

```python
#check if player p
#wins on the board b
def wins(b, p):
  return ((b[6] == p and b[7] == p and b[8] == p) or (b[3] == p and

#returns next best move for player 'o'
#given the current state of the Board

def nextmove(Board):
  #Checks if 'o' can win with the next move
  for i in range(9):
    b = Board.copy()
    #if space i is empty
    if b[i] == '':
      #try to make the move
      b[i] = 'o'
      if wins(b,'o'):
        print("AI: win")
        return i

  #Otherwise, blocks 'x' to win with the next move
  for i in range(9):
    b = Board.copy()
    #if space i is empty
    if b[i] == '':
      #try to simulate 'x' move
      b[i] = 'x'
      if wins(b,'x'):
        print("AI: blocking opponent win")
        return i


  #Otherwise, try to make a move
```

```prolog
next_move(4).
next_move(0). next_move(2). next_move(6). next_move(8)
next_move(1). next_move(3). next_move(5). next_move(7)

%I win if I complete a line (I already have 'o' in B a
win(A) :- o(B), o(C), different(B,C), tris(A,B,C).

%I prevent 'x' victory by choosing A
prevent_other_win(A) :- x(B), x(C), different(B,C), tr

%I create a fork if at the next move I can win in two
fork(A) :- o(B), o(C), different(B,C), tris(A,B,D), tr

%I prevent a fork if at the next move the opponent cou
prevent_fork(A) :- x(B), x(C), different(B,C), tris(A,

%I make a move, of course if it's empty
computer_move :- next_move(A), empty(A), assert(o(A)).
```

PROLOG



NN

MACHINE LEARNING AS EMERGING PROPERTY

Neural networks introduce a new approach in AI

A **neural network** can be modeled as a ***complex system***

- Very simple rules
- Global emerging behavior **NOT pursued** and **NOT linearly reconstructable** by means of the simple rules

Learning *emerges* in a complex way from simple rules of "neurons" and their connections

I SEE
It's your time to imagine the futures

Aim: to **recognize** behind the approaches **forms of disciplinary-like rationality/forms of reasoning**:

	IMPERATIVE APPROACH	LOGICAL APPROACH	MACHINE LEARNING
the programmer has to	explain to the machine exactly what to do for every possible situation through an ***algorithm***	make some logical statements and the machine will infer the output through an inference engine	collect examples of winning moves and train a neural network (NN) through a learning algorithm

INFORMATICS → IMPERATIVE APPROACH

MATHEMATICS-LOGIC → LOGICAL APPROACH

PHYSICS → MACHINE LEARNING

disciplinary knowledge could help in learning new disciplines or in **dealing with new problems that are not yet organized in a discipline**

Interdisciplinarity ← **Disciplinary authenticity**

New idea for O3: curricular interdisciplinary topic
Parabola in Mathematics and Physics

Mathematics: conic sections, different definitions and characterizations with different aims, physical problems induce evolutions and unification in mathematical theories

Apollonius
Archimedes

Kepler

Παραβολή

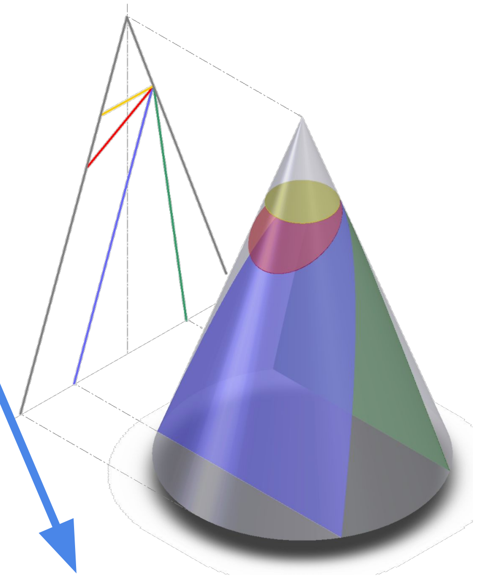
paràbola 1. = *lat.* PARÀBOLA dal *gr.* PARABOLÈ *azione di mettere al lato,* d'onde *comparazione,* e questo da PARABÀLLÔ *metto a lato* e quindi *ravvicino, paragono,* comp. di PARÀ *presso, in confronto* e BÀLLÔ *getto,* ed anche *pongo, metto* (v. *Balista*).

Paragone, Comparazione; Allegoria che conferma qualche verità importante, cioè Narrazione di un fatto comune con intendimento educativo di trarne un'analogia a circostanze di altro ordine, una norma per sapere ciò che sia da farsi in esse.

Deriv. *Parabolàno; Parabòlico; Parlàre; Paròla.* Cfr. *Problema.*

2. *In geom.* Figura prodotta da una delle sezioni del cono, tagliato da un piano parallelo ad uno de' suoi lati: così detta, perché in questa curva il quadrato dell'ordinata compara ossia agguaglia il rettangolo del parametro nell'ascissa, mentre è minore nella ellissi e maggiore nell'iperbola.

Abusivamente si usa per designare la Linea curva che segnano nell'atmosfera i proiettili delle armi da fuoco, detta più esattamente Trattòria o Traiettòria.



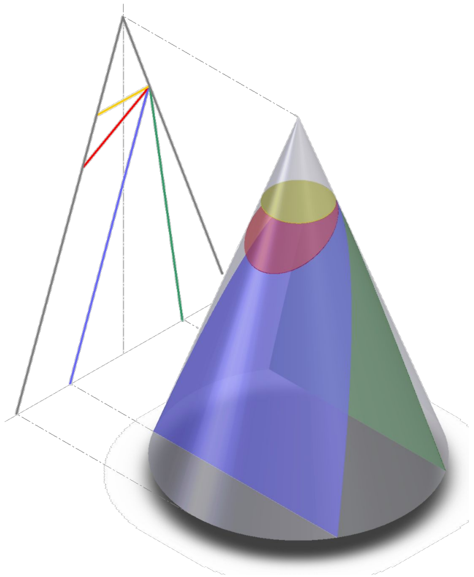
παραβάλλω

compare
put in parallel

ALMA MATER STUDIORUM
UNIVERSITÀ DI BOLOGNA

Euclide, *Elements* (IV-III a.C.).

