ALMA MATER STUDIORUM · UNIVERSITY OF BOLOGNA

School of Science
Department of Physics and Astronomy
Master Degree in Physics

Interdisciplinarity in Science Education in France: Analysis of Approaches and their Application to the Vibrating String Case Study

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to my grandmother Mara and
all the victims of COVID-19

à ma grand-mère Mara et
à toutes les victimes du COVID-19

a mia nonna Mara e
a tutte le vittime del COVID-19

svoji noni Mari in
vsem *žrtvam COVID-19*

» Le savant doit ordonner ; on fait la science avec des faits comme une maison avec des pierres ; mais une accumulation de faits n'est pas plus une science qu'un tas de pierres n'est une maison. »
Henri Poincaré, La Science et l'hypothèse (1908)

"The Scientist must set in order. Science is built up with facts, as a house is with stones. But a collection of facts is no more a science than a heap of stones is a house."
Henri Poincaré, Science and Hypothesis (1908)

ABSTRACT

This thesis explores the relations among STEM-disciplines in France, with a special focus on the interaction between mathematics and physics. It presents the French high school system that recently underwent a governmental reform called *BAC 2021*.

The work examines different approaches to interdisciplinarity in a context of science education, illustrating in particular the Julie Thompson Klein's taxonomy. Six French researchers working with interdisciplinary were interviewed. The analysis with the Klein's theoretical lenses of the definitions of interdisciplinarity collected in the interviews points out some practical criteria to compare and distinguish among methodological, theoretical, instrumental and critical interdisciplinarity.

The thesis claims that the collaboration of people having different backgrounds is enriching because experts of various domains know different epistemologies. Some critical opinion towards the new reform *BAC 2021* are expressed, especially about interdisciplinarity. Despite the introduction of a new subject of science education, it has emerged the fact that in the French education system the STEM topics seem still not to be present enough. Introducing the STEM-related problems in the teaching should be supported by an adequate university teaching research.

In the end, a historical case study of the vibrating string controversy is presented. Starting with a modern treatment of the problem, the thesis problematizes the fact that often it is presented as a simple exercise, whilst, it represented a very emblematic case of interdisciplinarity. Through the historical contextualisation with the biographies of the protagonists and the analysis of the debate it highlights the analysis of the interdisciplinarity between mathematics and physics in a theme proposed in the second-grade syllabus of the school subject "science education". The last part of the chapter develops the narrative of the debate and suggests some didactic implementations.

KEYWORDS

INTERDISCIPLINARITY, SCIENCE EDUCATION, FRENCH SCHOOL SYSTEM, THOMPSON KLEIN'S TAXONOMY, VIBRATING STRING

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INTRODUCTION

The idea for this thesis was born in November 2019, after the Kick-off meeting of the European Erasmus+ project *IDENTITIES*, *Integrate Disciplines to Elaborate Novel Teaching approaches to InTerdisciplinarity and Innovate pre-service teacher Education for STEM challenges* (for more information visit the website: https://identitiesproject.eu). The project is coordinated by the University of Bologna and involves partners from four different countries: France, Greece, Italy and Spain. The overarching goals are i) to designs novel teaching approaches on interdisciplinarity in science and mathematics to innovate pre-service teacher education for contemporary challenges and ii) to explore emergent advanced STEM themes (e.g. climate change, quantum technologies, artificial intelligence, nano-technologies) and curricular interdisciplinary topics (e.g. cryptography, parabola, non-Euclidean geometry and gravitation) as contexts to explore inter-multi-trans-disciplinary forms of knowledge organization and to design classroom activities and new models of co-teaching.

During the Kick-off meeting, the debates and working groups brought out different sensibilities and points of view on the issue of interdisciplinarity. The constructive dialogue, especially between the Italian and French group, was a stimulus to investigate further the question.

The Erasmus exchange experience in the Mathematics Department at the University of Cergy and Pontoise in France (*CY Cergy Paris Université*) was a great opportunity for an Italian master's student in Didactic and History of Physics to study as an external observer the interdisciplinary concepts and activities in a French-speaking environment.

This thesis aims to explore the relations among STEM-disciplines in France, with a special focus on the interaction between mathematics and physics. The goal is to analyse different approaches to interdisciplinarity in a context of science education and trough a case study to present a possible activity where these can emerge. The contents are developed in four main body chapters.

The first chapter presents the school system in France and the new governmental reform of the high school final exam *Baccalauréat 2021*. In order to have a better understanding of the contents studied in French high schools, the syllabuses of three scientific subjects (mathematics, physics/chemistry and science education) are analysed. This chapter has two different purposes. It gives an idea to a reader who does not know the French school system, on how it works, and it proposes an institutional analysis, although not in-depth, on how the French Government imagines science education in future.

The objective of the second chapter is to build a theoretical framework in interdisciplinarity. Some papers on interdisciplinarity were selected and examined. An attempt has been made to privilege texts that dealt with interdisciplinarity in a scientific or science education field, that were written in French, or that were translated into it, and quoted by French researchers. The second chapter presents this analysis. Julie Thompson Klein's taxonomy seemed to be the most suitable tool for classifying the interactions between the various disciplinary fields.

The third chapter presents the point of view of French researchers who worked with or are still working with interdisciplinary issues in science education or teacher training and an attempt to verify the applicability of the theory. For this purpose, an interview protocol was design and six interviews were made with the researchers. These six interviews cannot represent a statistically significant sample but they still allow to test the pulse to the university research situation. The objective of this chapter is to test the theoretical framework, proposed in the second, and to carry out an analysis of the data obtained.

The last chapter contains the case study taken from the history of science. The vibrating string controversy seemed to be a suitable case for this type of analysis. The controversy is mentioned in the syllabus of the French high school subject "scientific education" and could be an interesting content both for an Italian or a French science teacher. In the first part of the chapter, the modern treatment of the vibrating string problem is presented. It follows then the historical contextualisation with the biographies of the protagonists and the analysis of the controversy. The last part of the chapter develops the narrative of the debate and suggests some didactic implementations. The comparison between the French and the Italian education environment does not pretend to pronounce what the best school system is, but it would like, in a constructive spirit, to underline the strengths and weaknesses of each. The analysis of the problem of a vibrating string and the proposed activity is a personal didactic reworking of the research contents of the thesis.

In the final discussion and conclusion, the main results of this thesis are presented and commented on. Some considerations for future work are also given to stress the meaningfulness to include interdisciplinarity activities in science education. The conclusions highlight, for example, that the students could appreciate more the structure and the epistemologies of every single discipline and they would be forced to focus their attention on disciplinary forms of reasoning.

At the end of this introduction, I would like to clarify the point of the general situation during the writing of my thesis. This work was written mainly in France in a "special" period of the pandemic of Covid-19 when several health restrictions were introduced. From March to June 2020, both French and Italian universities did not do face-to-face teaching, but the so-called distance teaching was introduced. Due to the rules of the general confinement, the libraries and other university places were closed. This has also affected my research work because it was no longer possible to meet researchers in person or to consult the original books and papers as previously planned. All bibliographic material was found online.

1. The School System in France

The French school system is similar to the Italian one. The compulsory educational period is for pupils in the ages of 6 to 16. There is a pre-schooling institution, a kindergarten (*école maternelle*) for children in the age range from 2 to 6 years and a primary school (*école élémentaire*) for pupils in the ages of 6 to 11.

The secondary education system consists of two further educational stages: the compulsory middle school (*collège*) between the ages of 11 and 15, which ends with a national exam: *diplôme national du brevet* (*DNB*), and the second educational stage when the students can choose three different options. Students from the age of 15 to 18 can attend the high school (*lycée*) with two different paths: the general and the technical one, which have the first year (*classe de séconde*) in common and the following two grades (*classe première* and *classe de terminale*) separated, or a professional school.

After a three-year school path, the pupils have to pass a national exam called *baccalauréat* (*BAC*), which differs for the three different paths: the general high school (*lycée général)* ends with a *baccalauréat général*, the technical high school with a *baccalauréat technologique,* whereas the professional high school with *baccalauréat professionnel.* There is another two years option ending the educational path in a professional high school after the second year with an exam called ce*rtificat d'aptitude professionnelle* (*CAP*).

The *baccalauréat* or simply *BAC* is the basic requirement to continue the studies in higher education. The three levels are counted from the BAC: the bachelor's degree (*licence* or *licence professionnelle*) is equivalent to *BAC+3*, the master's (*master*) to *BAC+5* and the PhD (*doctorat*) to *BAC+8*.

1.1. The Government's Reform of *Baccalauréat: Bac 2021*

In the last years, the French Government and the Minister for the Public Education Jean-Michel Blanquer decided to reform the national final exam of the French high schools and as a consequence also the educational path. The Government found the final exam to be too complex and it needed to be simplified, therefore it was asked to a commission of experts, guided by Pierre Mathiot, to study the situation and to write a report with some advises. The report (Mathiot, 2018) was delivered to the Minister for Public Education on the 24th of January 2018.

The reform called *Baccalauréat 2021* was approved in summer 2018 and started to be implemented during the school year 2018/19; the major changes were made for the general high school, whereas the technical high school path and the professional one remained with only small changes basically the same. More information can be found on the website of the French Government: https://www.education.gouv.fr/un-nouveau-baccalaureat-en-2021-3098.

The *lycée général* had 3 different directions before the reform: the scientific (*série scientifique* labelled with *S*), the humanistic (*série litteraire* labelled with *L*) and the socio-economical one (*série économiques et sociales* labelled with *ES*). These three different choices were focused on different domains of knowledge and they offered students particular curriculums. The idea of the new reform was to abolish the different directions and to leave to the students the choice of the subjects they want to deepen.

The school subjects are divided into three different categories: **the compulsory subjects**, which are the same for all the students, **the specific subjects**, three in the penultimate year (*classe de première*) and two of these three in the last year (*classe de terminale*), and **the optional subjects** which should enforce the knowledge of other domains for possible future studies. The specific subjects are the most important ones because they are part of the final exam in addition to French and philosophy, therefore more school hours per week are and a new reformed syllabus which better enables students for higher education.

The reform aims to enable the students to choose a curriculum which fits best to their preferences and future ambitions, giving more school hours to the specific subjects and allowing a deepening of other study domains with the optional ones. There are some new subjects introduced in the study offer, for example, the subject ‘numerical science and technology’ (*sciences numériques et technologie*) in the first grade of high school with a focus on coding, and the ‘scientific education’ (*enseignement scientifique*) in the last two grades of the educational path. It should improve the scientific knowledge of the students and prepare them to understand the great challenges of the contemporary world. This subject is very multidisciplinary because it addresses issues that touch different disciplines like mathematics, physics, chemistry, biology, and geology.

The final mark at the final exam is composed of 40% on the effort evaluated with some tests during the last two school years (*contrôle continu*) and by 60% on the final exams. The new BAC consists of 4 written exams which, are the same for the whole of France, and an oral one. The French written exam takes place in June at the end of the *première* (the penultimate year), the exams for the two specific subjects are taken in the spring of the *terminale* (the last year) and the written philosophy exam and the oral one take place in June of the last year. The oral exam, which should last 20 minutes, consists in a presentation of a personal project, prepared during the last two years of the high school, concerning a topic of one or both specific subjects, and some questions related to the project asked by two commissioners. This should be the exam which allows the students to show their deeper understanding of the matter and to develop a personal interdisciplinary approach, as well.

1.2. The School Program

To better understand the curriculum, it is useful to have a look at the syllabus of some subjects. In this chapter, because of the interest of the thesis, it will be taken into consideration only the case of the general high school, the so-called *lycée général*. Choosing appropriate specific and optional subjects, the *lycée général* can be compared to the scientific high school (*liceo scientifico*) in Italy. The analysis does not pretend to be complete and detailed, it aims rather to give a reader a general idea of school contents in France.

In particular let us suppose analysing a curriculum of a student interested in science who has chosen in the second grade of high school (*classe de première*) the following specific subject: mathematics (*mathématiques*), physics/chemistry (*physique-chimie*), life sciences (*SVT*) and out of these three in the third grade (*classe de terminale*): mathematics and physics/chemistry, with a choice of advanced mathematic syllabus 'mathematics for experts' (*mathématiques experts*) as the optional subject.

An analysis of the French national syllabuses of the following subjects: mathematics, mathematics for experts, physics/chemistry and science education are presented below. More information can be found on the web page of the Ministry for National Education and Youth of the French Government which has been the main source for the analysis. The detailed syllabus of the subjects studied in first and second grade (*classe de seconde et de première*) one can find in the *Bulletin Officiel Spécial n° 1 du 22 janvier 2019*:
https://www.education.gouv.fr/pid285/bulletin_officiel.html?pid_bo=38502,
whereas the detailed syllabus of the subjects studied in the third grade (*classe de terminale*), one can find in the *Bulletin Officiel Spécial n° 8 du 25 juillet 2019:*
https://www.education.gouv.fr/pid285/bulletin_officiel.html?pid_bo=39051.

1.2.1. The National Syllabus of Mathematics

The number of teaching hours per week changes every year: there are 4 compulsory hours per week at the first grade of high school, 4 hours in the second one and 6 hours as a specific subject with the possibility to choose additional 3 hours with an optional subject: mathematics for experts (*mathématiques expertes*) at the third grade. The program aims to develop six mathematical competences: researching, modelling, representing, reasoning, calculating, and communicating.

Starting with the first grade, the syllabus is divided into 6 chapters: Numbers and Calculus (*nombres et calculs*), Geometry (*géométrie*), Functions (*fonctions*), Statistics and Probability (*statistiques et probabilités*), Algorithms and Coding (*algorithmique et programmation*), Set-Theory Language and Logic (*vocabulaire ensembliste et logique*). The contents are listed in Table 1.1.:

Table 1.1. Mathematics contents for the first grade (*classe de seconde*)

Chapter:	Contents:
Numbers and Calculus	Deepening of different types and sets of numbers, developing the numerical and algebraic calculus, working on inequalities, modelling problems with first degree equations and inequalities
Geometry	Introduction to 2D-vectors, set of points described by an equation (case of a straight line)
Functions	Notion of a function (dependence of one variable of the other), expressed in different languages: especially algebraic and graphical one; study of notions of variations of functions, maxima and minima
Statistic and Probability	Some elementary notions of statistics (average value, median, standard deviation, etc.) and of probability (probability, event, a sample of events, etc.)
Algorithms and Coding	Definition of a function, coding as a 'text writing' in the informatic language (describing some algorithms in natural and informatic language, writing some basic algorithms, interpreting, or modifying more difficult algorithms)
Set-theory and Logic	Elementary notions of set-theory and logic (writing some mathematical propositions, use of 'and' and 'or', basic negations, implications, logical equivalency, prepositions with quantifiers)

The second-grade syllabus (mathematics as specific subject) is divided into 6 chapters: Algebra (*algèbre*), analisys (*analyse*), Geometry (*géométrie*), Statistics and Probability (*statistiques et probabilités*), Algorithms and Coding (*algorithmique et programmation*), Set-Theory Language and Logic (*vocabulaire ensembliste et logique*). The contents are listed in Table 1.2.:

Table 1.2. Mathematics contents for the second grade (*classe de première*)

Chapter:	Contents:
Algebra	Algebraic and geometrical series and recursive relations
Analysis	Concept of derivative and applications, the study of functions (exponential and logarithmic), trigonometry
Geometry	Metrics (scalar product), notions of orthogonality, conics

Statistic and Probability	Conditional probability, formalization of independent variable, expectation value, variance, and standard deviation
Algorithms and Coding	More exercises in coding and use of more advanced algorithms
Set-theory and Logic	Advanced use of set-theory and logic in mathematics (negations of implications and propositions with quantifiers), necessary and sufficient condition

The third-grade syllabus (mathematics as specific subject) is divided into 5 chapters: Algebra and Geometry (*algèbre etgéométrie*), Analysis (*analyse*), probability (*probabilités*), Algorithms and Coding (*algorithmique et programmation*), Set-Theory Language and Logic (*vocabulaire ensembliste et logique*). The contents are listed in Table 1.3.:

Table 1.3. Mathematics contents for the third grade (*classe de terminale*)

Chapter:	Contents:
Algebra and Geometry	Cartesian product, elementary combinatory, geometry in 3D-space, the study of vectors in space, vector product, orthogonality in space, algebraic calculus in R^3
Analysis	Convergence, limits, derivates, integrals and study of a function, theorems of existence and unicity
Probability	Basic theory of probability (Bernoulli's law and schema, Poisson's law, additivity of variance for the independent variable, the law of large numbers, inequality of Bienaymé-Čebyšёv)
Algorithms and coding	No new elements are introduced: a consolidation of acquired knowledge is expected
Set-theory and Logic	Elements of set-theory (set, subset, complementary set, union and intersection), logic as mathematical language, reasoning ad absurdum, searching a counterexample for rejecting a law, demonstration of a propriety by recursion, reasoning by equivalence

In the third grade – *classe terminale*, it is possible to choose as an optional subject also an advanced course of mathematics, called mathematics for experts *(mathématiques expertes*). The syllabus is divided into 3 chapters: Complex Numbers (*nombres complexes*), Arithmetic (*arithmétique*) and Matrixes and Graphs (*matrices et graphes*). The contents are presented in the Table 1.4.:

Table 1.4. Contents of the subject mathematics for experts in the third grade (*classe de terminale*)

Chapter:	Contents:
Complex Numbers	Property of complex numbers from the algebraic and the geometrical point of view, Euclidean plan R^2 and complex numbers, complex numbers, and trigonometry (Euler's and de Moivre's formula), the n-root of unity.
Arithmetic	The fundamental results and applications of Arithmetic (divisibility tests, Diophantine equations, encryption problems)
Matrixes and Graphs	Basic theory of matrixes and graphs (linear systems, geometrical transformations, calculus with matrixes, Markov's chains)

1.2.2. THE NATIONAL SYLLABUS OF PHYSICS/CHEMISTRY

The syllabus is divided into 4 chapters for each grade: Constitution and Transformation of the Matter (*constitution et transformations de la matière*), Movements and Interactions (*mouvement et interactions*), Energy: Conversions and Transfers (*l'énergie: conversions et transferts*), Waves and Signals (*ondes et signaux*). As one can see, there are not conventional subdivisions like Mechanics, Thermodynamics, Electromagnetism etc., but there is rather a mixture of physical and chemical properties of a system. This can have some positive implications as some negative ones. The number of teaching hours per week changes every year: there are 3 compulsory hours per week in the first grade of high school, 4 hours in the second grade and 6 hours as a specific subject in the third grade.

