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IDENTITIES INTEGRATE DISCIPLINES TO ELABORATE NOVEL TEACHING APPROACHES TO INTERDISCIPLINARITY AND INNOVATIVE PRE-SERVICE TEACHER EDUCATION FOR STEM CHALLENGES

How to approach interdisciplinarity in school and in teacher education

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Outline

① Our experience with modelling and interdisciplinarity

a. The proposal of the Study and Research Paths and Fields of experience

b. Some examples of the use of SRP in interdisciplinary contexts

② Study and research paths for teacher education (SRP-TE)

③ Examples of the experiences of the SRP-TE

a. Case 1: SRP-TE about forecasting Facebook users' growth with in-service secondary school teachers

b. Case 2: SRP-TE about the cake box with pre-service primary school teachers

④ Overview to the phases and aims of 04

1. Previous modelling perspective using **Fields of experience**

- **Natural observations and school.** Halley comet (Giménez & Fortuny, 1986) Environmental growing (Alsina, Fortuny & Giménez, 1996).
- **Social environment situations**. Energy, cleaning and introduction of algebra (Gimenez, 1994) Consummation problems (Alsina, Fortuny & Giménez, 1992). Mathoscope idea (Alsina, 2001).
- **Project open work based on inquiry to understand the world.** Understanding of math processes (Sol & Giménez, 1999); The weight of suitcases (Vilatzara group 1994), (Sol, Giménez, Rosich, 2002).
- **Situations to understand Roman and Iberical constructions** (Vilatzara, 2000) Guidance towers (Vilatzara, 2002), Roman ruins (Sala, Font, Giménez, & Barquero, 2017); Treasure in Roman House (Sala & Font, 2019).
- **Understanding of some art movements.** Modernism and isometries (Vilatzara, 2001); Oldemburg & Dali (Badillo, Giménez, & Vanegas, 2009).

1. Previous approach to modelling and inquiry through the SRP

The Study and Research Paths (SRP) in the framework of the ATD

- **SRP** appear in front of the necessity of looking for moving towards the new pedagogical paradigm of questioning the world (Chevallard, 2015)
- **SRP** appear as an appropriate didactic device for teaching of mathematical modelling, inquiry where mathematics is conceived as a modelling tool
- The research problem at the stake is the **ecology** and **sustainability of SRP** and of modelling at different school institutions

Research questions: Under which conditions and constraints can SRP be integrated in regular courses of mathematics at different school levels? What conditions can favour and which constraints hinder the long-term sustainability of SRP?

The starting point of an SRP is an initial question, the **generating question** Q_0, which generates the inquiry process and which can open many derived questions

The main aim of an SRP is that the inquiry community (students and teachers) elaborate their **answers** to the **derived questions** in order to build up a final answer

STUDY AND RESEARCH PATHS (SRP)

The SRP can be synthesised as an **arborescence of questions and answers** that delimits to possible paths to follow. Most of the times, involving **more than one discipline**

Importance of the **collaborative inquiry work** to look for and build up answer/s. New roles and responsibilities of teacher/s and students

The development of an SRP promotes that **different kinds of knowledge** (in particular mathematical knowledge) becomes **articulated, connected**... with the aim to provide answers to the proposed questions

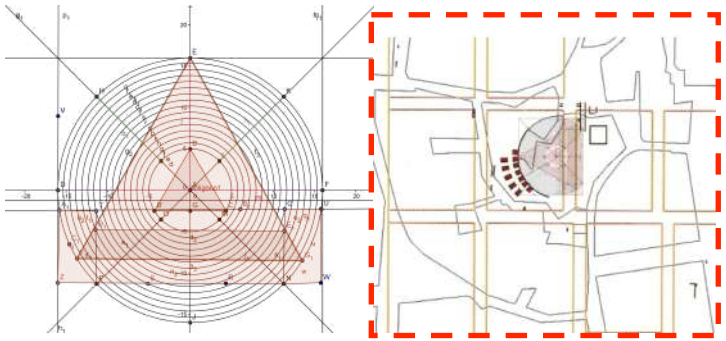
1. Previous approach to modelling and inquiry through the SRP

Level	Interdisciplinary Study and Research Paths (SRP)
Secondary - University	**Biology-Mathematics** ▪ Population Dynamics (Barquero, 2009)  **Economy-Mathematics** ▪ Sales forecast (Serrano, 2011)  ▪ Human Resources management (Serrano, 2011) ▪ Evolution of network users' dynamics (Barquero & Serrano, 2013) ▪ A urban bike sharing system (Barquero & Serrano 2014) ▪ Forecasting of Facebook users (Barquero, Monreal, Ruíz,Serrano, 2018) **Archeology-History-Mathematics** ▪ What are these ruins hiding? Investigating the Roman ruins of Baetulo" (Sala, Font, Giménez & Barquero, 2015)  **Engineering context** ▪ How to make a bike part? (EUSS, Florensa 2016-18) ▪ How to make slatted bad base? A kart chassis? (EUSS, Bartolomé 2016-18)

1. Previous approach to modelling and inquiry through the SRP

Level	Study and Research Paths for Teacher Education
Teacher Education	**Pre-service and in-service teacher training at P, S and U Education** ▪ Evolution of systems of numeration? (UCM, Sierra 2006) ▪ How to teach algebra at secondary level? (UAM, Ruiz-Olarría & Sierra 2015) ▪ How to teach real numbers at secondary level? (Licera, 2017) ▪ How to teach proportionality at secondary level? (UAM, Ruiz-Olarría 2015) ▪ How to teach modelling at Secondary school education? (CICATA, Barquero, Bosch & Romo 2014-2019) ▪ How to teach modelling in Primary school level? (UB, Barquero,2014-2019) ▪ How to predict a dengue epidemic? (EUSS, Lucas & Florensa, 2016)