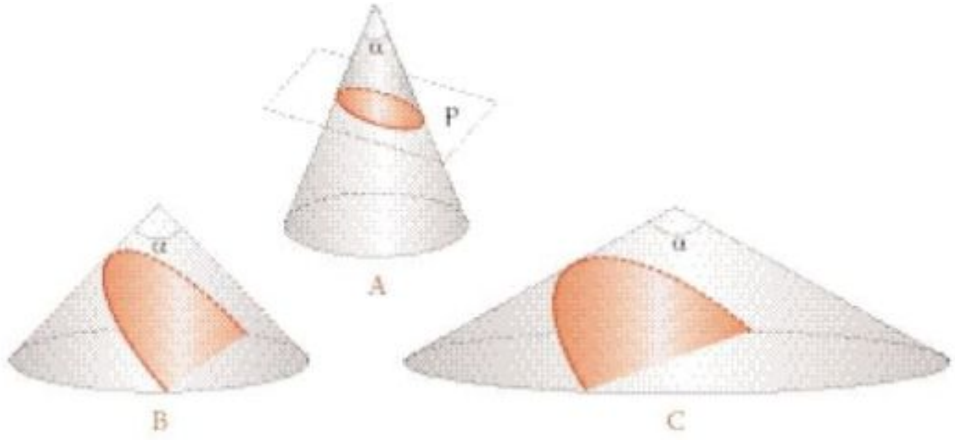
Elementi di Euclide, XI.18



1. acutangolo (*oxytoma*)
2. rettangolo (*orthotoma*)
3. ottusangolo (*amblystoma*)

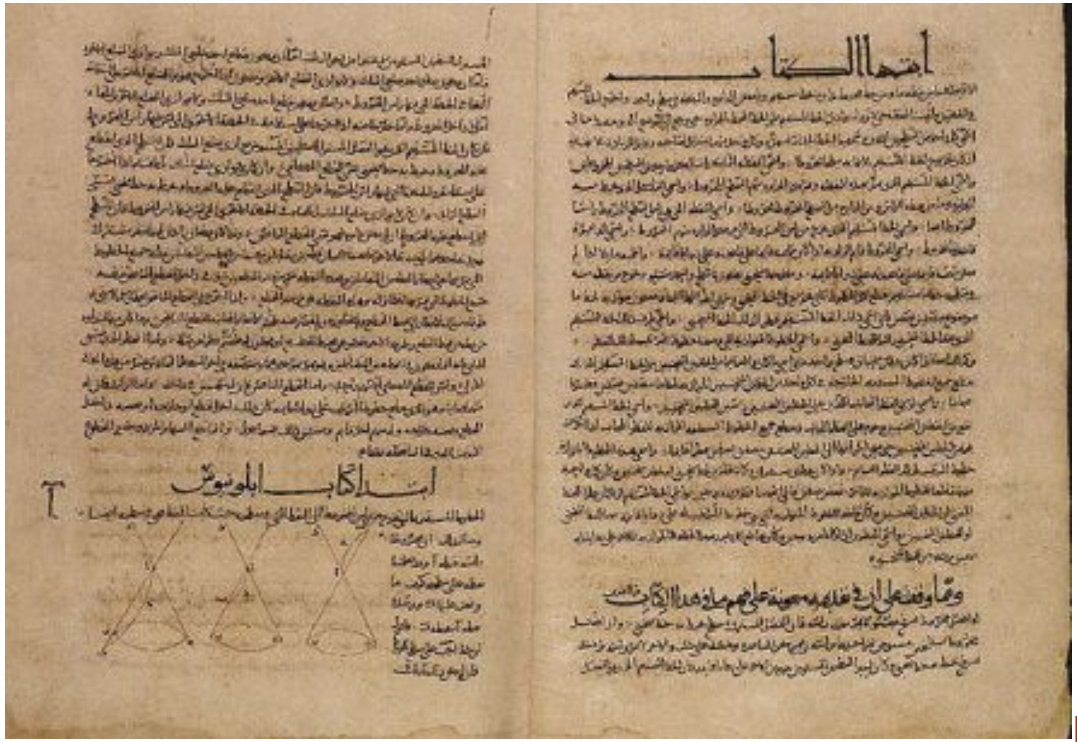
Dai termini greci: angolo acuto (ὀξύς)

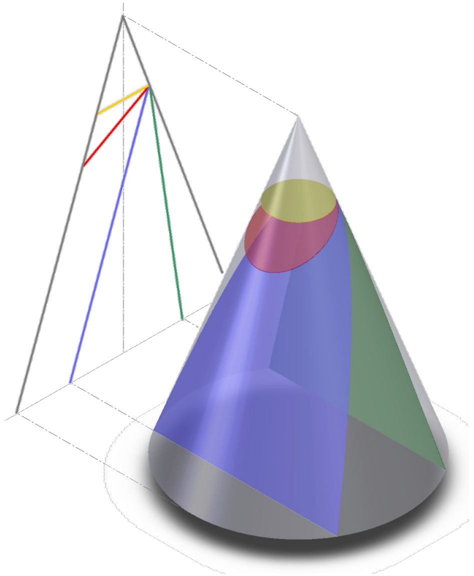
retto (ὀρθός) e ottuso (ἀμβλύς)



https://web.math.unifi.it/archimede/note_storia/Belle-Napolitani-Coniche.pdf

Apollonio di Perga, *Conic sections* (III-II a.C.).





Apollonio di Perga, *Conic sections* (III-II a.C.).

Coniche, proposizione I.11

Dato il cono ABC di vertice A e base BC si consideri un piano secante che generi una sezione il cui diametro PM sia parallelo a uno dei lati del triangolo per l'asse. Sia QV un'ordinata relativa al diametro PM.