The program aims to develop five scientific competences: analysing, reasoning, realizing, validating, and communicating.

The first-grade syllabus is divided into 3 chapters: The Constitution and Transformation of the Matter (*constitution et transformations de la matière*), Movements and Interactions (*mouvement et interactions*) and Waves and Signals (*ondes et signaux*). The contents are reported in the following Table 1.5.:

Table 1.5. Physics/chemistry contents in the first grade (*classe de seconde*)

Chapter:	Contents:
Constitution and Transformation of the Matter	Description and characterisation of the matter at microscopic scale: pure and mixed compositions, mass and volume composition of a mixture, aqueous solutions (solvent, solute calibration assay), molecular and atomic species, ions, electroneutrality of the matter, identifications of atoms and chemical elements, electron configuration, chemical stability of elements, mass quantity and definition of mole, models of transformations of matter and energy transfer (physical, chemical and nuclear transformations)
Movements and Interactions	**Kinematics:** description of a movement (dimensions, frame of reference, movement's relativity, point-mass description, position, trajectory, average speed, rectilinear movement), **Dynamics**: an action of a force on a system (Newton principles, characterisation of a force, examples of forces: gravitational force, weight force and tension of a rope, 1D free fall)
Waves and Signals	**Acoustic**: emission and perception of a sound (emission and propagation of a sound signal, speed of propagation, frequency and period, perception of a sound, sound intensity), **Optics**: vision and image (linear propagation of the light, light velocity, emission spectra, wavelength, Snell-Descartes' law, optical index, dispersion of the light in a prism, dispersion and focal lens, focal distance, enlargement, eye's model), **Signals and electronic instruments** (Kirchhoff's laws, characterisation tension-current of a dipole, Ohm's laws, electronic sensors)

The second grade syllabus of Physics/Chemistry as the specific subject is divided into 4 chapters: Constitution and Transformation of the Matter (*constitution et transformations de la matière*), Movements and Interactions (*mouvement et interactions*), Energy: Conversions and Transfers (*l'énergie : conversions et transferts*), Waves and Signals (*ondes et signaux*). The contents are reported in Table 1.6.:

Table 1.6. Physics/chemistry contents in the second grade (*classe de première*)

Chapter:	Contents:
Constitution and Transformation of the Matter	Evolution of a system after a transformation: initial composition of a system described by physical parameters, chemical evolution of a system (redox reactions), description of a final state, stoichiometric mixtures, titration, physical proprieties of a system (structure of molecules, polarity, Lewis' schema, the electronegativity of atoms, interactions among ions, solubility), dissolution of salts in water, equation of dissolution reaction, hydrophilicity, lipophilicity and amphiphilicity of an organic chemical species, characteristic groups associated with families of compounds (alcohol, aldehyde, ketone and carboxylic acid), the names associated to formulas, identifications of groups by infrared spectroscopy, synthesis of organic chemical species, conversion into the energy of organic matter, modelling of combustion, microscopic interpretation of a gas-phase interactions
Movements and Interactions	Fundamental interactions and introduction of the concept of field, electric charge, Coulomb's law, gravitational and electrical field, description of a static fluid state (temperature, pressure, volumetric mass), modelling of an ideal gas, Mariotte's law, pressure of fluid on the surfaces, fundamental law of static fluids, fluid dynamic.
Energy: Conversions and Transfers	Energy aspects in electrical phenomena: electric charge drivers, real and ideal charge and tension generator, generator in series, electrical power and energy, Joule's effect, converter efficiency, energy aspects in mechanical phenomena: kinetic energy, work done by a force, kinetic energy theorem, conservative forces, potential energy, non-conservative forces, mechanical energy and its conservation, energy gain or dissipation
Waves and Signals	Mechanical waves, wave's velocity, periodical waves (sinusoids), wavelength and frequency, **optics**: magnification, real and virtual images, real and reverse images, white light, colours of objects, additive and subtractive

	synthesis, absorption, diffusion, transmission, colour vision, electromagnetic waves, relations among velocity, wavelength and frequency of light, photons, qualitative description of matter-radiation interaction, absorption and emission, quantification of atomic energy levels

The third-grade syllabus for physics/chemistry as specific subject is also divided into 4 chapters: Constitution and Transformation of the Matter (*constitution et transformations de la matière*), Movements and Interactions (*mouvement et interactions*), Energy: Conversions and Transfers (*l'énergie : conversions et transferts*), Waves and Signals (*ondes et signaux*). The contents are reported in Table 1.7.:

Table 1.7. Physics/chemistry contents in the third grade (*classe de terminal*)

Chapter:	Contents:
Constitution and Transformation of the Matter	Acid-base transformations, analysis of a chemical system by physical methods, pH measurements, absorbance, Beer-Lambert's law, conductance Kohlrausch's law, infrared and UV-visible spectroscopy, analysis of systems by chemical methods, mass titre and density of a solution, titration with pH-metric and conductimetric monitoring, chemical evolution of a system (time evolution, cinematic factors, catalysis, velocity law of 1st order) macroscopic description of the transformation, description of a system by nuclear reaction (radioactive decays, nuclear reaction equation, conservation laws), natural and artificial radioactivity, applications of radioactivity, spontaneous evolution of a chemical system, dynamic equilibrium, quotient of a reaction Q, spontaneous transformation for a redox reaction, stack, half stack, slat bridge, no-load voltage, functioning of a battery, electrochemical reactions, acid-base reactions, acid constant, tampon solution, produced current in a chemical reaction, functioning of an electrolyser, stockage and conversion of chemical energy, synthesis of organic molecules (structures and proprieties), synthesis' optimisation and velocity, strategies of multi-stage synthesis

Movements and Interactions	Description of accelerated movement (uniformly accelerated rectilinear movement and circular movement), Galilean transformations, the equilibrium of a system, movement in a uniform field (gravitational and electrical), Kepler's laws, geostationary satellites, acceleration of charged particles, energy aspects, Archimedes' principle, movements of incompressible fluids, Bernoulli's relation, Venturi's effect
Energy: Conversions and Transfers	Ideal gas model, state-equation of ideal gases, first thermodynamic principle, internal energy, work, heat, thermic capacity of a system, heat flux and the thermal resistance, evolution of a system in contact with a thermostat, thermal balance of the Earth-atmosphere system, greenhouse effect
Waves and Signals	sound intensity, attenuation in dB, diffraction of a way passing through a split, characteristic angle of diffraction, interference of two waves (constructive and destructive), interference of two light-waves, the difference of optical path, conditions for constructive and destructive interference, Doppler's effect, astronomical optical lenses, magnification, description of the light as photon flux; energy, velocity and mass of photons, photoelectrical effect, photons' absorption and emission, the energy efficiency of a photovoltaic cell, the electrical current intensity in the variable regime, capacitors, the relation between charge and tension, RC circuit model, the characteristic time

1.2.3. THE NATIONAL SYLLABUS OF SCIENTIFIC EDUCATION

The 'scientific education' (*enseignement scientifique*) is the new compulsory subject introduced by the reform. It has 2 hours per week in school timetable of the second and third grade of high school. It aims to give a scientific background to every student. It is focused on scientific knowledge and how this influences human society. The main goal of the course is to present some themes that are important from the scientific point of view and that have a large impact also on human life. The multidisciplinary approach tries to develop a critical sense and to improve active citizenship in the students.

From the didactical point of view, it is suggested to pay attention to the following points: the teaching must present the real complexity of the themes and highlight the importance of the interdisciplinary approach, give a special role to Mathematics and its language, encourage observation of phenomena and laboratory experiences, underline the importance of the scientific thinking in human history and use numerical tools.

The course is going to develop seven interdisciplinary themes, in two years. There are three general aims of the lectures: understand the nature of science and its methods, identify, and act the scientific practices and finally identify and understand the influence of science and technological development on the society and of the environment.

The syllabus and the contents of the second grade are listed in Table 1.8., whereas the syllabus and the contents of the third grade are listed in Table 1.9. As in the prior paragraphs, there is no seek of completeness, the main goal is to give to the reader just an idea of the syllabus, taught in a general high school.

Table 1.8. Science education contents in the second grade (*classe de première*)

Chapter:	Description:	Contents:
1. A Long History of the Matter (*Une longue histoire de la matière*)	The incredible diversity of the matter in the Universe is composed of a small number of elementary particles organized hierarchically. In this chapter, it will be presented the history of the matter from the Big Bang to the birth of life.	1.1. Levels of organisation: the chemical elements (formation of elements from the hydrogen to the heavy ones, graphical representations of the abundance of different elements, fusion and fission processes, radioactive elements, half-life, graphical representations of radioactive law decays, C 14 dating) 1.2. Orderly buildings: the crystals (the solid-state, 3D-representation of NaCl, elementary geometrical structures of crystals: simple cubic, bcc and fcc, amorphic structure, identify some specimens of crystals and amorphic rocks 1.3. A complex structure: the living cell (discovery of cells using a microscope – historical approach, cell theory, cell components, membrane structure)
2. The Sun: Our Source of Energy	The Earth receives the essential energy for life from the Sun. This energy influences the	2.1. The solar radiation (solar energy production by nuclear fusion, Einstein's relation for mass-energy equivalence, solar radiation spectrum, black body model, Wien's law, solar irradiated power (depended

(*Le Soleil, notre source d'énergie*)	Earth surface and it permits the photosynthesis, a basic element for food production.	on daytime and seasonal variation, latitude), graphical representation of data) 2.2. The terrestrial radiation balance (total power of solar radiation, albedo, absorbed and emitted power, a model of the greenhouse effect, graphical representation of absorption in function of wavelength, the dynamic equilibrium between the emitted and absorbed power, average terrestrial temperature) 2.3. Biological conversion of solar energy: the photosynthesis (photosynthesis process, heating and evapotranspiration, biosphere as chemical energy stock, energy exchange in leaves, organic fuels: gas, carbon, oil) 2.4. The thermic balance of the human body (cell respiration, thermic power of human body radiation, qualitative diagram of energy exchange between the human body and environment, energy stocked in food)
3. The Earth, a Singular Star (*La Terre, un aster singulier*)	The Earth is one of the most studied objects: its shape, its age, its place in the Universe, its movement around the Sun. Analysis of historical debates and scientific knowledge: first terrestrial meridian measurements, solar system organisation, the study of age	3.1. The shape of the Earth (first measurements in Antiquity, triangulation method, spherical geometry, and geography) 3.2. History of the Earth's age (analysis of sediments, biological evolution, dating with radioactivity; analysis of historical documentation in debates) 3.3. The Earth in the Universe (the trajectory of the Earth around the Sun, historical debates: geocentric vs heliocentric system, the motion of the Moon)
4. Sound and Music, Information Carriers	The human being perceives the reality through some signals, some of them are sound	4.1. The sound as a phenomenon of vibration (the physical reality of the sound, periodic signals, sound frequencies and composed signals, harmonics, the power of a sound signal and its propagation through

(*Son et musique, porteurs d'information*)	signals. The study of harmony is present since the Antiquity in Arts, Music and also Mathematics. The music can be analysed with informatics tools.	the room, measurements of loudness, historical debates about the sound, analysis of a sound spectrum with informatics programs, the study of vibrating strings) 4.2. Music as the art of making the numbers heard (notes and intervals in music, Pythagoras' ranges, calculation of powers and quotients related to the cycle of fifths, construction of ranges at equal intervals with irrational numbers, the partition of an octave into twelve equal intervals) 4.3. Sound, information to code (discretization of an analogic sound signal, sampling and quantification, compression of audio files) 4.4. Listening to the music (outer and middle ear: receiving and transmitting sound vibration, sounds heard by humans, the functioning of the internal ear, specialized brain areas for processing auditory signals)

Table 1.9. Science education contents in the third grade (*classe de terminal*)

Chapter:	Description:	Contents:
1. Science, Weather, Society (*Science, climat et société*)	The primitive atmosphere of the Earth was different from today one. Its transformation was due to geological and biological events. After the industrial revolution, the human activity started heavily to influence the composition of the atmosphere and the results of this activity are	1.1. The terrestrial atmosphere and the life (the composition of the atmosphere, geological influence on the evolution, states of water-aggregation, graphical representation and its interpretation, first living organisms, photosynthesis, determination of the presence of oxygen in atmosphere through geological analysis, the formation of ozone and its destruction, interpretation of ozone's absorption spectrum, greenhouse effect, the biochemical cycle of carbon) 1.2. The complexity of climate system (climatic parameters, climatology and meteorology, studies of the variation of average atmospheric temperature,

	many and well visible. The future individual and societal choices should be based on scientific studies and technology.	gases causing the greenhouse effect, climate change and global warming, their effects) 1.3. The future climate (the climate models are based on a system of fundamental equations describing the mechanisms that act on the Earth and their numerical resolution methods, interpretation of the main results of numerical methods: increasing average temperature and of the sea level, extreme events, ocean acidification, impacts on terrestrial and marine ecosystems) 1.4. The energy, the choice of development and future climate (different forms of energy production, clean energy, units of energy measurements and their conversions, analysis of energy production data on individual, national or international scales, fossil fuels, the impact of pollution on general health, personal choices and pollution, future emissions of greenhouse gases and their consequences on ecological systems)
2. The Future of Energies (*Le futur des énergies*)	In the energy sector, electricity plays an important role in economy. The role of electrical current will be analysed through historical, economical, and political issues. It will be presented the history from the discovery of electromagnetism in the XIXth century through Einstein's photoelectric effect to the modern	2.1. Two centuries of electric energy (Faraday's and Maxwell discovery of electromagnetic phenomena, the quantum revolution, semiconductors and photovoltaic production, electric alternator) 2.2. The advantages of electricity (conversion of mechanical and thermodynamic energy: dynamo, wind, hydroelectric, geothermic and solar energy, nuclear power plants, efficiency, electrochemical conversions and electronic batteries, different forms of electric storage) 2.3. Optimisation of electric transport (Joule's effect, dissipation, the schema of a high-voltage electric circuit, a model of minimisation of Joule's effect in energy transport)

	ways of current's production.	2.4. Choice of energy and impact on the society (collective and individual choices in electricity consumption and their impacts on climate, environment, agriculture, health system, economy)
3. History of Life (*Une histoire du vivant*)	Many different organisms are populating the Earth. This biodiversity is very important and evolution plays a crucial role in its understanding. The mathematic models describe the dynamic of this complex system. The capacity of treatment and analysis of all this date are improved by artificial intelligence and other applications of informatics.	3.1. The biodiversity and its evolution (methods of measurements and classification of biodiversity, population dynamics, sampling methods and data interpretation, the theory of Hardy-Weinberg, evolution of genotypes with a program based on the Hardy-Weinberg's method, human impacts on biodiversity, management of an ecosystem) 3.2. Evolution as World reading grid (anatomic structures as results of evolution, the adaptation of prophylactic strategies: antibiotics and vaccines, agricultural impact on biodiversity) 3.3. The human evolution (Homo sapiens as a primate, construction of a phylogenetic tree, history of hominids) 3.4. Demographic models (linear model, graphic interpretations of data, exponential model, Malthus' model: predictions, graphical representation, calculation of some parameters, more elaborated methods: analysis of results) 3.5 Artificial intelligence (Turning's machine, computers and data analysis, coding in informatic languages, different types of files: text, sound, video, image and executive files, errors in coding and debugging, artificial intelligence (AI), machine learning, big data elaboration, ethic problems with AI, Bayesian inference and machine learning)

2. THE ISSUE OF INTERDISCIPLINARITY

When one thinks about interdisciplinarity, maybe one of the first thoughts is at its etymological meaning: the world is composed of “inter” that means “in-between, among” and “disciplinarity” which refers to the disciplines.

Edgar Morin, a French philosopher, frequently highlights that the disciplines as we know them today, are quite recent. Only in the 19th century with the birth of modern universities, the structure of the knowledge became disciplinary-organized. In his opinion, the scientific disciplines are organizing categories within scientific knowledge which establishes the division and specialization of work and responds to the diversity of fields that make up the sciences (Morin, 1994). A discipline naturally tends towards autonomy, by the delimitation of the borders, by the language which it constitutes, by the techniques which it is brought to develop or use and by the theories which are proper to it. Professor Marie-Anne Hugon intends the disciplines as social constructs that appear, evolve and even disappear (Hugon, 2004). Therefore, it is interesting to analyse the relations between them.

The study of the interactions among different disciplines started in the early ‘70s. In the book *Interdisciplinarity: Problems of Teaching and Research in Universities,* the distinction between interdisciplinarity, multidisciplinarity (called sometimes also pluridisciplinarity) and transdisciplinarity appeared. The publication is a result of an international conference held in Paris in 1970 and cofounded by the Organisation for Economic Co-operation and Development (OECD). Although some literature can be found even before that event, the systematic study of the interaction between disciplines can be traced back to that date and onwards. In the Figure 2.1. one can see the use of the term “interdisciplinarity” in the recent years.