1. Previous approach to modelling and inquiry through the SRP



What are these ruins hiding? Investigating the Roman ruins of Baetulo"

Designed by researchers with the collaboration of the history and mathematics teachers,

Experienced in a Secondary school in Badalona (Catalonia, Spain) in June 2015 and 2016 with 30 students (12-13 year old). 8h per week during two weeks

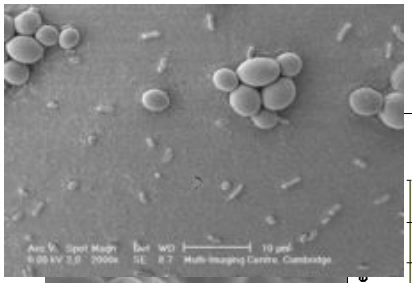
Collective work where the students with the two teachers and researchers elaborate during two weeks with 8 h/week a collective answer → URL blog: "*What are these ruins hiding? Investigating the Roman ruins of Baetulo*".

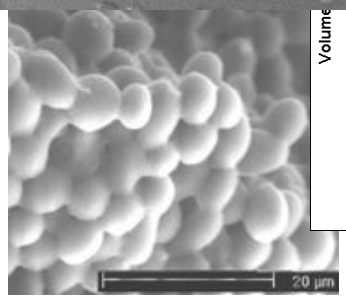
http://ruinesdebaetulo.blogspot.com/

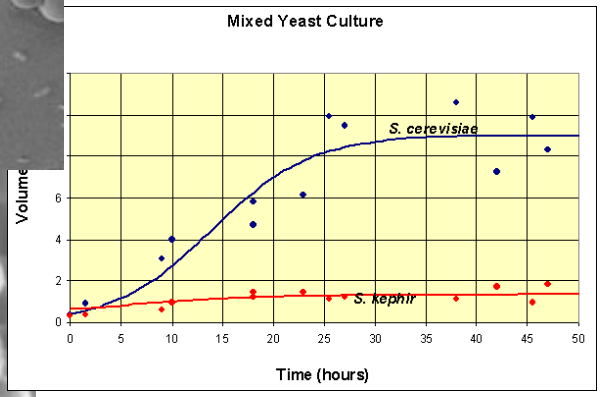
Sala, Font, Giménez & Barquero (2017)

1. Previous approach to modelling and inquiry through the SRP

Year	Population
1937	8
1938	26
1939	85
1940	274
1941	800
1942	1800







Initial question: Given the size of population over some time period,

- Can we predict its size after *n* periods? Is it always possible to predict the long-term behaviour of the population size?
- What sort of assumptions on the population and its surroundings should be made?
- How can one create forecasts and test them?

Barquero, Bosch & Gascón (2011, 2013)

1. Previous approach to modelling and inquiry through the SRP

OUR GENERATIVE QUESTION – Q_0

Given the size of population over some time period,

- Can we predict its size after *n* periods? Is it always possible to predict the long-term behaviour of the population size?
- What sort of assumptions on the population and its surroundings should be made?
- How can one create forecasts and test them?

Year	Population size
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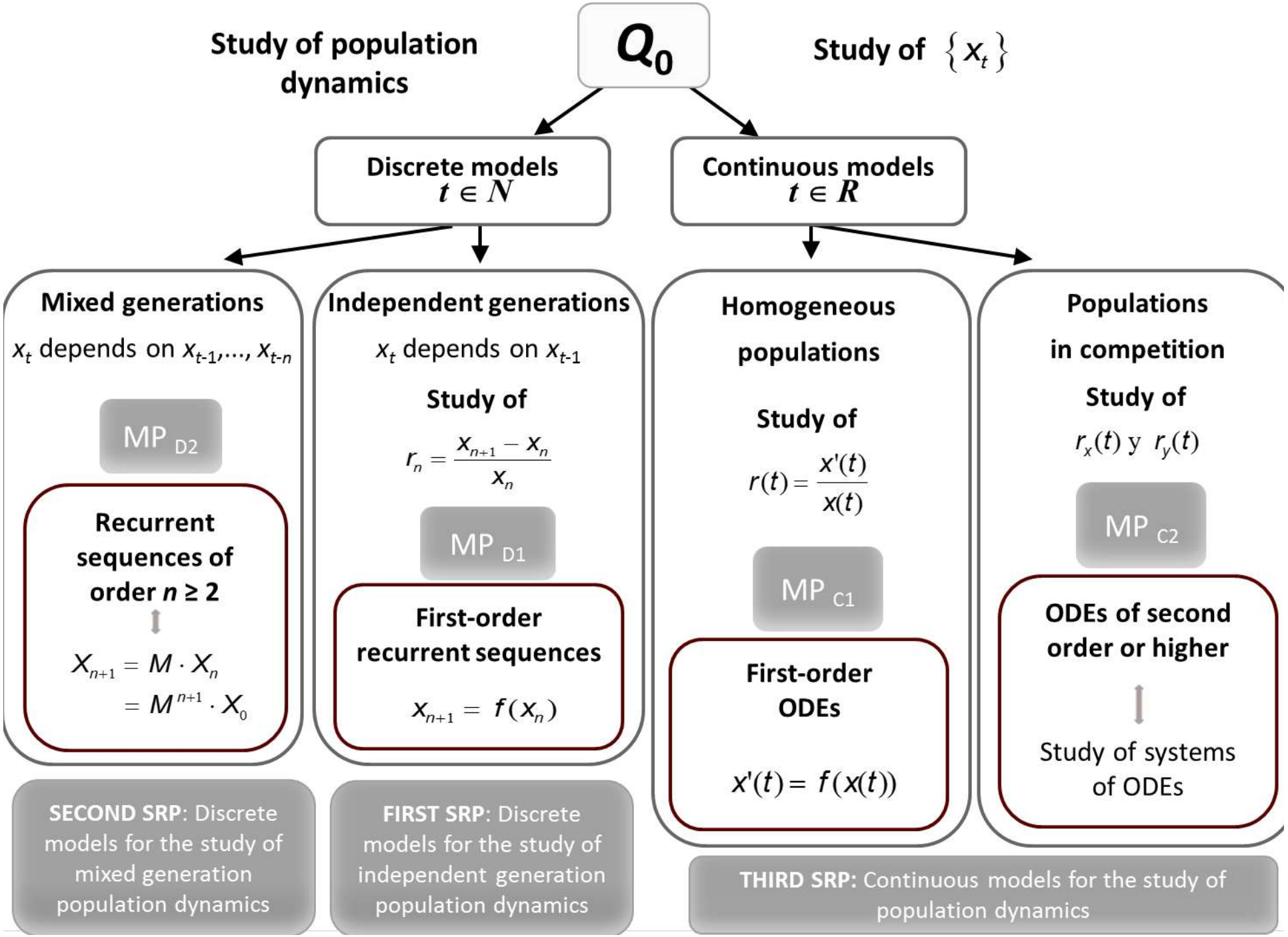
SOME CONDITIONS TO MathModelling