Se si traccia una retta PL perpendicolare a PM nel piano della sezione, tale che

$$PL : PA \quad = \quad BC^2 : BA \times AC$$

allora

$$QV^2 \quad = \quad PL \times PV \qquad (3)$$

La sezione così ottenuta si chiama parabola *e la retta fissa PL (rispetto alla quale si realizza l'uguaglianza fra il quadrato di una qualsiasi ordinata e il rettangolo costruito sull'ascissa e tale retta fissa) è detta* lato retto *della parabola.*

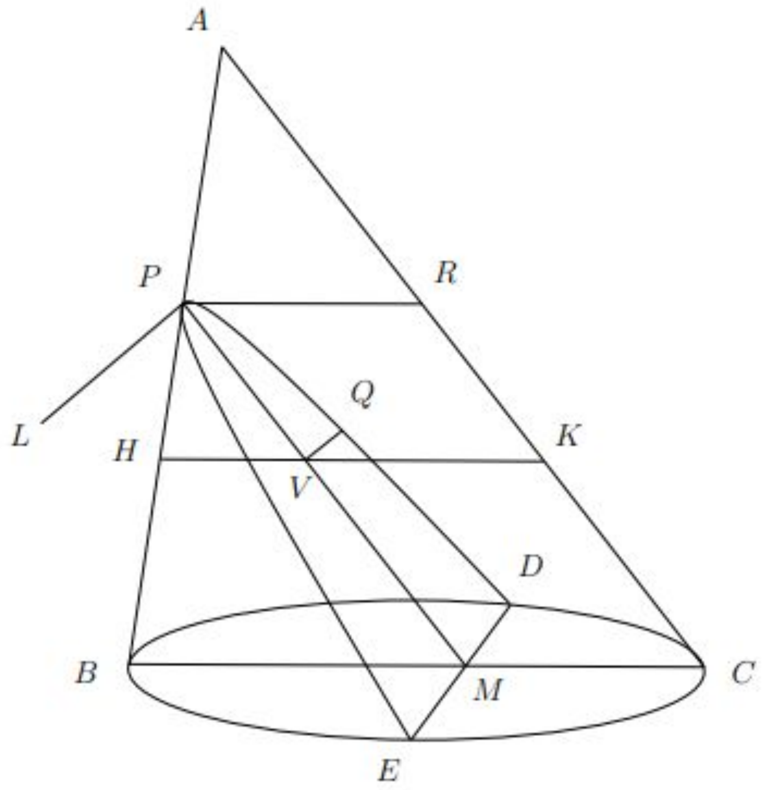


Figura 6: Il sintomo della parabola

http://old.unipr.it/arpa/urdidmat/Amici/GeoClassCap5.pdf

Apollonio di Perga, *Conic sections* (III-II a.C.).

Corollario (*Coniche*, prop. I.20)

Nella parabola i quadrati delle ordinate sono proporzionali alle ascisse.
Cioè, se Q_1 e Q_2 sono due punti sulla parabola e le rispettive ordinate sono Q_1V_1 e Q_2V_2, allora:

$$Q_1V_1^2 : Q_2V_2^2 = PV_1 : PV_2.$$

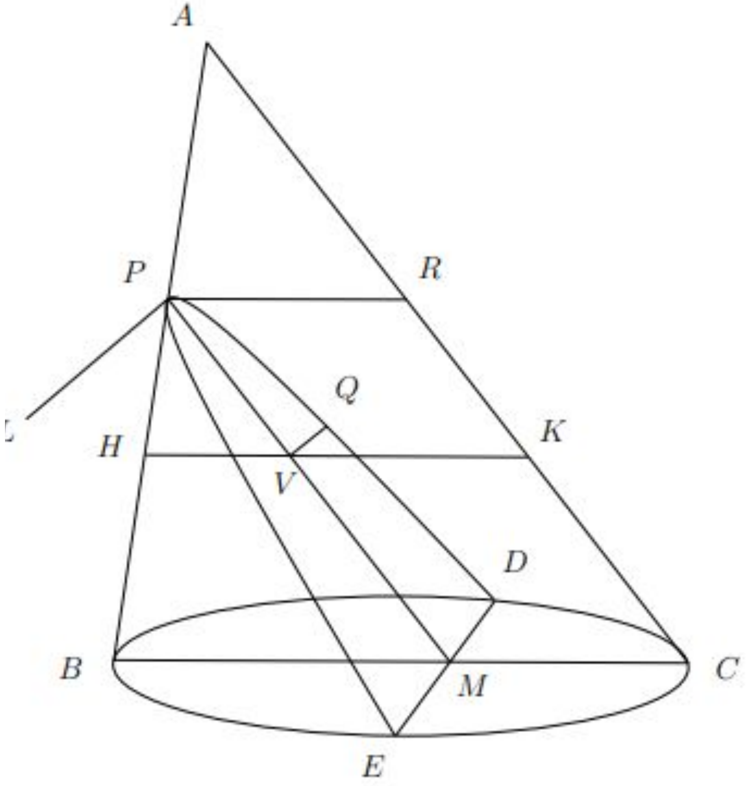


Figura 6: Il sintomo della parabola

Archimede, *Quadratura della parabola* (III a.C.).

Proposizione 2. *Dato un segmento parabolico, se conduciamo per il punto medio D della base AC la parallela all'asse, che interseca la parabola in B, e chiamiamo E l'intersezione tra la retta BD e la tangente alla parabola in uno degli estremi della base, allora EB=BD* (fig. 6).

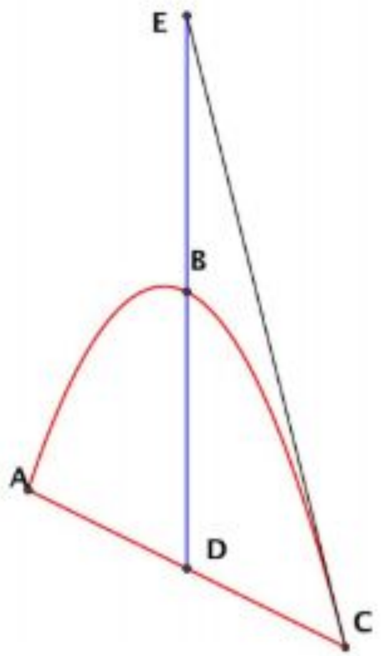


fig. 6

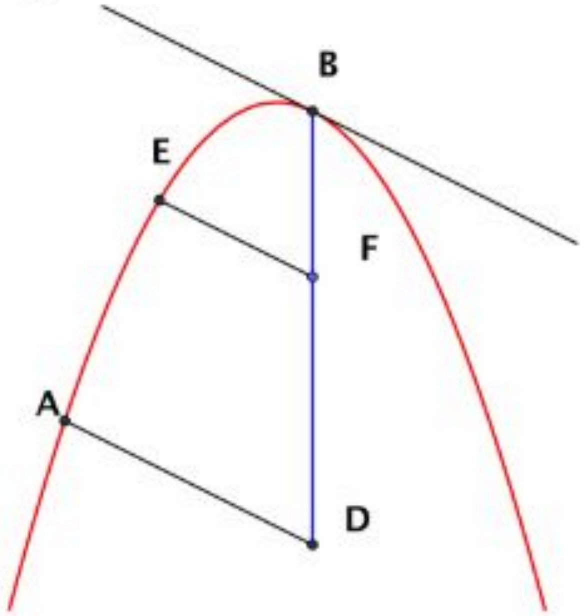


fig. 7

Proposizione 3. *Sia data una parabola, B sia un suo punto e BD una parallela all'asse ed F un punto di questo distinto da B e D. Se FE e AD sono parallele alla tangente in B, con A e E appartenenti alla parabola, allora* $\overline{BD}:\overline{BF}=\overline{AD}^2:\overline{EF}^2$ (fig. 7).