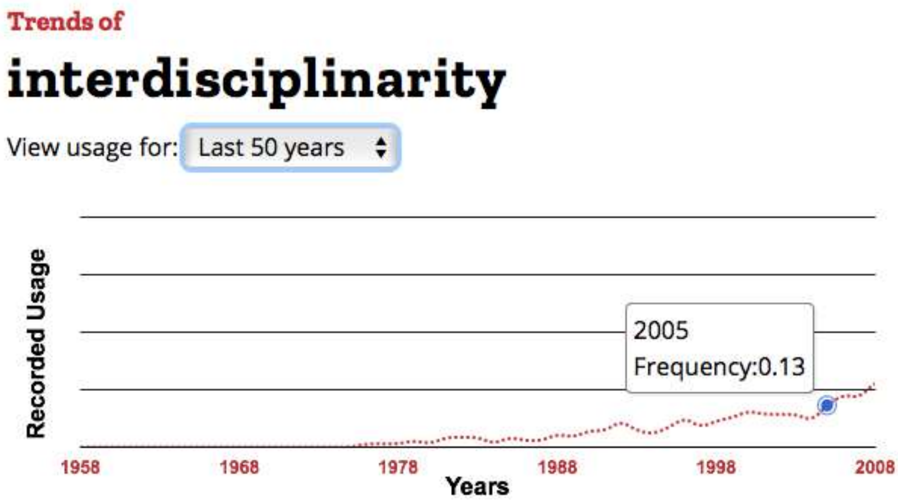


Figure 2.1. Use of the term “interdisciplinarity” in the period 1958-2008, Collins Dictionary (https://www.collinsdictionary.com/dictionary/english/interdisciplinarity)

2.1. An Overview of Interdisciplinarity Works in French Environment

In this first chapter, it will be presented the theoretical framework of some works that have influenced the study of interdisciplinarity in the French-speaking environment, therefore the analysis is focused on papers translated or written in French language. Generally, in the literature, it can be found the statement that interdisciplinarity necessarily implies relations among constructive elements of two or more disciplines like methods, techniques, contents and so on.

Without pretension of completeness, some works will be briefly illustrated to give the reader the idea of the richness of the literature on the theme. The choice was focused rather on the articles that have a certain relevance or an impact also on the scientific environment, ignoring many works that took into consideration case studies only of humanistic or political and economic disciplines.

For Pierre Giolitto and Maryse Clary the key ingredients of interdisciplinarity are the use, the association, and the coordination of appropriate disciplinary knowledge in an integrated approach to the problems (Giolitto & Clary, 1994). In their book, *Approches didactiques de l'interdisciplinarité*, in 2002 Alain Maigain, Barbara Dufour and Gérard Fourez define interdisciplinary as a way of putting in relation/linking at least two disciplines in order to elaborate an original representation of a notion, a situation, a problem. Édouard Kleinpeter quoting Pestre (2010) defines an interdisciplinary approach as an engine of scientific progress, a tool which improves a link among research and exploitation, a necessary element to allow a dialogue between society and science, the desired orientation in the formation of young scientists.

Yves Lenoir emphasizes that transversal skills and interdisciplinarity go well together, especially in a project approach (Lenoir, 2003). The realisation of the project should not, however, take precedence over learning, because the question of acquisition of the meaning is central. Interdisciplinarity must have an anchor in reality. The purpose of interdisciplinarity stems from the integration of learning processes and the resulting knowledge, by promoting the mobilisation of these and their application in everyday situations. An interdisciplinary work should be a process in which one develops a capacity of analysis and synthesis from the perspective of several disciplines. It is not possible to have interdisciplinarity without disciplinary knowledge.

It is interesting to notice that Yves Lenoir and Lucie Sauvé define another type of interdisciplinarity, called **school interdisciplinarity (SI)** (Lenoir & Sauvé, 1998). The main object of study of SI are the relations between school subjects, in fact, they define the school interdisciplinarity as the linking ("*mise en relation*") of two or three school disciplines which can establish some connections of complementarity, cooperation or integration of reciprocal actions under different aspects (study objectives, concepts or notions, final goals, learning approaches and technical skills, etc.). The final goal of school interdisciplinarity is to promote the integration of learning processes and knowledge among students. The School interdisciplinarity in a way ensures a reciprocal dependence between school subjects and their interrelations in terms of their content or their approaches (Lenoir & Sauvé, 1998). Although this kind of interdisciplinarity could be really interesting for this thesis, its claim in the literature has not been noticed, therefore it will not be used onwards. It should be also highlighted that interdisciplinary teaching is effective especially in a solution to a problematic situation in which different aspects of reality can be expressed.

For what concerns multidisciplinarity, normally it is understood as a process of juxtaposition when two or more disciplines or disciplinary approaches from different domains are present around a common theme. In Lenoir's opinion, it aims to study different points of view, sometimes very specialized, on the same topic in order to solve a problem in a complementary way, where each specialist retains the specificity of his concepts and methods. Sometimes one uses the term pluridisciplinarity as a synonym, though some authors as, for example, Nicole Rege-Colet claim that pluridisciplinarity juxtaposes pieces of knowledge that are closer as in multidisciplinarity (Rege-Colet, 1993).

Finally, one can encounter transdisciplinarity, when a study of a theme goes beyond single disciplines in particular within the framework of a project (Lenoir, 2003; Fourez, Maingain and Dufour, 2002). Transdisciplinarity has the ambitious objective of gathering knowledge beyond disciplines, refusing to approach the world, and its problems through the categories of disciplines, rather than trying to build its own content and its methods. This is maybe the focal point: in contrast with interdisciplinarity, where methods are borrowed from single disciplinary knowledge, transdisciplinarity builds its own methods. The definition of transdisciplinarity appears for the first time in 1972 (Kleinpeter, 2013) as seeking of a common system of axioms for a certain set of disciplines (Apostel et al., 1972). It can be considered as a big picture where a single discipline is a particular realisation. Its fundaments are, for Basarab Nicolescu, president of the International Centre of Transdisciplinary Research and Studies, the complexity, the multiple levels of reality and the principle of the third included (Klein, 2004). In his opinion it implies the creation of a common language, logic and concepts which permit a real dialogue.

There can be other distinctions of different types of interdisciplinarities that emerge if other aspects are analysed. If one takes a look at the disciplinary field, it is evident a distinction between narrow interdisciplinarity, when a link among the disciplines is established through common methods, themes and epistemologies, and wide or broad interdisciplinarity, when the epistemologies of single disciplines are not so close (as a discipline of hard science and a humanistic one). If one analyses the collaboration and the communication of researchers, it can be distinguished a shared interdisciplinarity and a cooperative one, where in the first one different researchers share the results but not the process, whereas in the second one the common work is necessary (Kleinpeter, 2013).

Another important paper is the empirical survey of Véronique Boix-Mansilla (2006) who interviewed 55 scientists working in interdisciplinarity institutes. She distinguishes three different epistemological approaches. The first is centred on concepts (**conceptual bridging approach**): it consists in analysing concepts, principles or laws which can correspond to different phenomena studied in many disciplines. The second is centred on explication (**comprehensive approach**): it tries to explain complex phenomena which components resound in different disciplines. The third is centred on results (**pragmatic ID**): it aims to find solutions at the problems concerning different domains.

In the end, it is worth mentioning some other results of the work of Nicole Rege-Colet (1993) who studied interdisciplinarity as it is taught in school, especially related to project works. She distinguishes between four different levels of interdisciplinarity. The first is the **pluridisciplinary level** in which the knowledge of different disciplines, although it is related to a common theme, is stacked and has a mosaic structure. The second is the level of **thematic interdisciplinarity**, where the disciplines are linked together by a common theme and it has a result a structuring of knowledge in a conceptual network. The third is the level of **instrumental interdisciplinarity** that allows several disciplines to converge and to find out a solution to a common problem; this project approach is based on the problem-solving. The last level is the **structural interdisciplinarity** in which the disciplinary frameworks are modified to constitute a new integrated reference framework.

Some of the works published in the last decades have been presented, but there is another relevant paper that analyses systematically the relations and the interaction between the disciplines, in order to establish a classification of different interaction and to create a possible taxonomy (Thompson Klein, 2011). In the present thesis, this work is taken as the theoretical framework for the analysis of different forms of interdisciplinarity.

2.2. THOMPSON KLEIN'S TAXONOMY

Julie Thompson Klein began a more systematic study in the '90s in order to explicit the epistemology of interdisciplinarity studies (Kleinpeter, 2013). In her article *Une taxinomie de l'interdisciplinarité* Thompson Klein (2011) distinguishes some kinds of interdisciplinarity: methodological (*MI*), theoretical (*TI*), instrumental (*II*), and critical interdisciplinarity (*CI*), and two processes Bridge Building and Restructuring.

The first approach can encountered when more than one discipline is present is **multidisciplinarity**, in some texts can be found also the word **pluridisciplinarity**. It consists of a **juxtaposition of knowledge** from disciplines that remain fundamentally distinguished but the process of juxtaposition favours the enlargement of knowledge, information, and methods. In this case, also the epistemology of each discipline remains intact and they maintain their own identity. Multidisciplinarity can be encountered in some publications, research projects, seminars, where different views on the same topic are presented in serial order. There are no interactions among disciplines and therefore no integration is possible.

We can consider some forms of multidisciplinarity which are sometimes classified as 'weak' or 'false' forms of interdisciplinarity such as **encyclopaedic interdisciplinarity**. It is present for example in some scientific books or journals. The topics are normally presented in sequential order, but there is no dialogue between different contributions. The results of single research groups are juxtaposed one to another and they remain in their own disciplinary field.

When some tools or methods are shared among different disciplines, one can talk about **pseudo interdisciplinarity**; this can be the case of the use of statistics in science. The method is common to each discipline, but there is no interaction between them.

The **dialogue** among disciplines is a fundamental condition in order to have real interdisciplinarity because they have to cooperate actively (Thompson Klein, 2011). The demarcation line between interdisciplinarity and multidisciplinarity is the presence of **interaction** and **integration** processes which favour the **reconstruction of knowledge**. Our goal at this point is to analyse and to classify the different interactions to obtain a 'taxonomy' of interdisciplinarity.

We can talk about **methodological interdisciplinarity** (*MI*) when a method or a concept is taken from one discipline and applied in another to verify a hypothesis, formulate a theory or answer to a research question. The main goal of MI is to improve the quality of results obtained in a single discipline. There is a contamination of epistemological knowledge, the borrowing of some theoretical tools from another discipline can give us a new structure of the original discipline.

Thompson Klein quotes Raymond Miller (1982) who distinguishes two kinds of methodological interdisciplinary: he talks about Share Components when the disciplines share research methods like for example the statistical inference, and about Cross-Cutting Organizing Principles, when the fundamental concepts or social processes are used to organize ideas and findings. In the shared methods could be included also surveys, interviews, samplings, case studies, cross-cultural analysis, and ethnographies, as methods of discourse analysis.

The **theoretical interdisciplinarity** (*TI*) is an evolution of the methodological one, it involves a more holistic general view and a more coherent epistemology. The main results of this approach are the elaboration of conceptual frameworks during the analysis of particular problems, the integration of propositions across disciplines and the new synthesis funded on the connection between models and analogies.

Within this type of interdisciplinarity, Julie Thompson Klein highlights that Margaret Boden (1999) distinguishes between the generalizing and the integrated interdisciplinarity, the first one occurs when a single theoretical point of view is applied to a variety of disciplines, whereas in the second one the concepts and the different point of view of one discipline help to the solution of problems or theories of another. In this case the original theoretical concepts and methods have developed and changed after the cooperation and interaction. A new conceptual framework and a unification method have been created.

Thompson Klein also exposes the thought of Lisa Lattuca (2001) who defines the conceptual interdisciplinarity as an implicit critique to the disciplinary understanding of solutions of problems and research questions that are not strongly linked.

There are two processes analysed in Thomson Klein's work that represent basic metamorphoses after the interaction between the disciplines: **Bridge Building** and **Reconstruction**. The Bridge Building takes place when one well-established discipline links with another one. It could be the case of biology and physics when these two disciplines create what we call "biophysics". Heinz Heckhausen (1972) names this process unifying interdisciplinarity (UI). The Reconstruction detaches and recombines parts of several disciplines creating a new coherent one. This can happen in two steps: the first is the specialisation when certain areas are selected and the second is reintegration when the areas are recombined and integrated into a new whole. Taking into consideration these two processes, Julie Thompson Klein can define two other types of interdisciplinarity.

Methodological interdisciplinarity becomes **instrumental interdisciplinarity** (II) when it serves some special needs of a single discipline. In the 80ies, instrumental interdisciplinarity gained visibility in informatics, biotechnology, or biomedicine. As Thompson Klein underlines, Peter Weingart (Stehr & Weingart, 2000) considers this type of activity a sort of strategic or opportunistic interdisciplinarity, when a tool is used in order to solve a problem. This kind of interdisciplinarity, based on sharing technology and instrumental equipment often led to the establishment of a new research field.

The **critical interdisciplinarity** (CI) questions the dominant structure of knowledge and the educational system to transform it. The instrumental interdisciplinarity does not raise fundamental questions as the critical one, which can destroy part of the system for reconstructing it later. The deconstructing process and the seeking for disciplinary limits are the base for a new epistemology. Asking critical questions and looking for a common answer is part of the process of building new correspondences. The questions and the disciplines put in correspondence have changed, the solidity of their borders crumbles and a common basis can raise.

The article *Une taxinomie de l'interdisciplinarité* (Thompson Klein, 2011) continues with the analysis of the transdisciplinarity, its history, and recent tendencies but this is outside the interest of this thesis, therefore it will not be reported.

The Figure 2.2., taken from the paper *Une taxinomie de l'interdisciplinarité* (Thompson Klein, 2011), reports the main interactions between the disciplines and their classification.

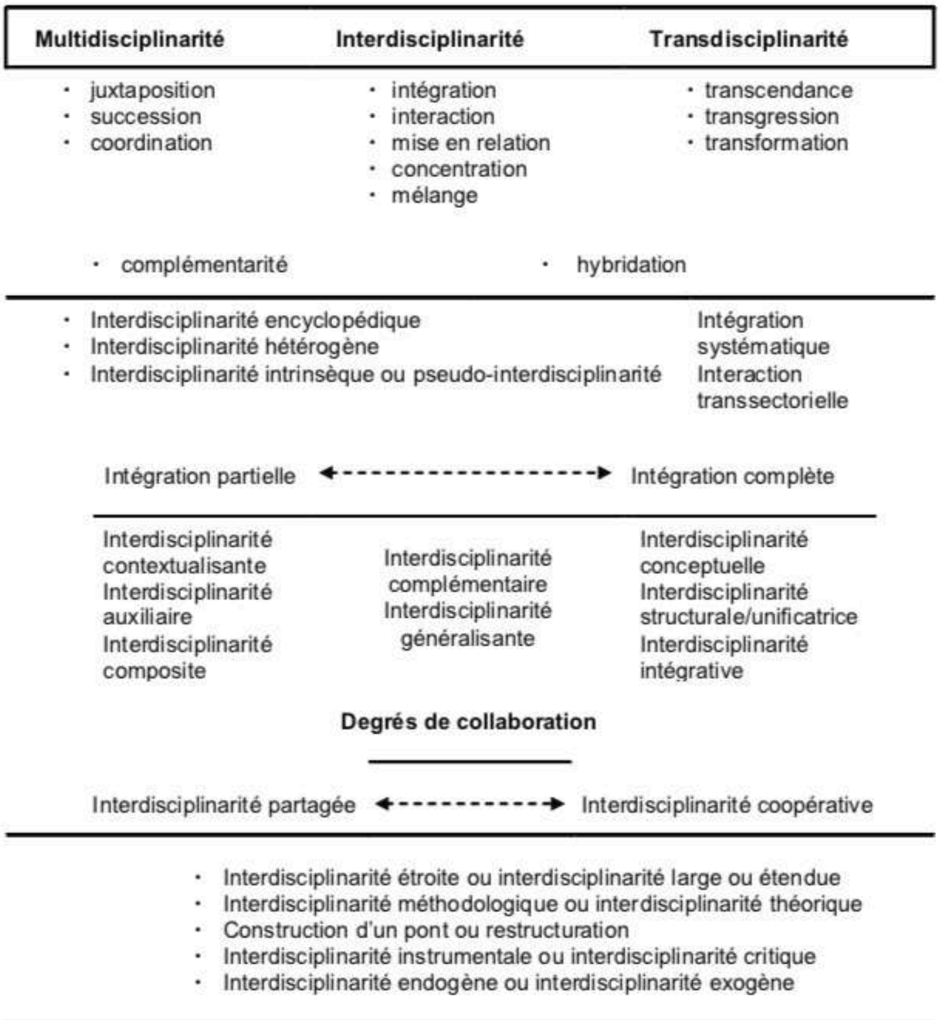


Figure 2.2. Characteristics of different types of interactions, taken from (Thompson Klein, 2011)

3. INTERVIEWS: AIMS AND STRUCTURE

For the present thesis, it was decided to build an interview protocol for collecting data among researchers and teacher trainers working on interdisciplinarity in the domain of science education in France. The protocol was design to investigate the three central themes of my thesis:

i. the definition of interdisciplinarity,

ii. its didactic implementation and

iii. an evaluation of the present situation in the French high schools.