- The study of the Q_0 covers most of the curricula contents of 1st year math course.
- Tree structure of the SRP: many possible paths depending on the models considered.
- Transfer of some responsibilities to the students: work in groups, planning, formulating questions, checking hypoth., weekly reports with temporary answers, 'secretary of the week', etc.

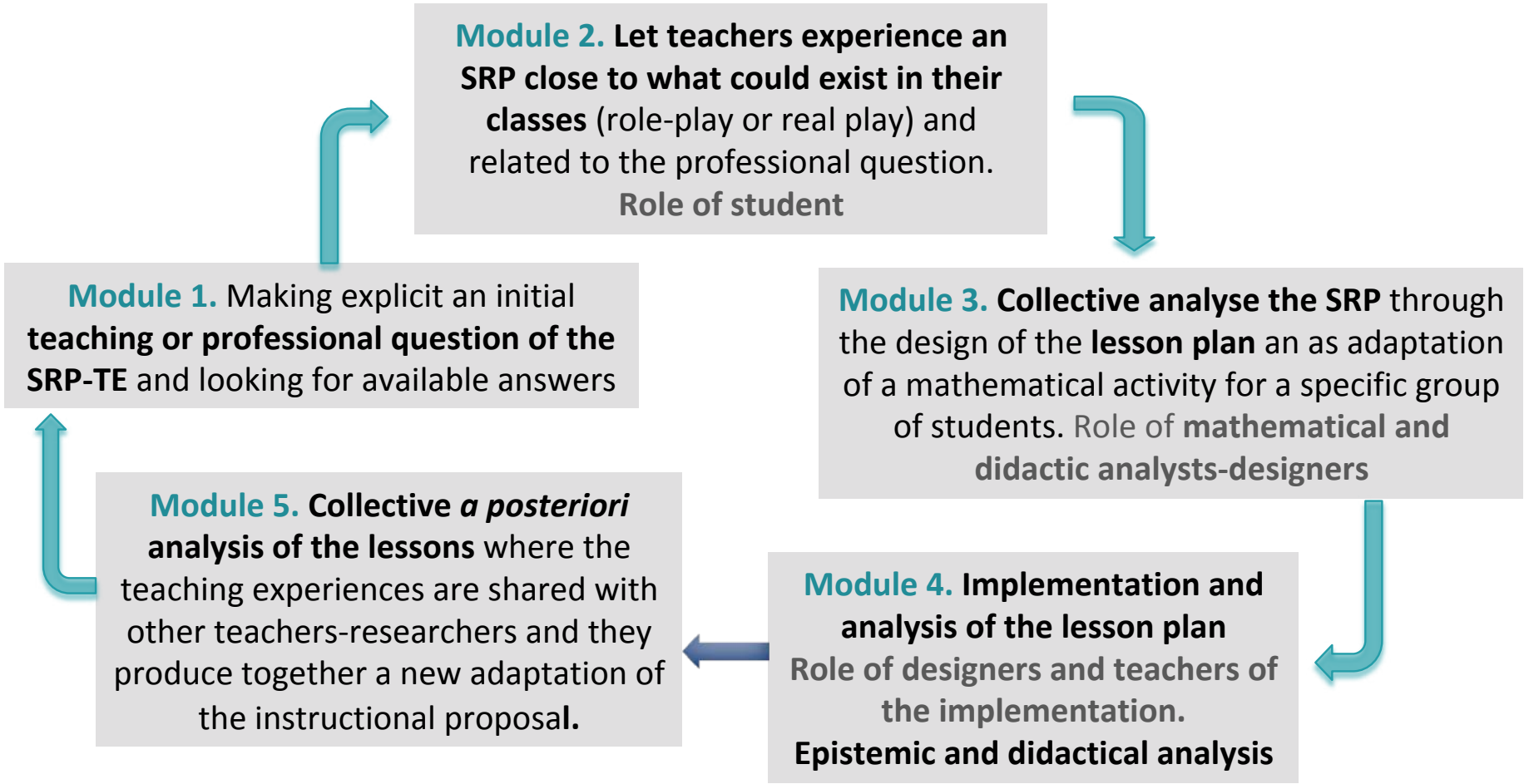
SOME OF THE CONSTRAINTS TO THE INTEGRATION OF MathModelling

- Necessity to break the rigidity of the classical structure 'lectures-problem sessions-exams' and integrate SRP in it.
- New devices to help the running of the new didactic contract.
- Some responsibilities are very difficult to transfer to the students.