IORUM
LOGNA

Apollonio di Perga, *Conic sections* (III-II a.C.).

- Book III

1. Focal points

   τὰ ἐκ τῆς παραβολῆς γενηθέντα

2. Geometrical characterization coming from physical optical properties:

   « le rette condotte da uno dei fuochi ai punti di contatto delle tangenti alla conica, formano, con tali tangenti, angoli uguali a quelli che con esse formano le rette condotte dall'altro fuoco agli stessi punti di contatto ».

http://www.mathesisnazionale.it/mathesisbkp/archivio-storico-articoli-mathesis/68_83.pdf

Johannes Kepler, *Astronomiae pars optica, Parapolimena* (1571-1630)

fuoco F_1 ed il vertice V_1 (entrambi su s). Tali coniche hanno perciò in comune anche la tangente t in V_1. Indichiamo con a la distanza F_1V_1 ed assumiamo, come parametro atto ad individuare la conica nella famiglia, la distanza x dell'altro fuoco F_2 dalla detta tangente t, valutata positivamente o negativamente a seconda che F_2 si trovi, rispetto alla t, dalla stessa banda di F_1, oppure dalla banda opposta. Facciamo crescere x. con continuità, da a ad ∞, poi da $-\infty$ a $-a$. Si riconosce subito che, per $x = a$, la conica si riduce alla circonferenza di centro F_1 e di raggio a, per $a < x < \infty$ la conica è un'ellisse, per $x = \pm\infty$ è una parabola, per $-\infty < x$, $x < -a$ è un'iperbole, infine per $x = -a$ essa si riduce alla retta t contata due volte.

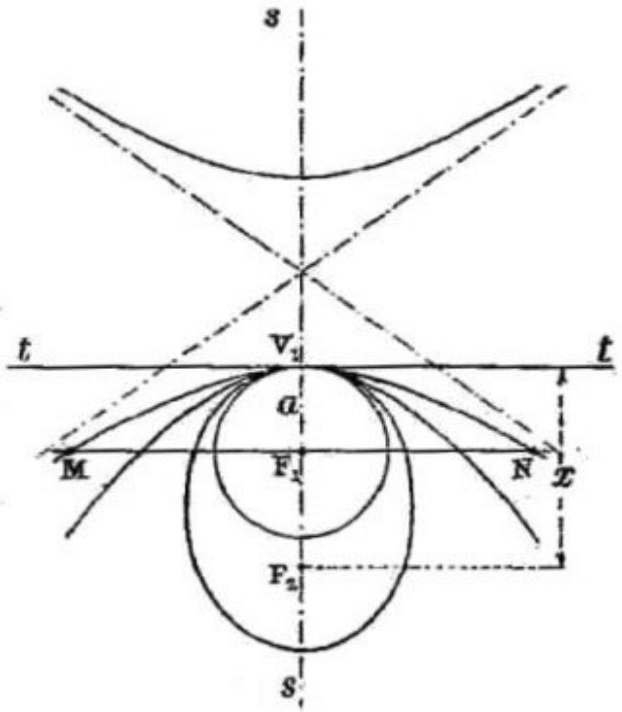


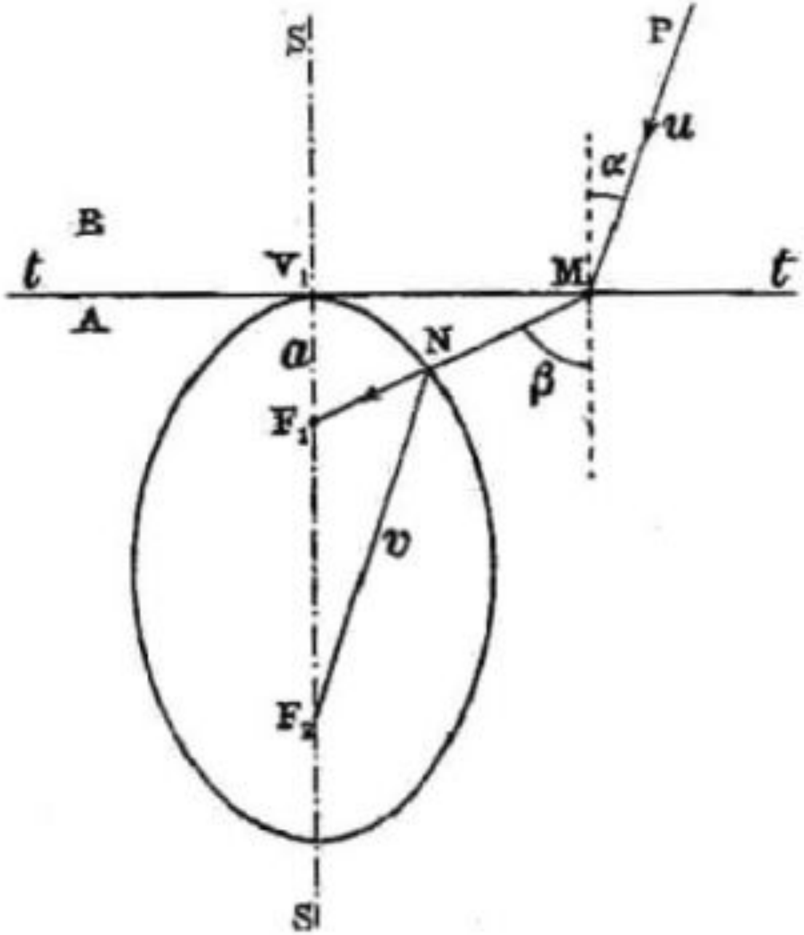
Fig. 1

http://www.mathesisnazionale.it/mathesisbkp/archivio-storico-articoli-mathesis/68_83.pdf