The protocol (Table 3.1.) consists of seven questions. The first two have a more general nature, gathering information on the interviewee's background and his/her research topics. The third question aims to investigate the relation of the interviewee with the issue of interdisciplinarity. The fourth requires a definition of interdisciplinarity. This question aims, on one hand, to explore the personal understanding and the meaning of interdisciplinarity and, on the other, to test whether Thomson Klein's taxonomy provides an effective categorization of the different types of interdisciplinarity. The fifth question aims to: a) explore their ideas about whether interdisciplinarity can be taught at school and how a project can be implemented to develop interdisciplinary skills; b) collect data on an evaluation of the school reform and the present situation in the French school system also in connection with interdisciplinarity teaching. Finally, it was asked the interviewee to add some free thoughts that did not emerge before.

In order to put the experts at ease and let them express themselves in the best way, the interviews were done in French or English language as one preferred.

Table 3.1. Interview protocol

n	English:	French:
1	**What is your educational path?**	Quel est votre parcours d'études ?
2	**What are your main research interests?**	Quels sont vos thèmes de recherche principaux ?
3	**When did you first encounter the issue of interdisciplinarity?**	Quand avez-vous rencontré pour la première fois la question de l'interdisciplinarité ?
4	**What do you mean by interdisciplinarity? Can you give me a possible definition? Which are the key ingredients? Can you give me some examples?**	Comment définiriez-vous l'interdisciplinarité ? Quels sont les ingrédients fondamentaux (les ingrédients clés) ? Pouvez-vous me donner quelques exemples ?

5	**In your opinion can interdisciplinarity be taught? Why/Why not? If yes, how?**	Selon vous, peut-on enseigner l'interdisciplinarité ? Si oui, comment ? Sinon, pourquoi ?
6	**Do you know the new reform of the high school program in France (Bac2021)? What do you think about it, also in connection with interdisciplinarity?**	Connaissez-vous la nouvelle réforme du baccalauréat en France (Bac2021) ? Que pensez-vous également de l'interdisciplinarité ?
7	**Do you want to add something?**	Souhaitez-vous ajouter quelque chose de plus ?

The selection of the interviewees has tried to take into account their geographical provenience and the domain of studies, including people working in mathematics, physics/chemistry, and computer science field. The aim of the job is not to do a detailed inquiry with statistical value but rather to test the pulse of the present situation in the academic environment, through a qualitative analysis. In this sense, the stratification and variety of the people involved in the sample was more important than their number.

During the interview, special attention has been paid to refrain from commenting and interfering in order to maintain as a listener a neutral position and not to influence the colloquium. In some cases, some clarifications or explanations were asked at the end and the interviewees were left free to send some papers or additional material that they thought could help a better understanding.

3.1. INTERVIEW: SUMMERY OF THE INTERVIEWS

Every single interview has been audio-recorded. In this section, it will be presented a summary of the interviews with the main results and the focal points that have emerged. These results have been sent back to interviewees to check their faithfulness and have their consensus to report them in this Dissertation.

3.1.1. EMMANUEL VOLTE

The professor Emmanuel Volte studied mathematics with some courses in informatics at the university (*École Normale Superieure*) and has a PhD in cryptography. His mains research interests are related to historical cryptography and didactic of informatics and mathematics.

His definition of interdisciplinarity is the following one: "*It is something that concerns many disciplines, it appears when a problem cannot be solved by a single discipline and one must use knowledge of different disciplines in mutual interaction*." It is important that the disciplines interact on the same level because otherwise there is no real interdisciplinarity. For instance, mathematics is frequently seen as a tool and not as a discipline with its own epistemology.

In his opinion, the interdisciplinary approach can be developed but the problem in the French school system is that there are very few possibilities for teachers of different disciplines to meet. Normally, a teacher teaches one single discipline, therefore the teaching is too dissected. According to Volte's advice, it is difficult to implement a real interdisciplinary approach if the teachers cannot meet around a problem.

In his opinion, the reform has eliminated some more room where it was still possible to develop interdisciplinary thinking, even if it tries to introduce a subject that should be intrinsically interdisciplinary. If one looks at the single teaching, for instance, mathematics and physics, one notices that they are too separated. Emmanuel Volte thinks that the reform does not encourage interdisciplinarity, for instance, in the syllabus for computer science there is not much effort on the complexity of algorithms. Students are also using Python as the coding language, but it is not expected any semantic analysis of a text. This could be an opportunity to do some interdisciplinary work between French-language teachers and computer science or mathematics ones, but unfortunately, this opportunity has been missed.

3.1.2. RITA KHANFOUR-ARMALÉ

Rita Khanfour-Armalé is a researcher in didactics of chemistry in *Laboratoire de didactique André Revuz* (*LDAR*) and teachers' trainer responsible for chemistry and physics education in the master program of teacher's formation at the University of Cergy-Pontoise. She did a bachelor in biochemistry, then a masters in chemistry/physics and another in science education and a PhD in the same domain. Her study interests are related to teaching practice, use of analogies, implementation of simulations. Mrs Khanfour-Armalé has encountered for the first time the issue of interdisciplinarity at her arrival in the *LDAR* research group when she has started to collaborate with a mathematics research group that was inquiring into modelling processes. Modelling (*modélisation*) is a term often used in the French environment of mathematics to indicate a type of interdisciplinarity.

As teachers' trainer she encourages interdisciplinary projects where trainee teachers of different scientific subjects meet together, analyse and discuss different school curricula and at the end, they prepare an interdisciplinary project. In her opinion, it is possible to develop interdisciplinary competencies in a classroom. The teachers must have the will to meet, share their disciplinary knowledge, discuss and build common language. In this way, they can create some links with different disciplinary concepts and contents and they can transmit this to their students.

Rita Khanfour-Armalé accepted Blanchard-Laville's definition of interdisciplinarity as co-construction of common meaning (Blanchard-Laville, 2000). To achieve this, it is necessary to work together, to know different points of view, to reason together. The same word can have different meanings and it is related to different disciplinary concepts. Therefore, the first step is building a group having a common language. When different disciplinary aspects of common concepts, definitions and themes are shared, one should look at the general meaning which is a synthesis of different sensibilities and approaches. After that, some activities can be done together. "*The fundamental ingredient for good interdisciplinarity is the sharing common concept, therefore the group needs gather, share, build science together; this work is hard but incredibly enriching. This is the secret of the co-construction of common meaning.*"

In her opinion, in recent years many reforms have tried to create some projects able to develop interdisciplinarity. Many types of research show that the teachers often juxtapose the different disciplines about a common theme. Professor Khanfour-Armalé thinks that it is important giving time to the teachers, so they can come up with a common and interdisciplinary project, sharing their knowledge.

3.1.3. MAHA ABBOUD

Mrs Maha Abboud is professor of mathematics education at the CY Cergy Paris University. After her masters in mathematics, she turned to didactic of mathematics with a PhD in this domain. She did her habilitation-thesis for the director of research in mathematical education. Her main research interests concern the teaching practices of mathematics in the secondary education system, teacher trainers' practices, and the use of technologies in mathematics education.

She encountered the issue of interdisciplinarity without considering it properly during her studies, and her practices as mathematics teacher and teacher educator. She began to be actively interested in the topic in recently, last four or five years, when she started to coordinate projects involving researchers in physics and mathematics education. In her opinion, it is meaningful trying to look for interdisciplinary approaches in the research work, although this is not her research topic.

Mrs Maha Abboud preferred not to give a proper definition of interdisciplinarity, but she gave rather her point of view on the theme. One can talk about interdisciplinarity when there is a sliding of two different disciplinary fields in the context of a common educational situation. One should look at how different actors or different disciplines interact in this situation. In her opinion, "*interdisciplinarity is always situated and linked to a concrete problem*". According to Professor Abboud, the key ingredient is not to have the supremacy of one discipline over the other. For example, physicists should not see mathematics as a "service" discipline, a tool for calculations, and on the contrary, mathematicians should not use physics just as a domain for applications. As interdisciplinarity is related to the learning environment, it cannot be taught directly but the teachers should encourage the students to revel interdisciplinary issues during a task resolution.

Although Professor Abboud did not know in detail the new reform and therefore she did not want to express a personal opinion about this question, she gave a more general point of view on science education in France. In her opinion, the teaching of scientific subjects would gain in efficiency, if one could do that in a more interdisciplinary environment. In the French education system, STEM topics are not so present as in other countries. She thinks that science teaching and learning should include more STEM-related problems. "*Perhaps the reforms, the new programs, and the institutions will allow the introduction and the development of STEM-approach in the future, but there is a lack in the research environment, in the French context, and this research theme should be introduced more explicitly.*"

3.1.4. Nathan Lombard

Nathan Lombard is a PhD student at the University of Montpellier where he studies with Professor Thomas Hausberger the relations between quantum mechanics and its mathematical structures as the vocabulary change in the transposition of knowledge. Although he is only a PhD student, his educational path is very rich: after a bachelor's degree with a thesis in high energy physics, he continued studying physics in the master program with experience abroad at the Columbia University in New York where he took some classes of pure mathematics. He did also a masters in general mathematics and went to Jena University in Thüringen where he was seeking for mathematical structures in Quantum Gravity comparing the Quantum Field Gauge Theory with General Relativity. His interests in mathematical structures in physics and how to teach them more efficiently brought him in Montpellier where Mr Lombard is analysing how knowledge circulates between laboratories and universities and its change of vocabulary.

As he admits, he is getting completely aware of the importance of interdisciplinarity just in recent time during his PhD studies, although he understood that something did not work in the French higher education system when his girlfriend was not allowed to take philosophy and physics courses within a curriculum in her undergraduate studies.

Nathan Lombard does not have a proper definition of interdisciplinarity between physics and mathematics, but in his opinion, it is a powerful tool that allows us to analyse from different points of view or different angels a problem to reach a final reformulation. The interdisciplinarity is "*an act* *of intellectual freedom that allows borrowing tools of other disciplines to solve a problem*." Therefore, it is worth to be taught in a classroom. Although Mr Lombard does not have a didactical theory on how to do it, he prefers to move practically with the help of some examples taken from the history of science.

He wishes the French school system were less technical and formal in using mathematical symbols and formulas and more focused on understanding the concepts behind the single formula. In his opinion, the syllabus is too rigorous and stringent. This fact is limiting for teachers trying to develop an interdisciplinary approach.

3.1.5. THOMAS HAUSBERGER

In 2001, Professor Thomas Hausberger discussed his European PhD dissertation in mathematics at the University of Strasbourg combined with the University of Bielefeld (Germany) and he became an Associate Professor at the University of Montpellier. At the beginning of his career, his research interests were focused on pure mathematics (Number Theory and Arithmetical Geometry) and, later on, on the relation between mathematics and philosophy, didactics, and epistemology of mathematics. As he said, he was interested also in "*finding the man behind the formula*". Since 2009 he has been an Associate Professor of Mathematics Education. He founded an IREM group (*Institut de Recherche sur l'Enseignement des Mathématiques* - Research Institute on Mathematics Education) at the University of Montpellier. In 2016, Prof. Hausberger became Full Professor defending his habilitation-thesis on teaching and learning of abstract algebra at university.

He defines interdisciplinarity as a form of collaboration of disciplines in order to better understand the world. In his opinion, it is a "*co-construction of a common point of view*". Different disciplines have different interests, but "*the co-acting can bring at the end to an integration process"*. It is a sort of reconstruction of knowledge.

Professor Hausberger thinks that it is possible to improve interdisciplinarity in schools, but this process requires time. When teacher trainers try to do a project together, they need first to build a common culture, an epistemological and a didactical approach because people coming from different backgrounds have different sensitivities and different interests. It is necessary for them to cross the barriers of their own disciplines, to build trust, and to make themselves to be understood what is not always simple. Physicists, for instance, talking about ontology have a different understanding of the term as mathematicians or philosophers, even when they use the same word. It is important for a teacher who shows the interdisciplinarity to his students to create more links during the didactic transposition process. Often these links have been eliminated from the text, so a teacher has to clarify and rearrange the content, making the links to re-emerge.

The reform introduces in the last two grades of the high school (*classe de première* and *de terminale*) a subject called "scientific education". This could be according to Professor Hausberger "*a great opportunity for mathematics teachers because they are forced to leave their ivory towers facing other contents.*" The syllabus of the subject is very nice and rich also from a historical point of view, but in his opinion, it is impossible to implement such a subject in a classroom with the means that are given to the teachers.

First of all, with the reform the idea of classroom changes; the students are free to choose their own courses, so the class population varies from subject to subject. In these conditions, it is very difficult to implement interdisciplinary teaching because of the different background of the students. Some of them are taking mathematics courses or/and chemistry-physics ones, but there could be others that are not studying any scientific subjects. The common knowledge of mathematics is very poor because it is just that of the first grade of high school (*classe de seconde*). Another problem that is still not clear is who is going to teach the subject of scientific education. The ideal situation would be involving mathematics, physics/chemistry, and biology teachers altogether, but, in reality, the hours would be probably shared, and it could happen that the teachers will not collaborate. Thomas Hausberger thinks that the success of the course depends on co-involving mathematics teachers. The syllabus is, in his opinion, too detailed and there are too many conditions and constraints that do not allow some freedom to the teachers, increasing the level of discussion and promoting interdisciplinarity. There are not the best conditions to engage the teachers in interdisciplinary teaching, although it is the only subject where this could be easily done.

The IREM group of the University of Montpellier prepares some teacher training projects, creating the best conditions possible to develop interdisciplinary skills. The trainees are divided into small groups, where at least a chemistry/physics and a biology or biotechnology teacher is present with two other mathematics ones. They have to collaborate to prepare a project connecting the disciplines and the contents found in the syllabus. They have to pay attention not to juxtapose the arguments, but rather to co-operate in order to deeply understand the connections. These activities are increasing the level of discussion. Enough time is given to the teachers to implement their project, which does not happen when they have to prepare a lot of lectures for their classes. Often there is not enough time to reflect on what they are going to teach. The IREM activities are a great opportunity for the trainees to learn about interdisciplinary also from the historical point of view and to see its evolution.

3.1.6. EMMANUEL ROLLINDE

Professor Emmanuel Rollinde studied physics at the university. Starting with the hadronic physics, he did a master thesis and a PhD in astrophysics. After 15 years of research in astrophysics, he moved to the teacher training with a focus on how to use astronomical topics in the school. 12 years ago, he became a researcher at the Didactic Laboratory André Revuz (*LDAR*) in Paris. His main research interests are connected with the teaching of astronomy in the school, especially using the body during the learning process.

Mr Emmanuel Rollinde encountered for the first time the issue of interdisciplinarity during his research in astrophysics. As he realised, the research in astrophysics was very interdisciplinary because it mixed physics with mathematics and informatics. When he started to do teacher training he understood the importance of the alliance between mathematics and physics; mathematics can describe the data, physics can give a possible interpretation. Nowadays also technology plays a big role in computer science.

Professor Rollinde studies a special type of interdisciplinarity the so-called *school interdisciplinarity*. "*When one has an object of study that can be a natural object or a research question, one can apply different vision frameworks. There are two different ways to do that; either one can analyse the object of study from each disciplinary framework separately or one can do an activity mixing the different disciplines at the same time*." For example, taking into account the solar system, in the first case one can do an art project on the solar system, in mathematics, one can study the equations, during physics one can learn more about the solar system, in technology one can use simulations; the students are studying the same topic from different points of view and they learn the disciplinary tools. In the second case, the students are asked to do an activity as going to the gym and simulate the movement of the planets. Each teacher will analyse the activity from own disciplinary vision framework: the sports teacher will put an effort on the coordination of movements, the physic teacher will analyse the orbits and the velocities, the mathematics teacher will derive the equation and the resulting graphs. Mr Emmanuel Rollinde thinks that this type of approach is more motivating because it is not a regular school lecture and the students learn things through their experience. The learning is done all together and the notions are linked by the same theme. One can consider the notion of velocity: the pupils have an experience of it during sports hours, but one can ask how the speed can be measured or maybe first what is a measurement. It can be asked to the students to measure some distances and then the time intervals the need to travel it. Putting the data in a chart and dividing the two quantities, one can obtain the average velocity. The measurements are physics activities, but the operation is a mathematical one.

Professor Rollinde thinks that "*scientific subjects are intrinsically interdisciplinary*", but to show interdisciplinarity it is important that every teacher looks at the theme with their own approach, asking to look at the disciplinary questions. One should choose one theme and develop it properly to deepen the different issues. Another possibility is to leave at each student the choice of an own study subject and to lead them during the interdisciplinary analysis. They will probably show more interest because they expressed their preference for the study topic.

Although Mr Rollinde does not know exactly the new reform, he thinks that one should develop the same study theme all year long. Doing so, one can deepen the subject and gain a good framework for the topic. Otherwise, it is difficult to show the links between the disciplines. The subject of science education has a too vast syllabus and there is not enough time to develop all the seven topics. In this case, the interdisciplinarity cannot emerge.

In the end, Professor Emmanuel Rollinde wanted to pose an open and provocative question: "*Is there an evaluation of the interdisciplinarity at the school?*" In his opinion, although many words were spent over the interdisciplinarity and interdisciplinary teaching during many conferences, there are not enough studies that reveal the impact of such teaching. "*It is still not clear if the students can establish links between different subjects and how they assimilate the interdisciplinary teaching they receive*." In his opinion, some more studies should be done on the effects that the school interdisciplinarity produces.

3.2. INTERVIEWS: ANALYSIS

The analysis of the interviews is here carried out and discussed according to the three main aspects of my research:

i. the definition of interdisciplinarity,

ii. its didactic implementation and

iii. an evaluation of the present situation in the French high schools.

Although the six interviews are not enough to do a detailed inquiry with a statistical value, which however was not the goal of this thesis, some qualitative results can be obtained and are going to be illustrated below.

The scientific background of the interviewees was varied, and different sensitivities did emerge. This shows the great variability of approaches to interdisciplinarity in France. Some researchers put much effort into the structure of the knowledge of their disciplines and their epistemology, others on the importance of the use of correct language for the transposition of knowledge. The importance of various possible points of view and the mutual contamination among the disciplines of scientific concepts and methods were also stressed.