Study of population dynamics
Q_0
Study of $\{x_t\}$
Discrete models
$t \in N$
Continuous models
$t \in R$
Mixed generations
x_t depends on x_{t-1},..., x_{t-n}
MP D2
Recurrent sequences of order $n \geq 2$
$X_{n+1} = M \cdot X_n$
$= M^{n+1} \cdot X_0$
Independent generations
x_t depends on x_{t-1}
Study of
$r_n = \frac{x_{n+1} - x_n}{x_n}$
MP D1
First-order recurrent sequences
$x_{n+1} = f(x_n)$
Homogeneous populations
Study of
$r(t) = \frac{x'(t)}{x(t)}$
MP C1
First-order ODEs
$x'(t) = f(x(t))$
Populations in competition
Study of
$r_x(t)$ y $r_y(t)$
MP C2
ODEs of second order or higher
Study of systems of ODEs
SECOND SRP: Discrete models for the study of mixed generation population dynamics
FIRST SRP: Discrete models for the study of independent generation population dynamics
THIRD SRP: Continuous models for the study of population dynamics

2. Study and research paths for teacher education (SRP-TE)

Modules structure of the SRP-TE (Ruiz-Olarría, 2015; Barquero, Bosch & Romo, 2018)



2. Study and research paths for teacher education (SRP-TE)

PRELIMINARY RESULTS AFTER SOME EXPERIENCES WITH SRP-TE

Previous research shows how **SRP-TE** seems to be useful tool to:

- Enable teachers' **epistemological and didactic questioning** of dominant paradigm (What does it exist? What can it be done differently? What new epistemological and didactic tools are needed?)
- **Transpose research tools** to **teaching practice** tools for the epistemological and didactic analysis
- Help teachers progress in the critical issue of **identifying institutional constraints** hindering a change of paradigm in current school systems to include modelling, interdisciplinarity, …

Adding the experience of curricular changes developed and analysed (2014-2017)

- The example of D'Ambrósio **curricular perspective** (literacy, matheracy, technoracy) used for justifying interdisciplinary units in Andorra as a way of maintaining coherence .
- Examples of **Epistemic** and **Didactical analysis** according suitability criteria for task design (Giménez, Font, & Vanegas, 2013) facing complexity (Giménez, 2019).
- Examples of teacher producing teaching units. The importance of the **sociocultural context**. Touristic knowledge; Heating at Andorra's homes ; Building a ski station ... (Giménez & Zabala, 2019)

3. Examples of the modules and activities of the SRP-TE

POSGRADO en línea de MATEMÁTICA EDUCATIVA

SRP-TE about forecasting Facebook users' growth with in-service secondary school teachers

Conditions for its implementation

- Online and in-distance course in Maths Education for in-service Secondary school teacher organized by CICATA (Mexico)
- 15-18 participants from Latin-America (Mexico, Uruguay, Paraguay, Argentina, …) and teams of 5 (to 8) instructors from Mexico and Spain
- The course runs over 4-5 weeks, with an expected participants' work of 80 h

Procesos de Institucionalización de la Matemática Escolar

Profesor: Berta Barquero
Profesor: Marianna Bosch Casabò
Profesor: Apolo Castañeda
Profesor: Josep Gascón
Profesor: Juan Gabriel Molina Zavaleta
Profesor: Avenilde Romo Vázquez
Profesor: Alicia Ruíz
Profesor: Mario Sánchez Aguilar

October 2013
October 2014

Naturaleza del pensamiento matemático

Berta Barquero
Marianna Bosch
Avenilde Romo
Mario Sánchez
Lidia Serrano

October 2015
March 2017
March 2018
March 2019

facebook

Dezigual

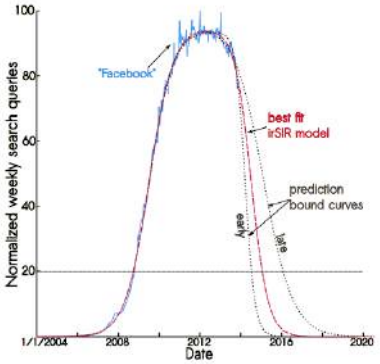
3. Examples of the modules and activities of the SRP-TE

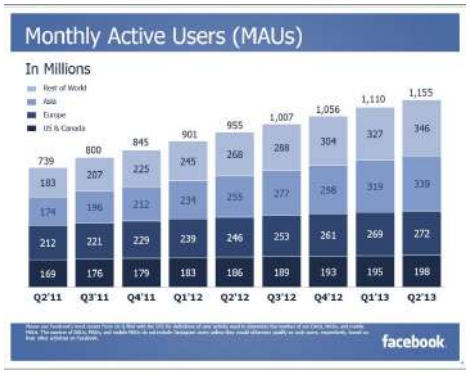
Module 2: Experience the SRP about Facebook users' forecasts and analyse it

Participants' role: Students of Secondary education

Educators' role: Mathematical guide of the team she/he supervises

Aim: Work in the activity to experience and to report on the activity. At the end, they start working on a first epistemological analysis of the activity.





Datos usuarios FACEBOOK a nivel mundial	
Q3-2011	800
Q4-2011	845
Q1-2012	901
Q2-2012	955
Q3-2012	1007
Q4-2012	1056
Q1-2013	1110
Q2-2013	1155
Q3-2013	1189
Q4-2013	1228
Q1-2014	1276
Q2-2014	1317

Datos usuarios FACEBOOK en US & Canada	
Q3-2011	176
Q4-2011	179
Q1-2012	183
Q2-2012	186
Q3-2012	189
Q4-2012	193
Q1-2013	195
Q2-2013	198
Q3-2013	199
Q4-2013	201
Q1-2014	202
Q2-2014	204

3. Examples of the modules and activities of the SRP-TE

Module 2: Experience the SRP about Facebook users' forecasts and analyse it

Phase 1: Teachers-students, working in groups of 3-4 persons and assuming the role of students, carry out the assignment to MC2 – Mathematical Solutions (as they received an external request of the report).