Johannes Kepler, *Astronomiae pars optica*, *Parapolimena* (1571-1630)

Rifraction and riflection

Curve/Flat mirror



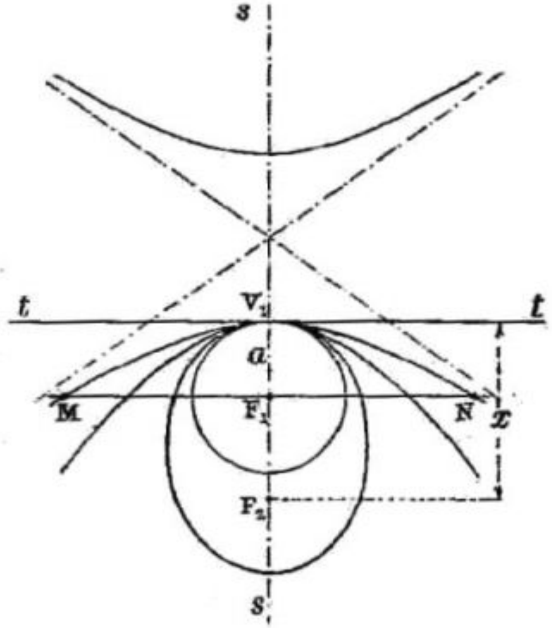
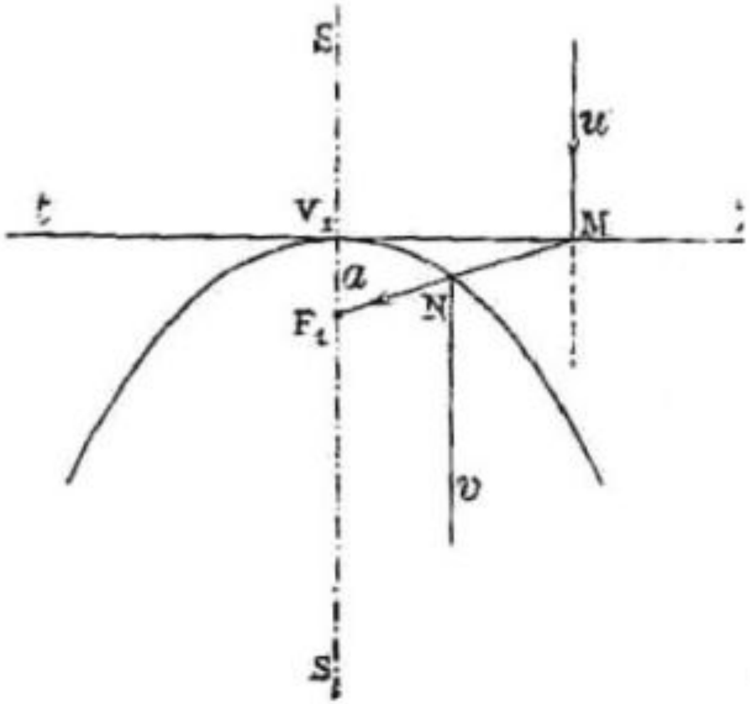


Fig. 1

mathesisbkp/archivio-storico-articoli-mathesis/68_83.pdf

Johannes Kepler, *Astronomiae pars optica, Parapolimena* (1571-1630)



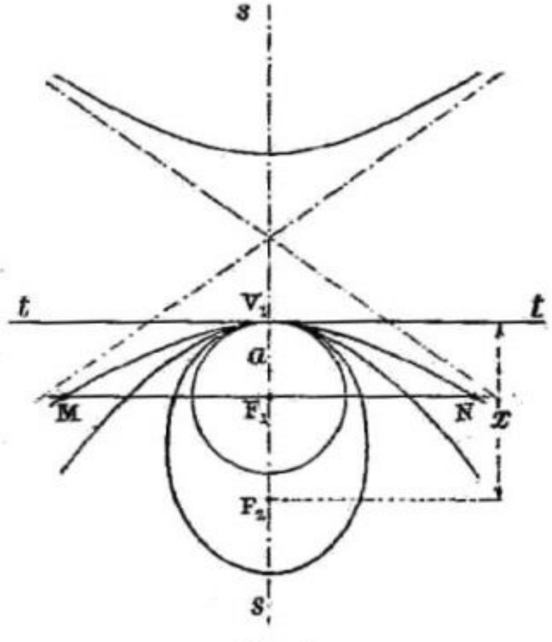
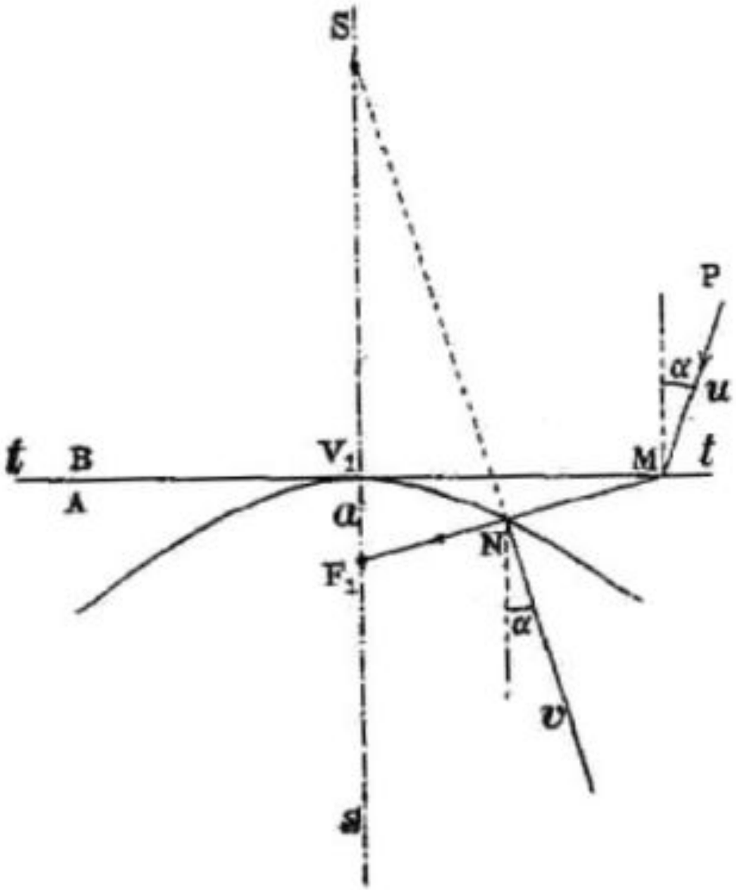