In order to address the first aspect and compare the different views of interdisciplinarity emerged from the interviews, Thompson Klein's taxonomy is used as inspirational tool.

As every kind of taxonomy, its application on a data corpus requires a deep analysis of both the categories and the data in a mutual-interaction. The aim is, indeed, not to evaluate or test taxonomy's completeness or efficiency, but to use it as a theoretical lens to explore and recognise the nuances present in the data.

In order to reach this goal, a debriefing activity has been organized within the research group in didactics of physics of the University of Bologna during a course called *Advanced Professional and Research Skills in Physical Sciences*. This new course is held in the second semester at the final year of the master program in Physics at the University of Bologna. The participants that attend the course are students at the end of their educational path and they are preparing to write their master theses in didactics of physics in one of the research groups at the Department of Physics and Astronomy.

The debriefing activity lasted about 1 hour and half. After an initial presentation of Thompson Klein's taxonomic system, the participants, divided in two groups were asked to analyse the excerpts (reported in Table 3.2.) about the definition of interdisciplinarity, taken from the interviews.

Table 3.2. Worksheet

Definition of interdisciplinarity	**Interviewee**
It is something that concerns many disciplines, it appears when a problem cannot be solved by a single discipline and one must use knowledge of different disciplines in mutual interaction." It is important that the disciplines interact on the same level because otherwise there is no real interdisciplinarity.	Emmanuel Volte
Interdisciplinarity is a co-construction of common meaning (…) The first step is building a group having a common language. When different disciplinary aspects of common concepts, definitions and themes are shared, one should look at the general meaning which is a synthesis of different sensitivities and approaches. After that some activities can be done together.	Rita Khanfour-Armalé

One can talk about interdisciplinarity when there is a sliding of two different disciplinary fields in the context of a common educational situation. One should look at how different actors or different disciplines interact in this situation. "Interdisciplinarity is always situated and linked to a concrete problem". The key ingredient is not to have the supremacy of one discipline over the other.	Maha Abboud
It is a powerful tool that allows us to analyse from different points of view or different angels a problem to reach a final reformulation. The interdisciplinarity is "an act of intellectual freedom that allows borrowing tools of other disciplines to solve a problem."	Nathan Lombard
It is a form of collaboration of disciplines in order to better understand the world. "Co-construction of a common point of view […] the co-acting can bring at the end to an integration process". It is a sort of reconstruction of knowledge.	Thomas Hausberger
When one has an object of study that can be a natural phenomenon or a research question, one can apply different vision frameworks. There are two different ways to do that; either one can analyse the object of study from each disciplinary framework separately or one can do an activity mixing the different disciplines at the same time.	Emmanuel Rollinde

In particular, they were asked to match the theoretical categories with the data. That would have implied both to use the Thomson Klein words to read the data and the date to interpret the theoretical categories. As result, sets of keywords have been pointed out to reflect on the data. After a valuable discussion, it emerged that the groups, taking into account different elements, came to contrasting interpretations of the Thomson Klein categories. For example, one problematic aspect concerned the interpretation of the instrumental interdisciplinarity because of the ambiguity of the word "instrument": some of them interpreted the word broadly as "conceptual tool" others as technological equipment, like a particles accelerator used in medicine. Such ambiguity led the instrumental and methodological interdisciplinary to be confused. Another ambiguity emerged about the distinction between methodological and theoretical interdisciplinarity, since they were not able to find out a demarcation criterion.

On the positive level, it emerged a consensus to recognise critical interdisciplinarity on the basis of the output of the process of inter-action between disciplines: whilst theoretical interdisciplinarity is said to lead to "new synthesis funded on the connection between models and analogies", critical interdisciplinarity leads to a reconstruction of the foundations of one or both the disciplines.

This debrief led, hence, to point out the following criteria to compare and distinguish among the various forms of interdisciplinarity:

a) the role and the relation between the different disciplines: in Thomson Klein's taxonomy, methodological interdisciplinarity seems to assume that one discipline in the locus of the problem formulation and the other the source of new methods; in all the other forms of interdisciplinarity seem to put the disciplines on a more comparable level; a key distinction is on the level of attention to preserve and mark single disciplinary identity or their "con-fusion" in common areas.
b) the type of interaction between the disciplines: in Thomson Klein's taxonomy, methodological interdisciplinarity seems to assume that one discipline has a sort of instrumental role; in the others the relationships are more of mutual interaction (co-construction, collaboration, co-action,…);
c) the type of output that results from interdisciplinarity: in Thomson Klein's taxonomy, methodological interdisciplinarity seems to lead to a new solution of a disciplinary problem, that is a problem that is formulated within one discipline and whose solution remains in that discipline; theoretical interdisciplinarity seems to lead to a new synthesis that belong to both the disciplines or to a common field; instrumental interdisciplinarity seems to lead, often, also to a new research field; critical interdisciplinarity seems to lead to foundational transformation of one or both the disciplines and an epistemological change.

The solution of such ambiguities led the taxonomy more effective to read and interpret the data. The results of the analysis are presented in Table 3.2.

Table 3.2. Analysis' results of the definitions of interdisciplinarity

Definition of interdisciplinarity	Analysis	Result:
Emmanuel Volte: It is something that concerns many disciplines, it appears when a problem cannot be solved by a	**Role of the discipline:** focus of the weaknesses of single disciplines and on the need to have them on the same	**All but methodological interdisciplinarity** The focus of the mutual interaction between

single discipline and one must use knowledge of different disciplines in mutual interaction." It is important that the disciplines interact on the same level because otherwise there is no real interdisciplinarity.	level (overcoming their borders) in mutual interaction **Relationship between the disciplines:** mutual interaction needed to solve an "common problem" that cannot be solved in a discipline **Output:** Not described	disciplines at the same level leads to exclude a methodological view. The lack of output does not allow to discriminate between the other kinds of interdisciplinarity.
<u>**Rita Khanfour-Armalé**</u> Interdisciplinarity is a co-construction of common meaning (…) The first step is building a group having a common language. When different disciplinary aspects of common concepts, definitions and themes are shared, one should look at the general meaning which is a synthesis of different sensitivities and approaches. After that some activities can be done together.	**Role of the discipline:** disciplines are on the same level in the process of co-construction of common meaning **Relationship between the disciplines:** Co-construction implies the share of common concepts, definitions and themes **Output:** A common meaning that is the synthesis of different sensitivities and approaches	**Theoretical interdisciplinarity** The focus of the mutual interaction between disciplines at the same level leads to exclude a methodological view. Moreover, the synthesis as output leads to read this definition as a typical example of theoretical interdisciplinarity.
<u>**Maha Abboud**</u> One can talk about interdisciplinarity when there is a sliding of two different disciplinary fields in the context of a common educational situation. One should look at how different actors or different disciplines interact in this situation.	**Role of the discipline:** Disciplines are treated as different fields that act as different lenses on the same problem, that necessary is formulates in a (school) context. Disciplines are different but act at the same level. **Relationship between the disciplines:**	**School interdisciplinarity** Disciplines interact but not in a mutual sense, since they are supposed to have their own strong identities that is not affected by the interaction. The emphasis of the context and on the educational

"Interdisciplinarity is always situated and linked to a concrete problem". The key ingredient is not to have the supremacy of one discipline over the other.	Interaction (non-mutual) to address a concrete problems from different lenses **Output:** Not described	relevance of the multiple perspectives lead to read this view as "school interdisciplinrity" which does not appear explicitly in the Thompson Klein taxonomy, but it was defined by Lenoir and Sauvé.
Nathan Lombard It is a powerful tool that allows us to analyse from different points of view or different angels a problem to reach a final reformulation. The interdisciplinarity is "an act of intellectual freedom that allows borrowing tools of other disciplines to solve a problem."	**Role of the discipline:** The evocative expression "interdisciplinarity as an act of intellectual freedom" lead us to think that one discipline represents the basis for the problem formulation, whilst the other is a "freedom field" to borrow a tool that problem. **Relationship between the disciplines:** There is an asymmetry between the two disciplines because of the different roles. **Output:** A new formulation of a problem/theme within one discipline.	**Methodological interdisciplinary** The asymmetric role of the two disciplines, as well as the type of output lead to see this definition as expression of methodological interdisciplinary.
Thomas Hausberger It is a form of collaboration of disciplines in order to better understand the world. "Co-construction of a common point of view […] the co-acting can	**Role of the discipline:** Disciplines identities are supposed to be fundamental for inter-disciplinarity. **Relationship between the disciplines:**	**Critical interdisciplinarity** (CI) The emphasis both on the integration process and on the type of output (re-construction) leads to

bring at the end to an integration process". It is a sort of reconstruction of knowledge.	Disciplines co-construct a common point of view. Interdisciplinarity is important when disciplines co-act to re-construct knowledge **Output:** Knowledge re-costruction	see this definition as expression of critical interdisciplinarity. The term "reconstruction" indeed seems to question the dominant structure of knowledge.
<u>Emmanuel Rollinde</u> When one has an object of study that can be a natural phenomenon or a research question, one can apply different vision frameworks. There are two different ways to do that; either one can analyse the object of study from each disciplinary framework separately or one can do an activity mixing the different disciplines at the same time	**Role of the discipline:** Disciplines are treated as different frameworks that act, with their own methods and views, to address the same object of study. Disciplines are different but act at the same level. **Relationship between the disciplines:** Interaction (non-mutual) to address a concrete problems from different frameworks **Output:** Not described	**All but methodological interdisciplinarity** The focus of the mutual interaction between disciplines at the same level leads to exclude a methodological view. The lack of output does not allow to discriminate between the other kinds of interdisciplinarity.

As for the second aspect, related to classroom implementation of interdisciplinarity teaching, all the interviewees seem to agree that interdisciplinary skills can be developed in a classroom with an analysis of examples of the history of science. More in general, all of them agree with the importance of interdisciplinary teaching, which, however, must be adapted to different situations. One of the points that emerge in quite all interviews is the need for collaboration of different disciplinary teachers. A significant interdisciplinary approach can be set up only in the comparison between experts of different domains. A single disciplinary teacher can also establish some links with other disciplines, but the effectiveness of the teaching will not be so good, as in the case in which the students have the opportunity to see approaches, methods and reasoning coming from various domains. The epistemology of every single discipline can only be illustrated by its experts. Therefore, the collaboration of people having different backgrounds is so enriching.

Some interviews show a critical opinion towards the new reform of the French government *BAC 2021*, especially about interdisciplinarity. Although some interviewees identify positive elements in the introduction of the new subject ‘science education’ (*enseignement scientifique*), the general opinion is that there are not enough means for teachers to develop interdisciplinarity and there is a concrete risk of remaining at a multidisciplinary level, with the juxtaposition of various notions of different disciplines. No deeper links are going to be established.

Some respondents stressed the importance of giving teachers enough time for dialogue and mutual confrontation before entering the classroom. A too rigid syllabus and a too detailed syllabus seem not to give the teachers enough space for creativity and freedom of thought to set up a truly interdisciplinary discourse.

A fact that has emerged is that in the French education system, the STEM topics seem not to be so present as in other countries. Introducing the STEM-related problems in the teaching should be supported by an adequate university teaching research. In addition, a way to assess the learning effectiveness of interdisciplinary activities should be introduced.

In the last chapter, an educational path analysing an example take of history of science will be developed, highlighting the analysis of the interdisciplinarity between mathematics and physics in a theme proposed in the second-grade syllabus of the school subject “scientific education”.

4. INTRODUCTION TO THE VIBRATING STRINGS CONTROVERSY

Although the vibrating strings controversy is rather unknown to most physicists, it is a clue episode in the history of science. This theme is just mentioned in the syllabus of the subject of scientific education and the syllabus does not explain how to deal with the topic. The goal of this part of the thesis is to face up with the problem of vibrating strings, to analyse its historical development and to show why and how it can represent a great opportunity to discover how interdisciplinarity acts.

To understand and appreciate the importance of the chosen case study, the chapter will be divided into five different parts. The first one will present the solution of a vibrating system, deriving the wave equation with a modern treatment as we got used to seeing it in an undergraduate course of mechanics. In the second part, the protagonists of the story will be introduced; short biographies will present the lives of four of the greatest scientists of the 18th century: Bernoulli, Euler, d'Alembert, and Lagrange. It is important to know the main contenders in the debate to fully appreciate the development of the controversy. The third part concerns the history of the debate and the protagonists' different positions will be analysed. The fourth part will be focused on the narrative of the debate, showing the emergence of interdisciplinarity and, finally, the fifth part will give some hints for a didactic implementation in a high school classes, during the university lectures or in teachers' training.

4.1. THE MODERN TREATMENT OF A VIBRATING STRING

The problem of a vibrating string could be encountered in high school physics classes, although its complete derivation is not given. The teachers usually show that a solution of the vibrating string movement is a sinusoidal function dependent on two variable: the position and the time.

Physics or mathematics students at the university can face up with a problem of vibrating string in a course of Mechanics or Analytical Mechanics. The problem is treated using the partial differential equations. Here it will be presented a possible treatment extrapolated from the 4th edition of the well-known book *Physics* by Halliday, Resnick and Krane (1991).

I show this approach to argue how it can be addressed as a simple exercise, whilst, as I will argue in the next of the chapters, it represents a very emblematic case of interdisciplinarity.

The goal of the problem is to find the equation of motion of a string lying in the (x, y) plane and oriented along the x axis. The string is perturbated with a transversal amplitude $y(x, t)$. One can denote the linear mass of the string with μ and its length with l. The extremities of the string in $y(0, t)$ and $y(l, t)$ are fixed and can be expressed as $y(0, t) = y(l, t) = 0$. They are the boundary conditions.

We will neglect the weight force and all other forces but the tension, called $\vec{T}$. Let us now consider an infinitesimal element δx of the string that is displaced from its equilibrium position. This element is under tension with the same magnitude from both sides but slightly different directions. We can denote the angle α between the horizontal axis and the direction of the tension on one left side and β on the right side of the element (see the Figure 4.1.).

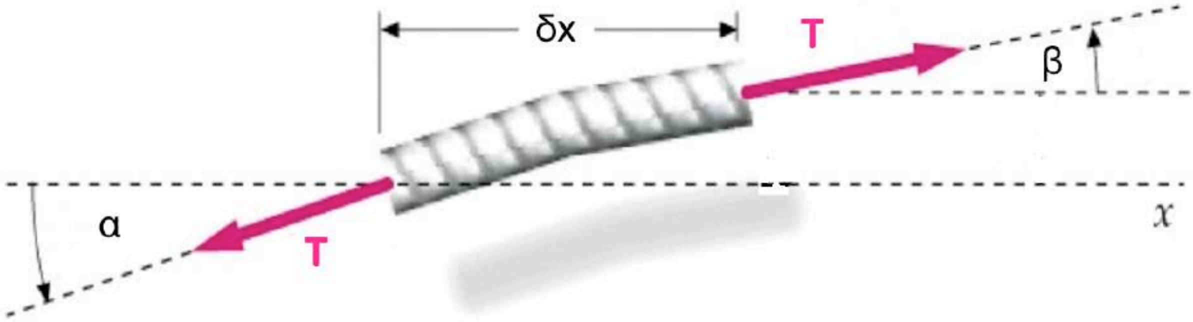


Figure 4.1. Representation of the infinitesimal element of the string

We apply the Second Principle of the Dynamics on the infinitesimal element δx with the mass $\delta m = \mu \delta x$. Since the perturbation is on the y axis, the net force in this direction is:

$$T_y = T \sin\beta - T \sin\alpha$$

There is no movement of the string element along the horizontal axis. Considering very small displacements from the equilibrium, the two angles α and β are small and therefore the approximation for small angles could be applied: $\alpha \approx \sin\alpha \approx tan(\alpha) = \frac{\partial y}{\partial x}$ and the same for β. And therefore:

$$T_y = T \tan\beta - T \tan\alpha = T\delta(\tan\theta)$$

with $\delta(\tan\theta) = \tan\beta - \tan\alpha$. Writing the acceleration on the y axis as a_y, one has:

$$T_y = a_y \, \delta m = a_y \, \mu \, \delta x$$

$$T \, \delta(\tan\theta) = a_y \, \mu \, \delta x$$

$$\frac{\delta(\tan\theta)}{\delta x} = \frac{\mu}{T} a_y$$

The transverse acceleration can be also written as $a_y = \frac{\partial^2 y}{\partial t^2}$ and the slope of the string as $\tan\theta = \frac{\partial y}{\partial x}$.

doing the limit for $\delta x \to 0$ of $\frac{\delta\left(\frac{\partial y}{\partial x}\right)}{\delta x} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}$, we obtain:

$$\boldsymbol{\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}}$$

or well-known general form of the wave equation:

$$\frac{\partial^2 y}{\partial t^2} = \mathbf{c}^2 \frac{\partial^2 y}{\partial x^2} \quad \text{with the velocity } c = \sqrt{\frac{T}{\mu}}.$$

At this point let us look at the possible solutions of the wave equation. Calling $u = x + ct$ and $v = x - ct$, one should notice that $y(x,t) = f(u)$ is the shape of the wave moving to the left with the constant velocity c, and $y(x,t) = f(v)$ is the shape of the wave moving to the right with the same constant velocity c. There is nothing more than translations in the time of the wave displacement amplitude y. The general formula for the traveling wave is $y(x,t) = f(u) + f(v)$. This fact is verified for a wave moving to left, but can be also easily verified for a wave moving to right:

$$\frac{\partial y}{\partial x} = \frac{df}{du}\frac{\partial u}{\partial x} = \frac{df}{du}, \quad \frac{\partial^2 y}{\partial x^2} = \frac{d}{du}\left(\frac{df}{du}\right)\frac{\partial u}{\partial x} = \frac{d^2 f}{du^2}; \quad \frac{\partial y}{\partial t} = \frac{df}{du}\frac{\partial u}{\partial t} = +c\frac{df}{du}; \quad \frac{\partial^2 y}{\partial t^2} = \frac{d}{du}(+c\frac{\partial u}{\partial t}) = +c^2\frac{d^2 f}{du^2}.$$

And finally: $\frac{d^2 f}{du^2} = \frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$. The solution to the problem of a vibrating string with this treatment is nothing more than a physics exercise that can be solved with some knowledge of calculus.