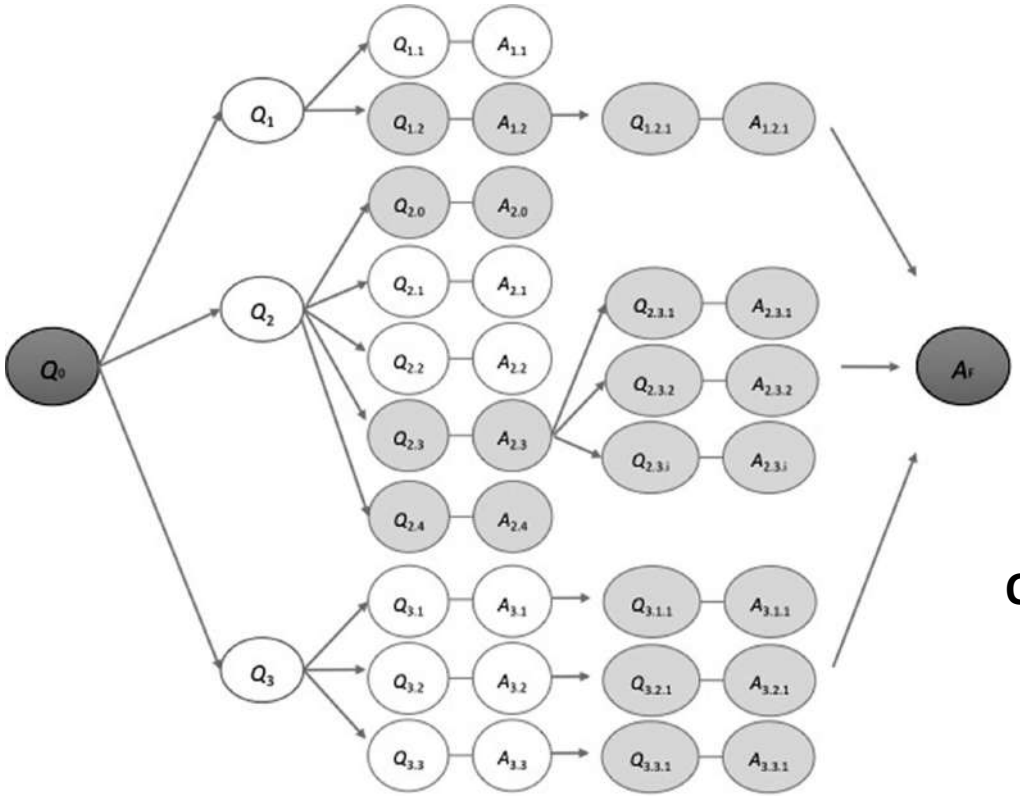
Phase 2: A new forum was created to interact with other working teams proposals and analyse the other participants' reports and contrast with your own. At the end they had to deliver a final common report to MC2.

Phase 2: Teachers-students were asked to analyse and describe the mathematical process followed and the difficulties found.

3. Examples of the modules and activities of the SRP-TE

Module 2: Experience the SRP about Facebook users' forecasts and analyse it

- Concerning the analysis of the modelling activity, the main tool provided to the participants took the form of a *questions-answers map* (Winsløw et al. 2013).



Questions–answers map from Activity 4 [Team A]

Barquero, Bosch & Romo (2018)

3. Examples of the modules and activities of the SRP-TE

Module 3: Lesson plan design

Participants' role: teacher going for a trip

Aim: Design of a (easy readable) lesson plan on the Facebook users' evolution

Phase 1: Elaboration of a **first individual version of the lesson plan** for the teacher who will be in charge of the implementation

Phase 2: Analysis of the team participants' lesson plans

Phase 3: Propose a **common lesson plan** agreed by the working team

In most of the previous implementation, there appear many difficulties in comparing and evaluating the lessons plans. Many questions appear:

- What are the necessary elements to include in the lesson plan? Which of them have been explicitly included? Which ones remain in the shadow?
- What institutional constraints are assumed for the teaching of modelling, of interdisciplinarity?

3. Examples of the modules and activities of the SRP-TE

Module 4: Implement the adaptation of a SRP under real school conditions

Participants' role: teacher **Aim:** Implement the initial phase of the activity "Comparing reality against forecasts: the case of FB users" with a group of students and analyse it.

Phase 1: Planning and a priori analysis

Phase 2: Experimentation

Phase 3: A posteriori analysis

Phase 4: Experimentation report

Experiencing institutional constraints from the local implementation

3. Examples of the modules and activities of the SRP-TE

Module 5: Joint analysis and final revision of the lesson plan

Participants' role: teacher (mathematical and didactic) analyst

Phase 1: Sharing of the reports of the experimentations

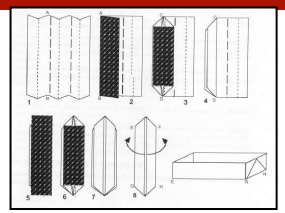
Phase 2: Reviewing of the lesson plan guide

Phase 3: Preparation of a joint final version of the lesson plan guide

Teachers-students have a lot of previous material to analyse and organise (reports of their experience with the activity, versions of the lesson plan, reports on the secondary school implementation, etc.)