Fig. 1

Johannes Kepler, *Astronomiae pars optica, Parapolimena* (1571-1630)



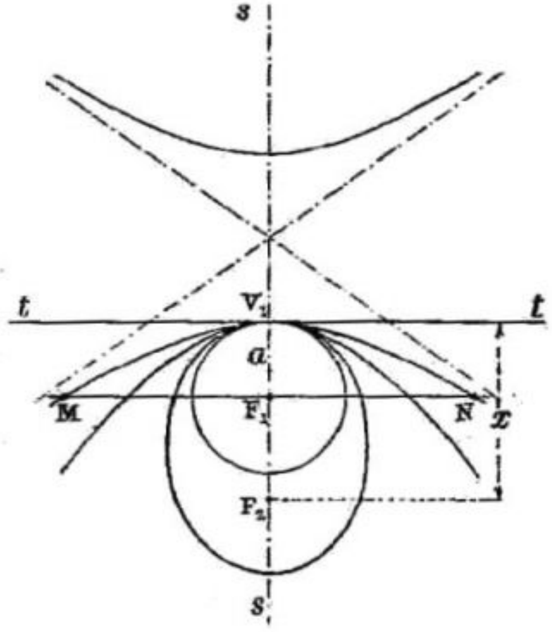


Fig. 1

Johannes Kepler, *Astronomiae pars optica*, *Parapolimena* (1571-1630)

4º) Il *valore del rapporto* $\frac{MN}{F_1V_1}$ (fig. 1), indicando con MN il segmento intercetto dalla generica conica della famiglia, sulla perpendicolare all'asse s, condotta per il fuoco F_1. Tale valore è 2 nel caso della circonferenza e cresce con continuità, passando per 4 nel caso della parabola e tendendo all'infinito nel caso della retta.

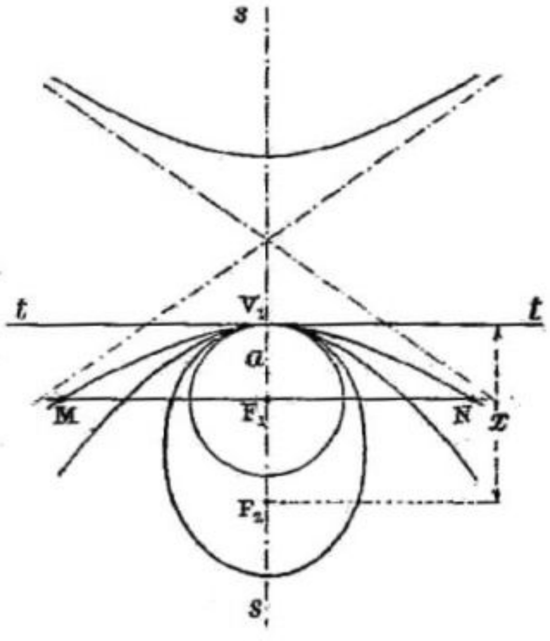
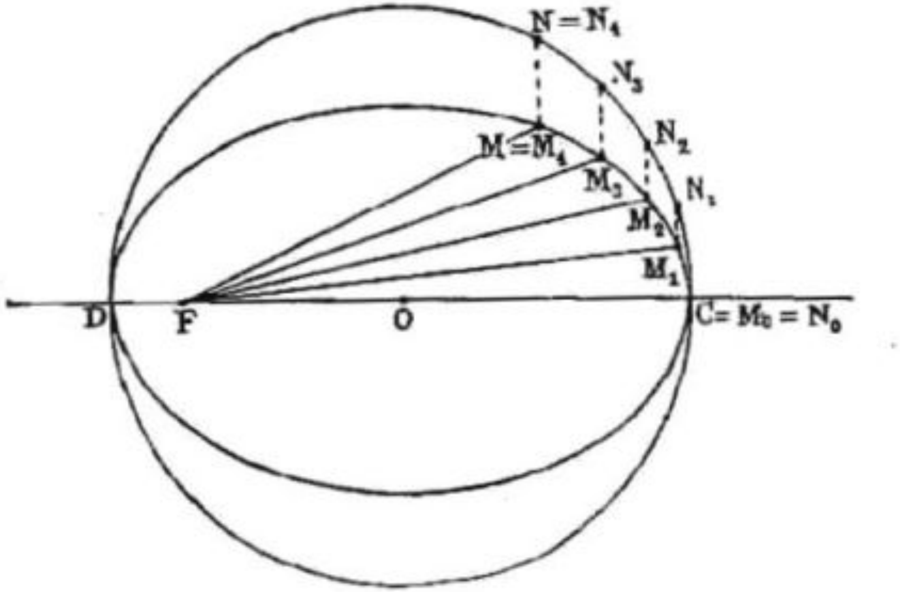
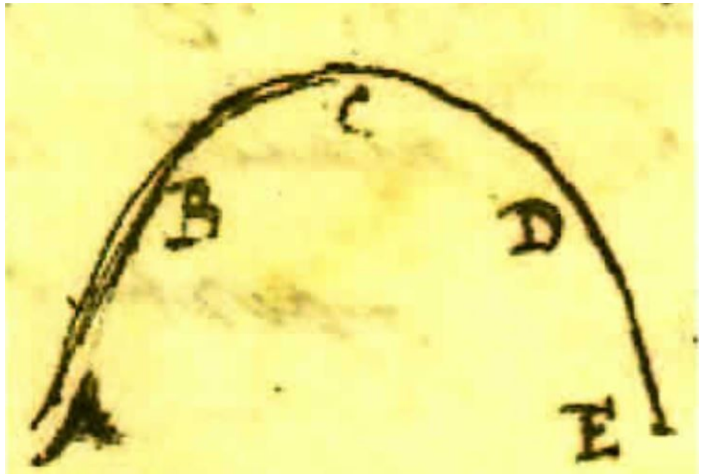


Fig. 1

Johannes Kepler: hypothesis of elliptic trajectory



Motion



Aristotele: natural and “violent” motion

Motion is linear or circular:

“Il moto locale, che è quello che noi chiamiamo ‘traslazione’ è sempre o rettilineo, o circolare, o misto di questi due: perché semplici sono questi due soli. E la ragione è che ci sono anche due sole grandezze semplici, la linea retta e quella circolare”.