4.2. The Biographies of the Protagonists

The vibrating strings controversy is an interesting chapter of history of the science, although it remains unknown to most scientists. It seems useful at this point, before entering in a discussion of the controversy, to look briefly at the biographies of the protagonists. Knowing better the main actors in the debate will allow us to understand their epistemological positions. The main source for this part was the online *MacTutor History of Mathematics Archive* of the University of St Andrews, curated by Professors John Joseph O'Connor and Edmund Frederick Robertson that can be found at the link: https://mathshistory.st-andrews.ac.uk/Biographies/.

4.2.1. Daniel Bernoulli (1700-1782)

Daniel Bernoulli was born on 8th February 1700 in Groningen (Netherlands) in a family of excellent mathematicians of the time. His father was Johann/Jean Bernoulli, the well-known Euler's professor. His uncle Jacob/Jacques Bernoulli and his older brother Nicolaus II Bernoulli were two affirmed mathematicians and this contributed to a rivalry among family members. After Jacob died in 1705, the family of Johann Bernoulli returned to Basel, where he had taken the place at the university left vacant by his brother.

Daniel Bernoulli started very young to study philosophy and logic at the Basel University in 1713; in 1715 he obtained his baccalaureate and a year after his masters'. During his philosophy studies, Daniel discovered the methods of calculus from his older brother and his father. Johann had tried to force Daniel into a business career, but he refused, starting to study medicine in Basel, Heidelberg, and Strasbourg. In 1720, he completed his doctorate at Basel University, writing a dissertation on the mechanics of breathing. In fact, during his medical studies, he did not lose his interest in mathematics and with his father studied the theory of kinetic energy and used the principle of conservation of energy in his medical studies.

After he had tried without success to obtain a chair at Basel University, he went to Italy to study practical medicine. In Venice, Daniel Bernoulli fell seriously ill and continued to study mathematics, publishing with Christian Goldbach's help his first mathematical work in 1724. *Exercitationes* (Mathematical exercises) was a book divided into four parts. The first one described with some notions of probability the game of faro, the second part concerned the flow of water from a hole in a container with an incorrect discussion of Newton's theory, the third the Jacopo Riccati's differential equation and the last part some geometrical figures bounded by two arcs of a circle. His medical studies on the pressure and flow of the blood brought him to study hydrodynamics. In 1725, he moved from Italy to Switzerland and he won a price of *Académie royale des sciences* of Paris with a work on an hourglass that could be used on the ships rolling in heavy seas.

Daniel moved to Saint Petersburg with his brother Nicolaus II where them it was offered a chair of mathematics, but unfortunately, his brother died of fever some months after the arrival. Johann Bernoulli sent Leonhard Euler to St Petersburg in 1727 to work with Daniel. The next six years were for both the most productive time.

Daniel Bernoulli started studying vibrating systems and elastic bodies. One of his most known results is the discovery of simple nodes and the oscillation frequencies of a system. He studied the movements of musical instruments strings, showing the superposition of an infinite number of harmonic vibrations. The study had an empirical nature.

The most important work in his period of stay in Saint Petersburg was *Hydrodynamica*, a study on fluid dynamic, the first draft was finished in 1734, but he continued working on it also between 1734 and 1738. The correct analysis of water flowing from a hole in a container was presented on the principle of energy conservation, studied with his father in Basel during his medicine studies in 1720. An important study is also the kinetic theory of gases that contained the main ideas of the equation of state derived exactly only by Johannes Diderik van der Waals in his article *Over de continuiteit van den gas - en vloeistoftoestand* in 1873 and for which he gained the Nobel Prize in 1910.

But in Saint Petersburg Daniel Bernoulli was not limited to physical problems. He published an article on probability and political economy, applying his deductions to insurance.

Despite the possibility to work with Leonhard Euler, Daniel Bernoulli was looking for a post in Basel and although neither a post in mathematics nor in physics was available, Daniel preferred giving lectures in botany rather than remaining in St Petersburgh. He left Russia in 1733 and after a journey through Europe returned to Basel in 1734. Daniel Bernoulli gave an application of his ideas to astronomy in order to entry for the Grand Prize of Paris Academy for the year 1734, but the same did his father. Both were declared winners therefore his father broke the relationship with him.

After he departed from Saint Petersburg, Daniel Bernoulli maintained a correspondence with Euler on vibrating systems. The collaboration was very fruitful, indeed, Daniel showed a great physical intuition, whereas Leonhard was able to put Bernoulli's ideas in a rigorous mathematical form. Daniel Bernoulli continued working on *Hydrodynamica*, adding another chapter on the force of the water on an inclined plane and in the force of reaction of a jet of fluid, applying the results to the propulsion of ships. In 1737, Bernoulli won the Paris Academy prize jointly with Giovanni Poleni presenting the best shape for a ship's anchor. During his life, Daniel Bernoulli was able to win 10 times the Grand Prize for nautical and astronomical topics: in 1740, jointly with Euler, for work on Newton's theory of the tides, in 1747 presenting a method for a method to determine the time at sea, in 1751 for an essay on ocean currents, in 1753 presenting the effects of forces on ships and in 1757 for proposals to reduce the pitching and tossing of a ship in high seas. In the years 1743 and 1746, he presented essays on magnetism.

In 1738, *Hydrodynamica* was ready to be published and a year after his father published *Hydraulica*, pretending to demonstrate that his book was based on a work in 1732. In doing so Johann Bernoulli tried to discredit his son Daniel.

In 1743, Daniel Bernoulli started to teach physiology and in 1750 he finally gained the chair of physics in Basel and he held it until 1776. His teaching was based on experiments. Bernoulli was able to anticipate some properties which were verified by certain laws only some years later, for example, Charles Augustin Coulomb's law in electrostatics.

Although Daniel Bernoulli abandoned a rigorous study of pure mathematics, his contribution is important for the mathematization of physics during the 18th century. He maintained an experimental position, but he combined Newton's theories with tolls deriving from Leibniz's calculus. His principle of conservation of energy was gained integrating Newton's basic equations.

Daniel Bernoulli was a great physicist in 18th century and this brought him to be a member of the most important scientific societies of his epoch. It is worth mentioning Bologna, Turin, Paris, London, Berlin, St Petersburg, Bern, Zürich, and Mannheim. Daniel died in Basel on 17th March 1782.

4.2.2. LEONHARD EULER (1707-1783)

Leonhard Euler was born on 15th April 1707 in Basel in Switzerland, although he spent his childhood in Riehen, not very far from Basel. His father was Paul Euler, a Protestant minister, and his mother, Margaret Brucker, was a daughter of a Protestant minister. Paul was one of Jacob Bernoulli's students and he was a Johann Bernoulli's friend. Leonhard Euler had, therefore, an opportunity to learn simple mathematics at home. Despite this fact, Leonhard had a poor mathematics teaching at the school and he was more a self-taught, reading alone mathematics texts.

At the age of 14, Leonhard as his father started preparing himself to become a Protestant minister at the Basel university and he had the same mathematics professor, Johann Bernoulli who discovered his great talent. After he had gotten his degree in philosophy with a work on Newton's and Descartes' ideas in 1723, he started studying theology, but soon changed to mathematics. Leonhard Euler concluded his studies in 1726, publishing a paper on isochronous curves in a resisting medium. A year after he wrote an article on reciprocal trajectories.

After the Nicolaus II Bernoulli's death in the summer of 1726, Johann Bernoulli proposed to Leonhard Euler to take his place in Saint Petersburg. Euler accepted the place in November, but he travelled to Russia the next spring. Meanwhile, he published an article on acoustics, trying to gain visibility in Basel for a chair in physics at the university. Peter the Great's wife, Catherine I, founded in 1725 the Saint Petersburg Academy of Sciences. Although Leonhard Euler joined the new institution in the mathematical-physical division, he served for three years as a medical lieutenant the Russian navy. In 1730, he became a professor of physics at the Academy and with this position also a full member of the institution.

The stay in Russia was for Euler very productive. He collaborated profitably with Daniel Bernoulli who had the chair of mathematics. He studied many different themes as magnetism, cartography and shipbuilding, machines, and fire engines, but also other topics more related to mathematics such as number theory, the new emerging analysis, calculus of variations, differential equations, etc. After Bernoulli's departure to Basel in 1733, Euler could take his mathematics chair at the Academy. In January of 1734, Euler married a daughter of a Swiss painter, Katharina Gsell, and they had 13 children, but only five reached their adulthood. In 1735, Euler began to suffer health problems after a severe fever when he almost lost his wife Katharina. In 1738, he started losing the sight and by the year 1740, he had completely lost one eye.

Leonhard Euler continued publishing many articles and a book on Newtonian dynamics based on mathematical analysis *Mechanica* (1736-37), which was a great contribution to the birth and development of rational mechanics. He won the Grand Prize of the Paris *Académie des sciences* in 1738 and 1740. He received an offer of the court of Frederick the Great and after a period of reflection, he decided to move to Prussia. Even after he arrived in Berlin in 1741, Euler continued to collaborate with the Russian Academy, writing some scientific reports and educating young Russians. This permitted him to maintain part of his salary from Russia.

The Berlin Academy of Science was founded in 1744 with Pierre-Louis Moreau de Maupertuis as its president and Leonhard Euler as director of the mathematics department. Euler had some duties at the Academy and some others at the Frederick's the Great court as an advisor for insurance, pensions and annuities, lotteries, and artillery. Euler spent 25 years in Berlin publishing around 380 articles and writing books on the calculus of variations, analysis, astronomy, ballistics, navigation, and shipbuilding.

After Maupertuis died in 1759, he assumed the responsibility of leading the Berlin Academy, but he had never received a nominee for President. The king did not appreciate Euler as during the first years at his court. Leonhard Euler was disappointed when Frederick offered in 1763 the presidency of the Academy to Jean le Rond d'Alembert because he had argued heavily on scientific matters with the French mathematician in those years. However, d'Alembert turned down a few times the King's offer, and after the Euler departure to the court of Catherine II of Russia in 1766, Joseph-Louis Lagrange succeeded him as Director of the mathematics department.

After an illness, Euler became almost blind losing the eyesight at the remaining eye. After a fire that completely destroyed his home in 1771, he became totally blind, but this fact did not prevent him to continue his work on algebra, lunar motion and even optics thanks to his incredible memory. Despite his blindness, he was able to publish almost half of his total life work with the help of his collaborators as his sons Johann Albrecht Euler, a professor of physics at the Academy, and Christoph Euler who worked in the Russian army and other members of the Academy as W.L. Krafft, the mathematicians Anders Johan Lexell and the Swiss Nicolaus Fuss.

Leonhard Euler died on 18th September 1783 in St Petersburg, when he was 76 years old, but the Saint Petersburg Academy continued to publish his unpublished works some decades after his death.

Leonhard Euler was one of the greatest mathematicians of the 18th century and certainly the most prolific of all the times. It is difficult to summarise his immense contribution to mathematics. He continued and developed the work done before by Newton and Leibniz in infinitesimal analysis, by Cardan and Bombelli in complex numbers, by Fermat in arithmetic and number theory, establishing new links between the fields.

Euler defined the beta and the gamma function first in 1729. In 1735 he calculated the limit of the series $\lim_{n\to\infty}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \ln(n)\right]$ and, in 1737, he demonstrated the connection between the function zeta with the series of prime numbers giving the famous relation: $\zeta(s) =$

$\sum(\frac{1}{n^s}) = \prod(1 - p^{-s})^{-1}$. In 1740, he began the study of the calculus of variation, publishing his book *Methodus inveniendi lineas curvas*. Euler studied linear differential equations with constant coefficients and their solutions.

In 1748, in his *Introductio in analysin infinitorum* Euler defined a function and claimed that mathematical analysis was the study of functions rather than od geometrical curves. In the same book were published the most known Euler's formulas $e^{i} + 1 = 0$ and $e^{ix} = \cos x + i \sin x$. In 1751, Euler had published his full theory of logarithms of complex numbers. He began a study of the calculus of finite differences, publishing in 1755 *Institutiones calculi differentialis*. In *Institutiones calculi integralis* (1768-70) he studied and calculated the previously introduced gamma function or Eulerian integral, generalising the factorial for no integer values.

Euler also proofed the Fermat's Last Theorem for the case of $n = 3$. He was interested in the analytical and differential geometry, trigonometry, and can be considered as the founder of topology with his study of characteristics of a polyhedron. Euler contributions to differential geometry included the theory of surfaces and curvature of surfaces; some of his results, that he did not publish during his lifetime, were rediscovered independently by Johann Carl Friedrich Gauss. During his study of vibrating membranes, Euler found some solution known nowadays as Bessel solution.

Euler also wrote many papers on artillery, cartography, music, mechanics, elasticity, three-body problem, acoustics, wave light theory, hydraulics, astronomy. Euler invented his theory of music and tried to reconducted it to mathematics, publishing his work *Tentamen novae theoriae musicae* in 1739. With his work *Theoria motus corporum solidorum seu rigidorum* (1765) the foundation for rational mechanics was laid.

Leonhard Euler invented some of notations that are still in use nowadays: in 1727, e for the base of natural logarithms, in 1734, $f(x)$ for a function f of the variable x , π for the irrational value pi, in 1755, Σ for the summation, in the infinitesimal analysis Δy and $\Delta^2 y$ for finite differences, in 1777, i for the square root -1, and many others.

4.2.3. JEAN LE ROND D'ALEMBERT (1717-1783)

D'Alembert was the illegitimate son of an artillery officer Louis-Camus Destouches and Madame de Tencin, who was previously a nun, until the papal dispensation in 1714. He was born on 17th November 1717 in Paris. Jean d'Alembert was found as a newly born child on the steps of the church of St Jean Le Rond and he was therefore baptised with the same name.

Although his parents did not recognise him officially, Louis-Camus gave the baby in care to Madame Rousseau, who became in d'Alembert's eyes his mother. He arranged for the illegitimate son his education in a private school. After his death in 1726, the Destouches family continued to look after Jean's education, financing his studies in the Jansenist *Collège des Quatre Nations*. Jean changed his surname from Daremberg to d'Alembert. In this school, Jean le Rond d'Alembert started to be interested in mathematics visiting the excellent school library and discovered Descartes' physics ideas, against which he would fight his enter life. Although the Collège aimed to form scholars, experts in Jansenist theology, after his graduation in 1735, d'Alembert decided to study law and to continue working on mathematics in his spare time.

In 1738, Jean Le Rond d'Alembert became an advocate, but he left his career and started studying medicine. He did not demonstrate a real interest in the subject, his real enthusiasm remained mathematics which he had studied mainly on his own. The first paper read to the *Académie royale des sciences* was on some errors found in Charles René Reyneau's *Analyse démontrée*. His second paper on the mechanics of fluids was more successful and was praised by Alexis Clairaut. In May 1741, d'Alambert was admitted to the *Académie royale des sciences* at the age of 24 years. He was working at the Academy his entire life, without moving a lot.

D'Alembert was often involved in controversies and he hardly admitted that he might have been wrong. But his stubbornness and determination in arguing had helped him to give a fundamental contribution to a solution of some of the controversies of the time. The competition with the mathematician Alexis Clairaut pushed him to seek a solution to the problem of conservation of kinetic energy. An improvement of Newton's definition of force was published in his *Traité de dynamique*, in 1743. This work contains also his principles of mechanics with which he tried to reconduct the so-called rational mechanics into domains of mathematics. In his opinion, the solutions to the problems should be sought by purely mathematical methods. A reason for his beliefs could be found in his philosophy; d'Alembert was convinced that mechanics was not based on experimental evidence but rather on metaphysical principles. The laws of motion should have been logical necessities. A year later in 1744, d'Alambert published his results in fluid dynamics *Traité de l'équilibre et du mouvement des fluides*, entering in conflict with one of the greatest experimentalists of that time, Daniel Bernoulli.

D'Alembert was a pioneer in the study of partial differential equations and their use in physics. He won a prize of Prussian Academy with an article published in 1747 on the generation of wind: *Réflexions sur la cause générale des vents*. Although this work was mathematically outstanding, it suffered from poor connections with physical evidence. For instance, the heating of the atmosphere played a minor role. Clairaut attacked heavily d'Alembert's methods. Despite this fact, D'Alembert was elected member of Berlin academy of science (Hug & Steiner, 2015) and he started a correspondence with Euler on various subjects. In the same year, his article on vibrating strings was published, which contained the wave equation, but again its results were at odds with physical observation. The content of this article will be developed in a more comprehensively in the next chapters of the thesis.

Another dispute over the Principle of Least Action took place at the Berlin Academy in 1751, involving Jean le Rond d'Alembert, his friend Pierre-Louis Maupertuis, Leonhard Euler and Samuel König. When d'Alembert was invited to become the President of the Berlin Academy of Science in 1752, the relations between him and Euler degenerated. Euler was accused by d'Alembert of stealing his ideas on vibrating strings without quoting him. Euler stopped the d'Alembert's publication of mathematical articles in the Berlin journal; those articles would be published by d'Alembert in his *Opuscules mathématiques* in eight different volumes between the years 1761 and 1780.