+ A more explicit guide for the epistemological and didactic analysis of the lesson plan is shared between teacher-students and educators

3. Examples of the modules and activities of the SRP-TE



SRP-TE about the cake box with pre-service primary school teachers

Conditions for its implementation

- The course on *Didactics of Mathematics II* (6 ECTS) runs over the whole first semester for **pre-service Primary school teachers**
- The implementation of the SRP-TE is carried out during one-month and a half (4 hours/week), with two groups of about 60 participants from 2012-13 to 2018-19 and 1 instructor

Important differences between Case 1 and Case 2:

- In-service teachers education offers more convenient and varied conditions for the implementations and the analysis of conditions and obstacles or constraints for implementing modelling in school
- Different activities are designed to cover some of the modules of the SRP-TE

3. Examples of the modules and activities of the SRP-TE

Module 1: Making explicit an initial teaching or professional question of the SRP-TE and looking for available answers

Aim: Presenting the initial professional question/s to pre-service teachers and ask them to analyse the available answers in different media (curriculum, textbooks, innovation material, etc.)

Initial professional question:

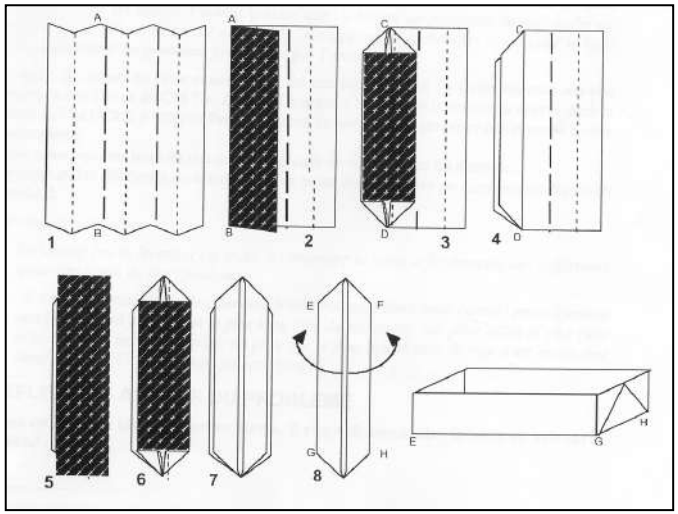
- How can we **teach mathematical modelling** in Primary school?
- How to **analyse, develop** and **integrate** a learning process related to **mathematical modelling** in our teaching practice?
- How to **institutionally sustain long-term learning processes** based on modelling?

3. Examples of the modules and activities of the SRP-TE

Module 2: Experience the modelling activity of the cake box and analysing it

Participants' role: students at Primary school level. In the classroom there were some students with the role of observers and reporter.

Aim: Work and report on the activity to be able to perform an epistemological analysis afterwards



Q_0 : How can we build a box to help a baker to pack her/his cakes? What kind of relation there is between the sizes or dimensions of the sheet and the dimensions of the resulting box?

Chappaz & Michon (2003)

Ruiz-Higueras (2008)

3. Examples of the modules and activities of the SRP-TE

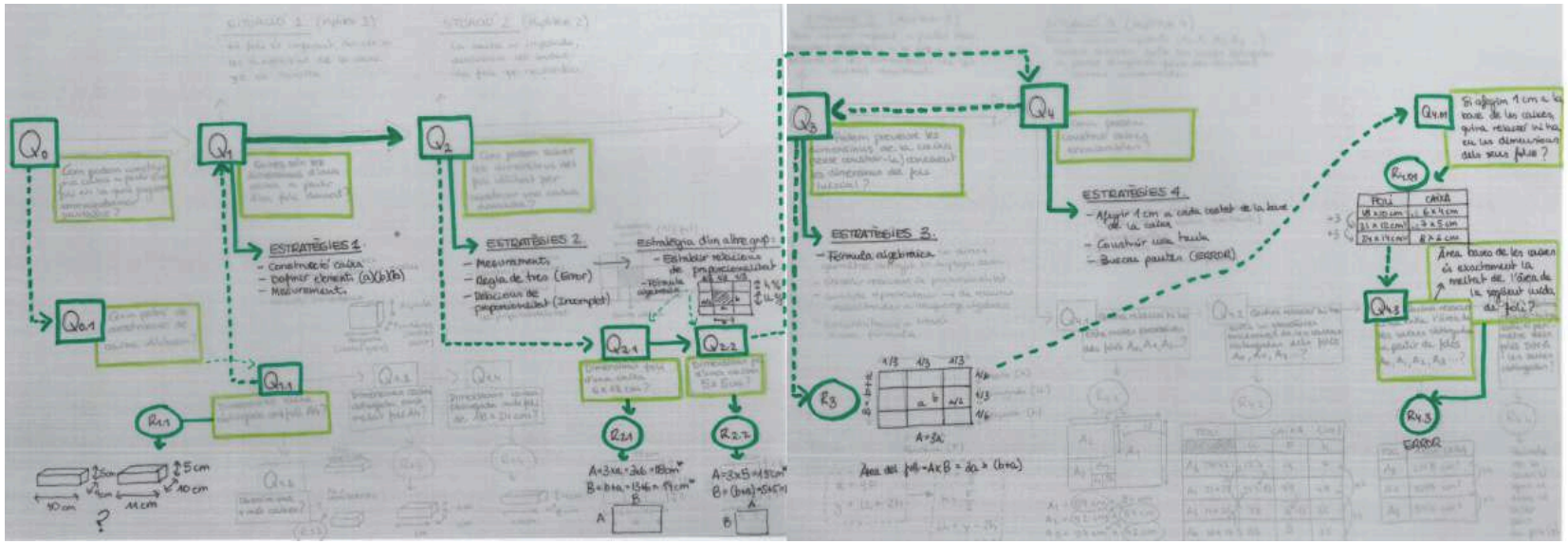
Module 3: Analyse the modelling activity as M-D analysts

Participants' role: mathematical and didactic analyst

Data collected: session reports and class debates

Phase 1: Build up a questions-answers map (Q-A maps) from the whole mathematical process that students and the teacher have experienced.