Guidobaldo Dal Monte (‘500): symmetry (the same behavior going up and down), the trajectory is a curve like the form of a chain under the effect of gravity, but reflected.

Motion

Description of phenomena on the Earth using Mathematics was something new

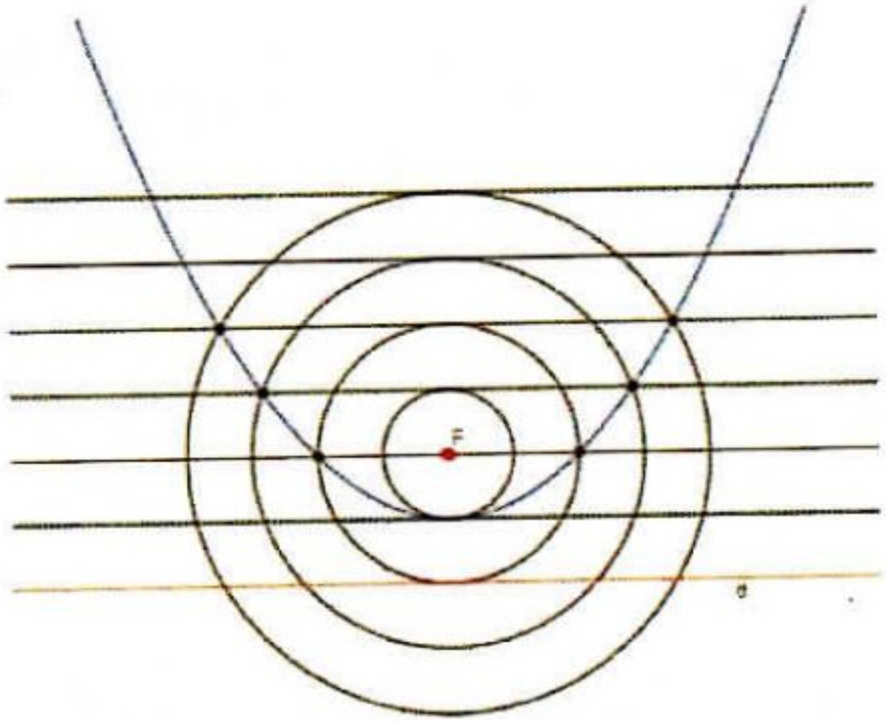
From the Sky to the Earth and back, using Mathematics

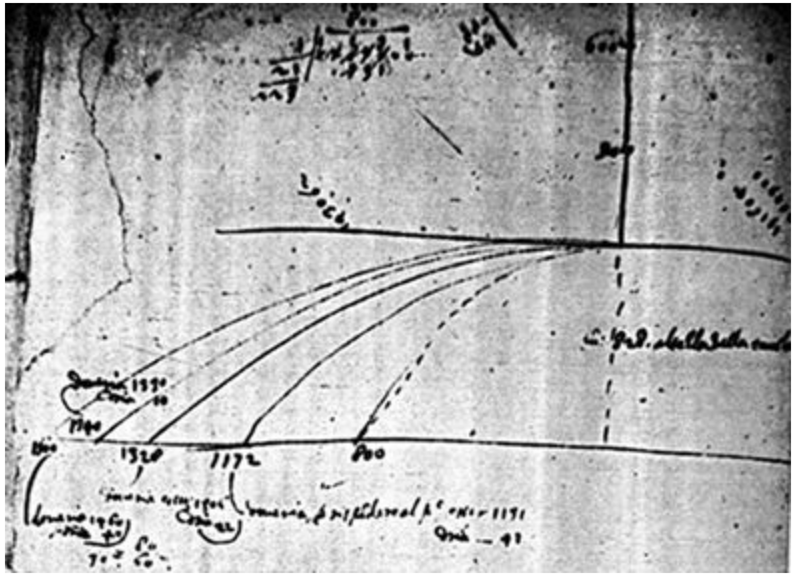
Galileo Galilei (‘600):

“La esperienza di questo moto si po’ far pigliando una palla tinta d’inchiostro, e tirandola sopra un piano di una tavola, il qual stia quasi perpendicolare all’horizonte, che se ben la palla va saltando, va però facendo li punti”

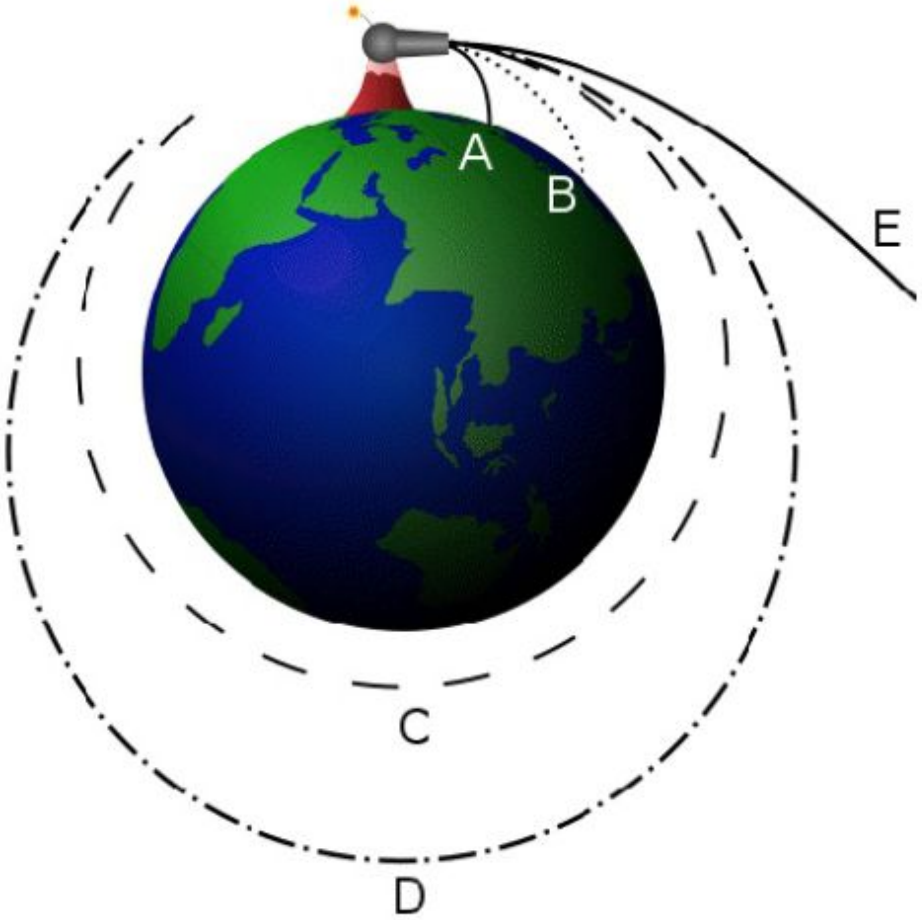
A marvellous way to draw a parabola: the trajectory (physical object) is identified with the mathematical object “curve”.