Despite the opposition of Euler, the Prussian King Frederick II tried one again to persuade d'Alembert to become the President of the Berlin Academy, but d'Alambert, after a three-months visit of the Academy in 1764, declined the offer and suggested Euler as president. D'Alembert had declined also the invitation from Catherine II to become a tutor for her son.

Around 1746, d'Alembert was involved by Diderot in editing their *Encyclopédie* for the domains of mathematics and physical astronomy, but his contributions spread over more fields. The Preface of the first volume published in 1751 had been written by d'Alambert and it became a great success. He wrote most of the mathematical articles contained in 28 volumes. In an article of the 4th volume with the title *Différentiel* he perfected the theory of limits, he highlighted the importance of functions and he defined the derivative of a function as the limit of a quotient of increments.

D'Alembert wrote also some articles on literature and philosophy, published in *Mélanges de littérature et de philosophie* between the 1753 and 1767, claiming his skepticism concerning metaphysical problems. The d'Alembert's election as a member of French Academy took place on 28th November 1754, whereas in 1772 he became its perpetual secretary.

His health conditions in the last years were very bad, so he abandoned the study of mathematics in 1765. He died of a bladder illness on 29th October 1783 in Paris. Jean le Rond d'Alembert was at that time considered as an unbeliever, therefore he buried in a common unmarked grave.

4.2.4. Joseph-Louis Lagrange (1736-1813)

Joseph-Louis Lagrange, known also under the Italian name Giuseppe Lodovico Lagrangia, was born in Turin on 25th January 1736 in Sardinia-Piedmont kingdom. His father Giuseppe Francesco Lodovico Lagrangia was a high officer, a Treasurer of Public Works, and Fortifications in Turin. His mother, Teresa Grosso, was a daughter of a doctor in Cambiano. The couple had 11 sons and daughters, but only the first-born Joseph-Louis and another one survived till adulthood.

Joseph-Loui Lagrange did a classical high school and he preferred Latin to mathematics. His father wanted him to become a lawyer. After having read Halley's book on the use of algebra in optics and discovered physics, he decided to become a mathematician. He did not have contacts with the great mathematicians of that time and was self-taught. His first work in 1754 was not what we could call a masterpiece, it was a paper written in Italian that he signed as Luigi De la Grange Tournier.

Lagrange started to work on a curve called *tautochrone*, which has a property that every weighted particle arrives at a fixed point at the same time independently of its initial position. This study brought him to discover some important properties of the calculus of variations, a new field of study in mathematics at that time. In August 1755, Lagrange sent his discoveries using the method of maxima and minima in Berlin to Leonhard Euler who was impressed by Lagrange's ideas. At the age of 19, Lagrange became a professor of mathematics at the *École royale d'Artillerie* of Turin in September 1755. He demonstrated the Wilson's and de Bachet's theorem and he founded the theory of quadratic forms, publishing his results in *Recherches arithmétiques* (1775).

A year later, Lagrange sent Euler his results of the application of the calculus of variations to mechanics and Euler was again impressed by his young friend. The Lagrange's results were a more general form of Euler's, therefore he decided to show them to the president of Berlin Academy, Pierre Louis Moreau de Maupertuis. Maupertuis discovered that Lagrange was a great supporter of his principle of Least Action, therefore he offered to the young mathematician a position at the Academy of Science in Berlin, which Lagrange kindly refused, preferring to remain in Turin.

In September 1756, Lagrange was successfully proposed by Euler for election to the Berlin Academy. In 1757, Lagrange became a founding member of the Académie des sciences de Turin which started to publish in 1759 a scientific journal *Miscellanea Philosophico-Mathematica Taurinensia*, known also as *Mélanges de Turin*. The articles were published in Latin or French.

The first volume contained some important Lagrange's works: his outstanding results on the calculus of variations, an article on probability, another article on the foundations of dynamics, based on the principle of Least Action and conservation of energy, and his study of sound propagation and vibrating strings. For this thesis the most important articles are *Recherches sur la nature et la propagation du son* (Michellanea Taurinensia, t.I, 1759), *Nouvelle recherches sur la nature et la* *propagation du son* (Michellanea Taurinensia, t.II, 1760-61) and *Addition aux première recherches sur la nature et la propagation du son* (Michellanea Taurinensia, t.II, 1760-61). His solution of vibrating string problem using a discrete model will be discussed later on, here it is important to underline that Lagrange solution was accepted as the conclusive and it seemed to confirm Euler's solution with some further details.

The third volume, published in 1766, contained an article on the integration of differential equations applied on fluid mechanics and orbits of Jupiter and Saturn. It was the first time that the Lagrangian function appeared and there were presented also his new methods to solve a linear system of differential equations using the characteristic value of a linear substitution.

Joseph-Luis Lagrange left Turin in 1763 visiting Paris during a journey in Europe with Marquis Caraccioli, an ambassador from Naples who was directed in London. A year after his return to Turin, Jean le Rond d'Alembert visiting the Berlin Academy of science in 1766, thanks to his friendship with Frederick II of Prussia arranged a position for Lagrange, but it seemed he was going to refuse the offer. D'Alembert who became a great friend of Lagrange wrote him another letter, saying that Euler was leaving Berlin, returning to St Petersburg. Joseph-Louis Lagrange accepted the Euler's succession as Director of Mathematics at the Berlin Academy at the end of the year 1766. Lagrange remained in Berlin 20 years. Many works of him won the prize of Paris *Académie royale des science*; in 1772 jointly with Euler for the three-body problem, in 1774 for the motion of the moon and 1780 for the perturbation by the planets of the comet orbits. His research interests were focused on the foundation of the calculus, astronomy, probability, mechanics, dynamics, and fluid mechanics.

In those years Lagrange was working also on the number theory, studying properties of integers and prime numbers. In 1770 he published his important work, *Réflexions sur la résolution algébrique des équations,* on solutions of equations of degree greater than 4. His reflections could be considered the first step in the direction of a group theory, developed by later mathematicians as Paolo Ruffini, Augustin-Louis Cauchy and Éveriste Galois.

Some Italian universities regretted that Lagrange left Turin and tried to convince him to leave Berlin, but he turned down the offer. After his wife's death in 1783, he fell into depression but decided to remain in Berlin until Frederick II died in 1786. In May 1787, Lagrange left Berlin moving to Paris where he became a member of the Paris Academy. During his stay in Berlin, he decided to orient his studies in the field of mechanics, starting to write a work based on the Principle of Virtual Velocities: *Traité de mécanique analytique*. But only after his arrival in Paris, he published his book in 1788, titling *Mécanique analytique*. This book contained a resumé of all the work done since Newton's time but using differential equation formalism. With his masterpiece, Lagrange wanted to integrate mechanics in the domain of mathematical analysis. As a member of the *Académie royale des sciences* he was involved in the standardization of weights and measures in 1790. The French metric system founded on the decimal base became several years after the International Metric System.

The political events afflicted the work in the *Académie royale des science*, when in 1793 the Reign of Terror suppressed many societies. Some important members of the Weights and Measures Commission as Pierre-Simon Laplace, Antoine-Laurent de Lavoisier, Charles Augustin Coulomb, Jean Baptiste Joseph Delambre and others were expelled, whereas Lagrange became its chairman. On 8 May 1794, the revolutionary tribunal condemned 28 people to death and guillotined them, among them the most known was the chemist Lavoisier. Lagrange with Lavoisier's help managed to escape from the arrest.

In 1794, the *École Centrale des Travaux Publics,* which became later *École Polytechnique,* was set up and Joseph-Louis Lagrange assumed the chair of analysis. The *École Normale* was established just a year after with the goal of training school teachers, Lagrange became a professor of elementary mathematics there. One of his first students at the *École Normale* was Jean Baptiste Joseph Fourier. Lagrange was not a good lecturer, his reasoning was to abstract for the young students, but he wrote two volumes of his lectures: *Théorie des fonctions analytiques* in 1797 and *Leçons sur le calcul des fonctions* in 1800.

Emperor Napoleon Bonaparte appreciated Lagrange's work, nominating him senator, and named him to the Legion of Honour, giving him in 1808 the title of Count of the Empire. In 1813 some days before his death, Lagrange received *the Grand Croix of the Ordre Impérial de la Réunion*. On the 10th April 1813, Joseph-Louis Lagrange died in Paris when he was 77 years old.

4.3. THE VIBRATING STRING CONTROVERSY

The importance of the controversy is not only in its solution, the wave equation is, indeed, a fundamental ingredient in many domains of classical and modern physics, but also in the development of the scientific reasoning that had as consequence the mathematization of physics. For this part, the main source was a paper written by Wheeler G. F. and Crummett W. P. and published in American Journal of Physics in January 1987.

As one may notice, the physics in eighteenth-century was still predominantly an experimental science and its arguments were mainly based on the empirical descriptions of the phenomena. Daniel Bernoulli had as physicist this position in the debate. He observed musical instruments and he tried to describe what he was looking at.

On the other hand, the domain of mathematics being a part of natural philosophy was much larger as nowadays, it also contained mechanics, optics, astronomy, acoustic beside arithmetic, algebra, analysis, geometry and game theory (what we nowadays call statistics or theory of probability). Jean le Rond D'Alembert faced the problem of vibrating strings as a pure mathematician with a refined mathematical reasoning but losing or neglecting the sense of reality. It seems that he was rather seeking an elegant mathematical solution, as to describe reality.

Between their opposite positions we can found Leonhard Euler and Joseph-Louis Lagrange, two scientists who played a fundamental role in the mathematization of physics and as its consequence the birth of the mathematical physics. As stressed by Wheeler and Crummett (1987), they could be described with modern words as a theoretical or mathematical physicist.

The first who started studying the problem of the vibrating string was Jean le Rond d'Alembert, although Leonhard Euler and Daniel Bernoulli had made some more general studies on vibrating systems during their collaboration period in Saint Petersburg.

D'Alembert preferred to approach the problem from a purely mathematical point of view. In his paper *Recherches sur la courbe que forme une corde tendue mise en vibration* (1747), he analysed an idealised physical situation. Considering a unitary wave speed, he obtained the partial differential equation[5], that nowadays is called d'Alembert's wave equation:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = \frac{\partial^2 y(x,t)}{\partial x^2}$$

and found a general solution of two completely arbitrary functions:

$$y(x,t) = \varphi(x+t) + \psi(x-t),$$

which impose the boundary conditions: $y(0,t) = y(L,t)$, reducing the solution to:

$$y(x,t) = \varphi(x+t) + \varphi(x-t).$$

D'Alembert wanted to show that there were many curves in addition to the sinusoids, that could satisfy to the problem of a vibrating string; he pretended that the tension and the vibration of the string should have been very small (infinitesimal) so that the arc of the curve AM had to be equal to its projection on the x-axes AP (Hug & Steiner, 2015) (see Figure 4.2. below). This requirement made the function φ to be odd, periodic, and everywhere differentiable. D'Alembert was not interested in the physical meaning of the solution, neither in the significance of the interval between 0 and L. His main goal was to develop pure mathematical reasoning, looking for the general solution of the

[5] The notation used by d'Alembert is not the modern one presenting in the text, but rather this one:

$$\frac{dd\, y(x,t)}{dd\, t} = \frac{dd\, y(x,t)}{dd\, x}$$

differential equation. The importance of d'Alembert's formal mathematical treatment in seeking for the solution is his application of the method of separation of variables.

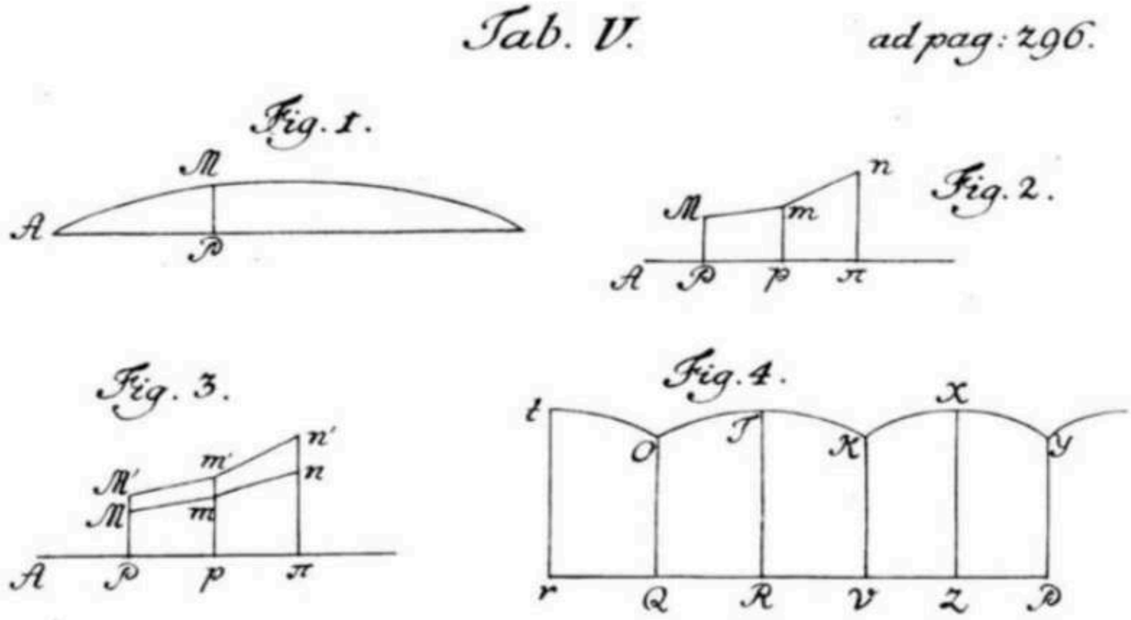


Figure 4.2. Annex from the article *Recherches sur la courbe que forme une corde tendue mise en vibration*, d'Alembert (1747)

Leonhard Euler was a great mathematician, but he was also interested in some physics problems. The physical reality represented for him often the starting point for a more refined mathematical reasoning, although he returned to the natural phenomenon at the end, interpreting his mathematical results. His first solution to the problem was presented in Berlin on 16 May 1748 at the Academy of Science (Hug & Steiner, 2015). In 1748 and 1749, he published the same article first in Latin (*De vibratione chordarum exercitato*) and then its translation in French (*Sur la vibration des cordes,* Figure 4.3.). Euler was interested in establishing an equation of motion balancing the forces in an infinitesimal element of the string. On the contrary of d'Alembert, he took into account the physical phenomenon. In his works he derived the general wave equation:

$$\frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2},$$

finding its solution:

$$y(x,t) = f(x+ct) + g(x-ct).$$

SUR LA VIBRATION

DES CORDES,

PAR M. EULER.

Traduit du Latin.

I.

Quoique tout ce que Mrs. *Taylor*, *Bernoulli*, & quelques autres, ont dit & découvert juſqu'à préſent au ſujet du mouvement vibratoire des cordes, ſemble avoir épuiſé la matiere, il y reſte néanmoins une double limitation, qui la reſtreint tellement, qu'à peine y a-t-il

Figure 4.3. First page of the article *Sur la vibration des cordes*, Euler (1749)

Euler stated that the solution was well defined only within the interval $0 < x < L$, but it was possible to extend it to the interval $0 < x < 2L$ (Garber, 1999), and so on till covering the entire real axis. As d'Alembert, Euler applied the boundary conditions $y(0,t) = y(L,t)$ and founded the solution

$$y(x,t) = f(x + ct) + f(x - ct).$$

The big difference with the French mathematician was the interpretation of the function f that could be deduced knowing the initial conditions, namely the initial position $Y(x, t_0)$ and the initial velocity $V(x, t_0)$ of the string:

$$y(x,t) = \frac{1}{2}[Y(x + ct) + Y(x - ct) + \frac{1}{2}\int_{x-ct}^{x+ct} V(s)ds].$$

Euler had in mind the real case of the plucked string and in his opinion, it was not required to the functions $Y(x)$ and $V(x)$ to be ordinary functions, but any curves, also drawn by hand, in the interval $(0, L)$ with odd periodicity. These curves could be considered as solutions, also having corners. Euler imposed that they should be extended along the real line.

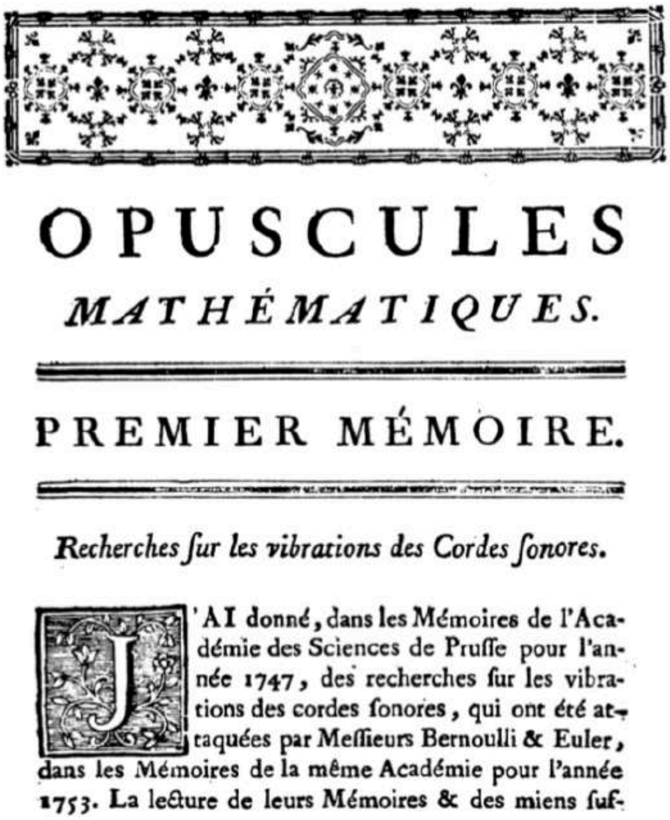
Daniel Bernoulli was raised in a family of known mathematicians and he probably knew the Brook Taylor's work *De motu Nervi tensi* (1713). In 1753, Bernoulli published his work *Reflexions et éclaircissemens sur les nouvelles vibrations des cordes exposées dans les Memoires de l'Academié de 1747 et 1748*. He based his solution upon the observation of the physical phenomenon of a vibrating string, as an experimentalist would do. As Taylor, he described the fundamental amplitude of the wave as $y(x) = A \sin\left(\pi \frac{x}{L}\right)$ and the solution should be a sum of the fundamental and higher harmonics:

$$Y(x,t) = A \sin\left(\pi \frac{x}{L}\right) \cos\left(\pi \frac{ct}{L}\right) + B \sin\left(\pi \frac{2x}{L}\right) \cos\left(\pi \frac{2ct}{L}\right) + \ C \sin\left(\pi \frac{3x}{L}\right) \cos\left(\pi \frac{3ct}{L}\right) + \cdots$$

Daniel Bernoulli examined the oscillation of a system of several particles, and he used for the first time the Principle of Superposition without demonstrating it. It is not clear how one should calculate the coefficients. In fact, his argumentation was based exclusively on the observations, he gave a very poor mathematical support.