Phase 2: Use of this Q-A maps to: **(1)** Analyse other groups productions and **(2)** Define new elements to evaluate their work



3. Examples of the modules and activities of the SRP-TE

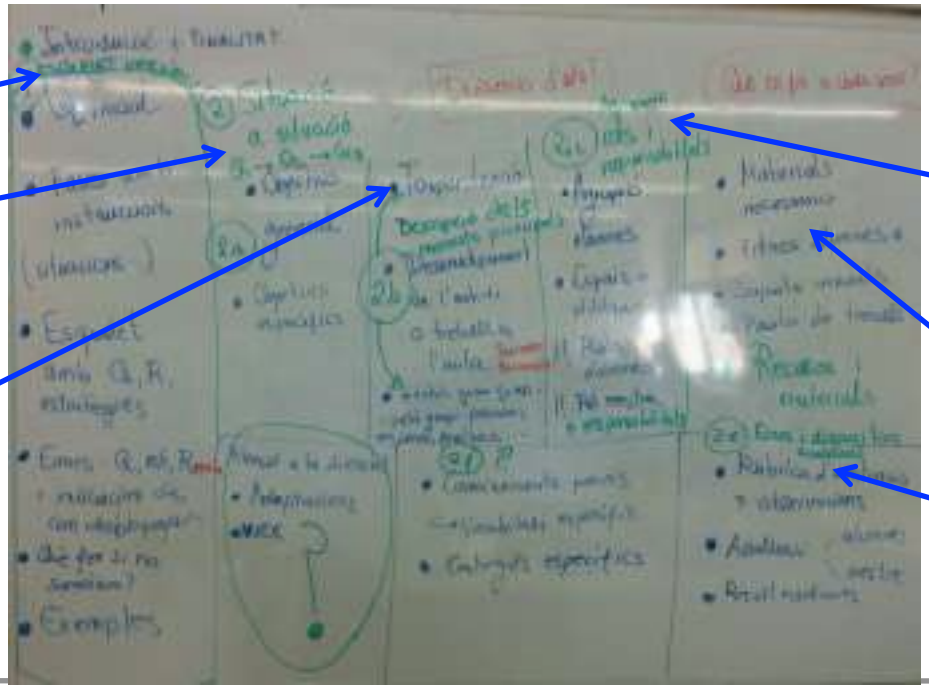
Module 4: Lesson plan design

Participants' role: teacher going for a trip

Aim: Design of a (easy readable) lesson plan on the Cake Box Activity

Phase 1: Discuss on the necessary elements to include in the lesson plan

Phase 2: Design the lesson plans accordingly to the activity they select



Q-A maps

Generals aims and specific aims of each 'situation'

Time distribution

Roles and responsibilities

Resources and its use in each step

Instruments and devices for assessment

3. Examples of the modules and activities of the SRP-TE

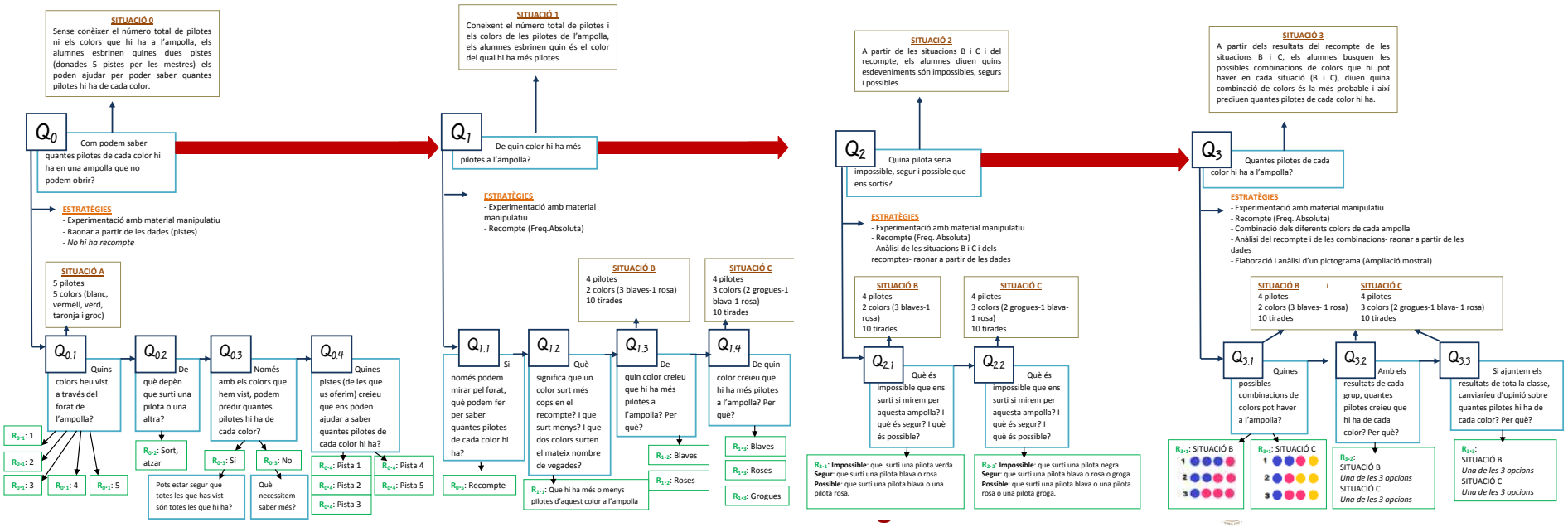
Module 5: Implement and analyse a SRP

Participants' role: Mathematical and didactic designer (but also Primary school teachers, analysts,…)

Aims and tasks: Some activities are propose by the lecturer of the course.