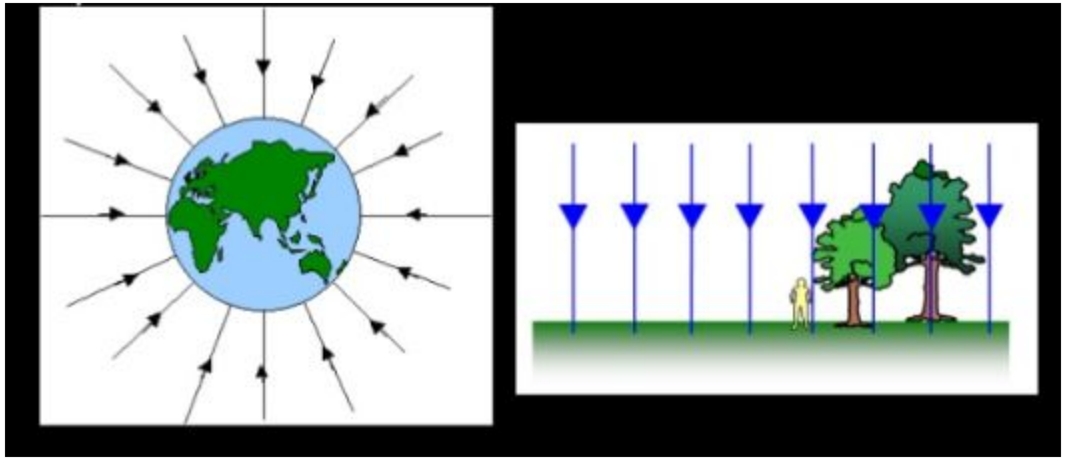
Motion

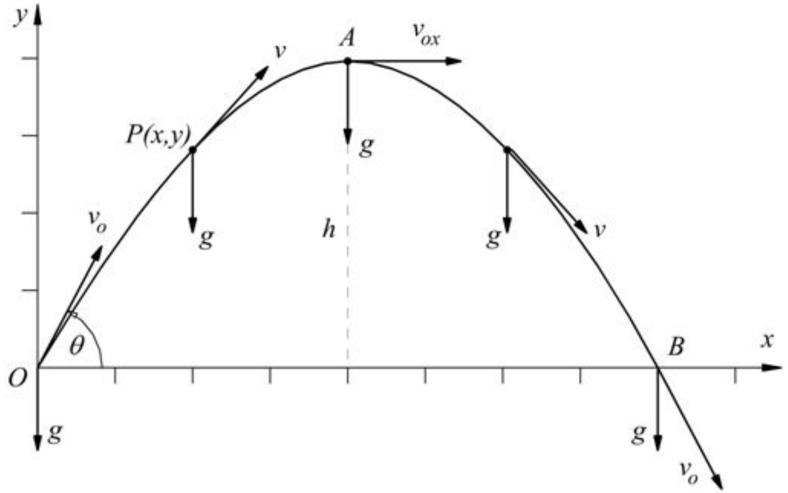




Motion: Newton's laws







New idea for O3: curricular interdisciplinary topic Parabola in Mathematics and Physics

Mathematical epistemological issues:

1. different possible formalizations and definitions of the same mathematical object
2. rigorous use of ratios in Geometry and proofs compared to Descartes' Analytic Geometry: synthetic/analytic paradigm
3. infinity: unification provided by the unifying concept of "improper point", "point at infinity"
4. mathematical machines: curves incorporating properties due to their construction

New idea for O3: curricular interdisciplinary topic Parabola in Mathematics and Physics

Physical epistemological issues:

1. qualitative/quantitative analysis of motion
2. mathematics embedded in natural phenomena/modeling approach in which mathematics is a tool
3. curve vs trajectory
4. can a trajectory be a conic section?

New idea for O3: curricular interdisciplinary topic Parabola in Mathematics and Physics

Interdisciplinary epistemological issues:

1. different possible **formalizations** and definitions of the same mathematical object have different “power” in physical problems (see Newton’s proof of a conic trajectory)
2. rigorous use of ratios in **Geometry** and proofs compared to Descartes’ **Analytic Geometry**: exact proofs vs approximation/limits/analysis
3. **infinity**: parallel lines as models for Sun light rays and for gravity field locally, unifying Earth and Sky with local modeling
4. mathematical machines: **curves** incorporating properties due to their construction vs **trajectories**

New idea for O3: curricular interdisciplinary topic
Parabola in Mathematics and Physics

5. Different roles of mathematics in the description of phenomena on the Earth and in the Sky:

- Guidobaldo Dal Monte: the trajectory of a ball due to gravity must resemble a "gravitational" phenomenon → chain curve
- Galileo: the trajectory of a ball *is* a parabola, "a really marvellous way" to draw it
- Kepler: there must be analogies between different phenomena, mathematical formalization show and induce unification; conic sections in Optic and not in trajectories (hypothesis of elliptic curve, approximation)
- Newton: derivation of trajectories from formal arguments using physical constraints and mathematical properties of curves