- Groups had one month to: (1) Develop their a priori designs; (2) Its experimentation and (3) A posteriori analysis.

A2-Recorregut d'Estudi i Investigació per a **Cicle Inicial**



3. Examples of the modules and activities of the SRP-TE

Dealing with institutional constraints (Barquero, Bosch & Romo, 2018)

- The structure and guidance of the SRP-TE's modules and activities help to make important constraints emerge and be visible for teachers:
  - the need of new tool to analyse mathematical activity

    → The role of the Q-A maps
  - experiencing institutional constraints from the local implementation of a modelling activity and comparing them

    → Too much better achieved with in-service teacher, but we are trying different alternatives with pre-service teachers
- This **role-play** appeared to be a successful strategy to make different institutional constraints emerge

4. Assessment ideas (under construction)

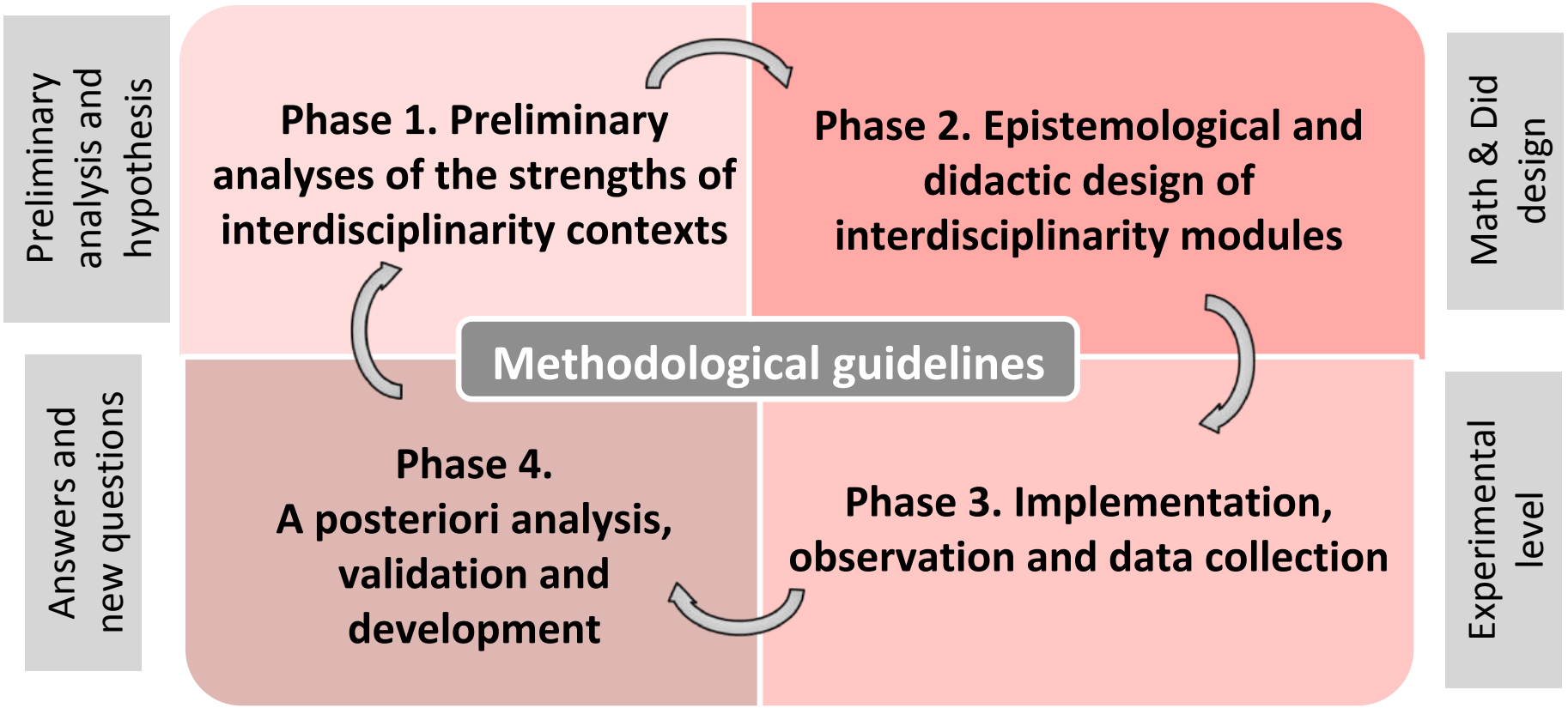
- Tools for teachers for reflecting on the conditions and constraints for interdisciplinarity and tools for experiencing non-conventional activities → Modules 1 &2
- Tools for **Epistemological and Didactic analysis** → Modules 3 & 4
- Tools for **Collaborative analysis and reflection** about experiences under real Secondary school conditions → Module 5
- Tools for all group reflection (summer schools)

4. Overview to the phases and aims of 04

- O4 consists in providing **methodological guidelines** along the project to support teams with:
  - The design and redesign of teaching modules on curricular interdisciplinary STEM projects,
  - The implementation and analysis with pre-service secondary school teachers.
- O4 provides an **analytical viewpoint** to the work developed in O2-O3
- O4 will also guide the development of O5:
  - Selection of themes, modules, case studies, etc.
  - Redesign of the teacher education modules under the common lines,
  - …

4. Overview to the phases and aims of 04

- The different stages of the project could correspond to the different stages of a global Didactical Engineering Methodology (Artigue, 1990, 2014):



4. Overview to the phases and aims of 04