These are the different solutions found by the three scientists. The controversy was going on some years initially between d'Alembert and Euler on the definition and the properties of the function f, the solution of the wave equation, and later between Bernoulli and the other two protagonists on his solution based on trigonometric series. The debate ended, after Joseph-Louis Lagrange gave his solution, although the final and complete one was given in Fourier series only in 1807 by Joseph Fourier.

D'Alembert did not accept Euler's definition of the solution; in his opinion, Euler was too general in admitting also functions with corners and hand-drawn function. The discussion with Euler and all his remarks were collected in his *Opuscules mathématiques*, published between the years 1761 and 1780.



OPUSCULES

MATHÉMATIQUES.

PREMIER MÉMOIRE.

Recherches ſur les vibrations des Cordes ſonores.

J'AI donné, dans les Mémoires de l'Académie des Sciences de Pruſſe pour l'année 1747, des recherches ſur les vibrations des cordes ſonores, qui ont été attaquées par Meſſieurs Bernoulli & Euler, dans les Mémoires de la même Académie pour l'année 1753. La lecture de leurs Mémoires & des miens ſuf-

Figure 4.4. First page of the article *Recherches sur les vibrations des cordes sonores* from the first book of *Opuscules mathématiques*, d'Alembert (1761)

D'Alembert pretended the solution to be an odd periodic function, but in his opinion, the function could not be defined by pieces, as he wrote in 1750. He refused to take as an initial condition a function represented by two joined pieces of a parabola. He refused also that the function φ could change its shape (Jouve, 2017). D'Alembert's concept of a function was, first of all, a formal expression.

In 1761, d'Alembert in his *Recherches sur les vibrations des cordes sonores* (Figure 4.4. and 4.5.) objected to Euler the use of physical arguments in constructing his solution and not a rigorous mathematical treatment. The function representing the string could not make any "curve jumps" (*sauts de courbe*) (Jouve, 2017). This requirement could be translated in modern language as everywhere differentiable, indeed d'Alembert defined his function as continue in modern sense[6].

A function describing a plucked string in a general point p in the interval $(0, L)$, excluding the extremities and the middle point $\frac{L}{2}$, had two different slopes at its left and right side, the result was then:

$$\left.\frac{\partial y(x,t)}{\partial x}\right|_{p^+} \neq \left.\frac{\partial y(x,t)}{\partial x}\right|_{p^-}.$$

This meant that it could not exist the second derivative in that point. If $\left.\frac{\partial^2 y(x,t)}{\partial x^2}\right|_p$ does not exist, then cannot be equal to $\frac{\partial^2 y(x,t)}{\partial t^2}$, a well-defined quantity.

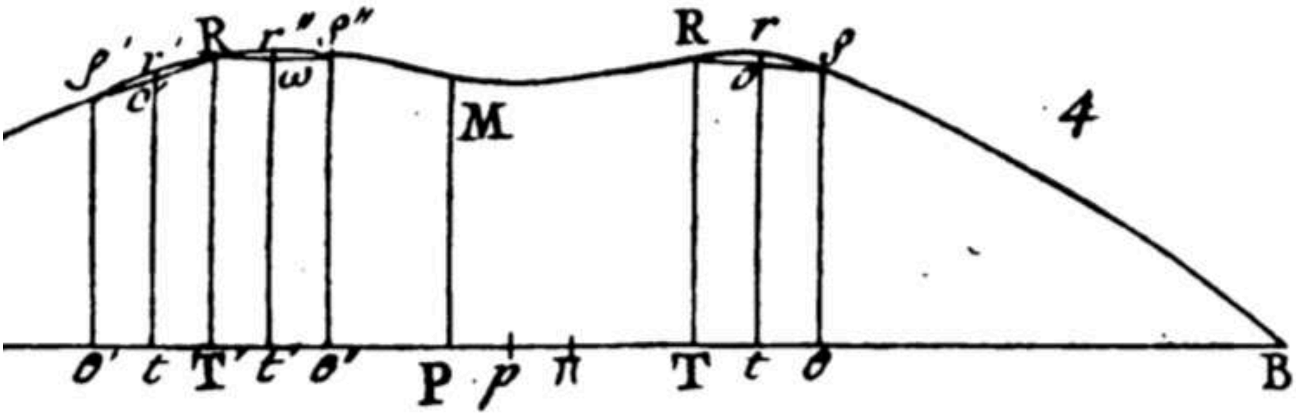


Figure 4.5. A figure taken from the article *Recherches sur les vibrations des cordes sonores,* first book of *Opuscules mathématiques*, d'Alembert (1761)

[6] In the eighteen century, a "continue function" designed a function that did not change its expression.

Leonhard Euler replied to d'Alembert in two papers (Wheeler&Crummett, 1987), *Sur le mouvement d'une corde, qui au commencement n'a été ébranlée que dans une partie* (1765) and *Éclaircissemens sur le mouvement des cordes vibrantes* (1766), rejecting his criticism as weak. In his opinion, the considered displacements were so small that the error introduced by assuming the function differentiable was infinitesimal. Euler definition of a function was less restrictive. In his opinion, a function was to be intended as a dependence relation between two variables (Jouve, 2017). This could be expressed in many different ways, also in a graphical one (Hug & Steiner, 2015). D'Alembert was not satisfied with Euler's answers.

It is interesting to have a look at Euler's more general definition of a function, given in his book *Institutiones calculi differentialis* (1755):

> *"If certain quantities depend on other quantities in such a way that if the others change, these quantities also change, then we are accustomed to calling these quantities functions of the latter; this denomination has the greatest extent and contains in itself all the ways by which a quantity can be determined by others. If therefore, x denotes a variable quantity, then all other quantities which depend on x in any way, or which are determined by x, are called functions of x.*[7]"

The discussion between d'Alembert and Euler on the definition of a function brought to a clarification of its very concept. D'Alembert continued to argue that functions defined in pieces necessarily lead to curvature jumps (Jouve, 2017). He changed his mind only at the end of his life, as can be read in the unpublished paper *Sur les cordes vibrantes*, written in the years 1779 and 1783 (Hug & Steiner, 2015).

Jean le Rond D'Alembert attacked Daniel Bernoulli because his solution was based only on physical observations and he did not give a mathematical explanation. Bernoulli used the idea of the Principle of Superposition, although at that time it was not mathematically demonstrated, and for that reason, d'Alembert did not accept that the motion of the string could be composed of motion of separate distinct modes. In his opinion, only one frequency should have been associated at vibrating conditions and the trigonometric decomposition with its multiple frequencies was to reject as a solution.

Leonhard Euler accused Bernoulli that his trigonometric solution was not general enough to represent an arbitrary odd periodic function f as d'Alembert and he had proposed. In his work *Sur le mouvement d'une corde, qui au commencement n'a été ébranlée que dans une partie* (1765), Euler

[7] Si certaines quantités dépendent d'autres quantités de telle manière que si les autres changent, ces quantités changent aussi, alors on a l'habitude de nommer ces quantités fonctions de ces dernières ; cette dénomination a la plus grande étendue et contient en elle-même toutes les manières par lesquelles une quantité peut être déterminée par d'autres. Si, par conséquent, x désigne une quantité variable, alors toutes les autres quantités qui dépendent x de n'importe quelle manière, ou qui sont déterminées par x, sont appelées fonctions de x. (reported by Jouve, 2017)

highlighted that Bernoulli's series could represent just a wave produced by snapping the string at one of its ends.

On the contrary, Daniel Bernoulli replied to Euler and d'Alembert, accusing them to abuse of "abstract analysis" (Jouve, 2011) and not listening to the sound of a vibrating string. In Bernoulli's opinion, the other two scientists were completely neglecting the acoustics; every function sinus represented a vibration harmonic and the produced sound was a juxtaposition of many harmonics, therefore the solution had to be a linear combination of functions sinus.

However, none of the three disputants realised that the solution needed to be defined only in the interval $(0, L)$ and not on the entire real line. Euler and d'Alembert had disagreed over the degree of generality of the solution function but both could not accept Bernoulli's trigonometric series. In their opinion, the problem was the periodicity; in fact, they could not understand the geometrical periodicity of the function f inside the interval. Moreover, Euler disagreed with Bernoulli's definition of an odd function.

Euler's criticism of Bernoulli's solution missed the most important point: the generality of the solution. Euler's solution was more general than Bernoulli's. In fact, Bernoulli considered a particular case as an initial condition a string with zero velocity, whereas Euler's solution function includes also strings with no zero initial velocities.

In 1759, Joseph-Louis Lagrange published his work *Recherches sur la nature et la propagation du son* in the first volume of *Mélanges de Turin* and the following two years also *Nouvelles Recherches sur la nature et la Propagation du son*, and *Addition aux premières recherches sur la nature et la propagation du son* (Guin, 2016). He approached the problem from physics mathematical point of view. He rederived the wave equation, supporting Euler's arguments against d'Alembert with whom Lagrange had a frequent correspondence. In Lagrange's opinion, Euler did not use rigorously the infinitesimals, although the functions with corners were perfectly admitted. As Euler, he considered Bernoulli's trigonometric solution, not enough general.

Lagrange considered in his solution of the problem string formed by a discrete system of n equally spaced masses, connected by a massless cord. The discretisation of the problem was one of the greatest ideas that brought Lagrange to a quasi-definitive solution. He derived n differential equations: $\frac{d^2y}{dt^2} = K\,\frac{(y_{k-1})-2y+y_{k+1})}{r}$[8]. Lagrange solved the system for a string with a discrete number of masses and find the linear combination of n sinusoids. He did the limit for n tending to infinity and the distance of the masses to zero. His integral solution had the following form:

[8]Wheeler&Crummett (1987): $\frac{d^2y}{dt^2} = c^2(y_{k-1} - 2y + y_{k+1})$

$$y(x,t) = \frac{2}{L}\int_0^L dsY(s)\ \left[\sin\left(\frac{\pi s}{L}\right)\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi ct}{L}\right) + \ \sin\left(\frac{2\pi s}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi ct}{L}\right) + \cdots\right]$$

$$+ \frac{2}{\pi c}\int_0^L dsV(s)\ \left[\sin\left(\frac{\pi s}{L}\right)\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi ct}{L}\right)\right.$$

$$\left. + \frac{1}{2}\sin\left(\frac{2\pi s}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\cos\left(\frac{2\pi ct}{L}\right) + \cdots\right]$$

where $Y(x)$ and $V(x)$ are respectively the initial position and the initial velocity of the string. Lagrange came very close to the definitive Fourier's result, written in series. He, differently from Bernoulli, wanted to show that the laws of nature can be derived in an analytically way, based on the principles of mechanics (Guin, 2016). Modifying the expression of $y(x,t)$ in terms of temporal sinus for small angles and using the integral calculus combined with trigonometrical relations, he obtained after many pages of calculations the expression:

$$y(x,t) = \Phi(x + t\sqrt{c})\ \Phi(x - t\sqrt{c}),$$

where $\sqrt{c}$ depended on the geometrical and physical properties of the string (Guin, 2016). If the initial displacement $\mathrm{Y}(x)$ is included in Lagrange's equation for the $t = 0s$, one obtains the odd Fourier series:

$$Y(x) = \frac{2}{L}\ \sum_{n=1}^{\infty}\left[\int_0^L Y(s)\ \sin\left(\frac{n\pi s}{L}\right)ds\ \right]\sin\left(\frac{n\pi x}{L}\right).$$

Nobody understood that Lagrange's solution would have been the Bernoulli's one with the coefficients given by the integrals. The controversy ended with Lagrange's solution. In 1807, Joseph Fourier studying the heath propagation discovered his famous series and with his solution, the vibrating string problem was definitely solved. Finally, one can notice ad posteriori that every disputant in the debate was partially right: d'Alembert found the general solution of the wave equation, Euler's solution using the boundary conditions was perfectly acceptable and Fourier showed that Bernoulli's solution with trigonometric series was valid in case of the zero initial velocity.

4.4. THE NARRATIVE OF THE DEBATE ON THE VIBRATING STRINGS

One of the greatest debates in the history of science ended with a birth of new branches of analysis: the harmonic analysis and the study of partial differential equations. The wave equation is a powerful tool, used in classical and modern physics. The treatment of the vibrating string cannot be reduced to a simple exercise in a course of Mechanics without losing its richness in ideas, concepts, and methods.

The goal of this part is to show that this case study can be used in a classroom to explain how interdisciplinarity acts. Also here, Thompson Klein taxonomy can help us to analyse different possible interactions between mathematics and physics in the narrative of the vibrating strings controversy.

There is a problem in the domain of mechanics that cannot be solved. From a deep analysis of the literature, a lack in the knowledge of how to find a rigorous description of a vibrating string emerged. At this point, there are two options as we have seen, both can give a good solution.

Bernoulli chose the position of an experimental physicist, observing the phenomenon and describing it in terms of acoustics observations. He used the results of Taylor's theory, remaining in the domain of experimental physics. One could see a kind of methodologic interdisciplinarity in his reasoning. Making an analogy between Taylor's case and the vibrating string, he proposed his trigonometric series solutions. He did not develop a mathematical treatment but he was convinced of the validity of his results because he relied on his previous experience in the study of acoustical phenomena. The mathematical series was a useful tool to describe a superposition of fundamental frequency and higher harmonics. Although he could not give a rigorous demonstration of the fact, he understood that it worked. The asymmetry between the role of mathematics and the role of physics was often observed in his works.

Euler and Lagrange took another way: they preferred to describe the problem analytically. The physical phenomenon was the starting point and after some dynamical considerations, the wave equation could be found. The next step was to find a solution to the wave equation. A physical problem is treated by mathematical formalism and after finding a solution, the physical interpretation of the results is needed. Mathematics and physics co-act at the same level, solving a problem in a more general way. In this case, one could find some properties of a theoretical interdisciplinarity. The synthesis of their solution was also confirmed by Fourier's series that nowadays belong to both field: from the foundational point of view to the harmonic analysis (mathematics) and from the practical use to the signal analysis (physics). The relation between physics and mathematics, thanks to Euler's and Lagrange's work, brought to the mathematization of physics and to the definition of a new disciplinary field, the mathematical physics, where the two disciplines are so connected that it is impossible to split them up.

In the discussion between Euler and d'Alembert on the acceptability of the solution, there are some elements of critical interdisciplinarity. The study of differential equations of the physical problem of a vibrating string brought to a transformation of pure mathematical analysis, that through the development of topics, led to a reconstruction of the differential calculus theory. The discussion of the properties of a derivable (continue) function questioned the fundaments of analysis and led to a more rigorous definition of the last one.

Nowadays it seems strange to question the admissibility of some classes of functions in courses of physics, normally these types of questions are left to courses of mathematical analysis, but in the eighteenth century, these types of questions had not a certain domain. Without the problem of acceptability of a solution to a physical problem, also mathematical analysis could not improve its knowledge. D'Alembert's approach to the mathematics treatment has certainly some elements of critical interdisciplinarity.

D'Alembert focused his attention on the issue of the nature of the mathematical function, the definition of continuity, and the differentiability (Garber, 1999). He used the physical problem as a general starting point for his mathematical treatment. He saw a good opportunity to develop the calculus and to explore a possible solution to partial differential equations of a certain type.

Finally, one can understand that d'Alembert's researches on the vibrating strings had three important effects (Jouve, 2017) on the evolution of mathematics. The first was the emergence of the first theory of partial derivatives equations and their solutions, the second is the introduction of the notion of "jumps" (*sauts*) of the curve and the consequential modern mathematical definition of the continuity of a function and the third the move of the study of partial differential equations from the domain of the so-called mixed mathematics to the domain of the modern mathematical analysis.

On the contrary, nowadays no one would try to do physics without a rigorous mathematical treatment and this is largely due to Euler's and Lagrange's mathematization processes when results of physics were proofed by analytical demonstrations and vice versa. What today is seems obvious, in eighteenth-century was not, and the history of the debate on the vibrating string can partially explain how the two disciplines started to be linked together.

4.5. DIDACTIC IMPLEMENTATION

As one can notice in the narrative of the controversy on the vibrating strings there are different levels where interdisciplinarity emergences.

At the high school, it is impossible to do a treatment with the formalism of partial differential equations neither to demonstrate the equivalence of the Bernoulli's and Euler's solution using Fourier series as it can be done at the university, but the discourse of continuity and differentiability of a function can be developed.

In France, the topic would be treated during scientific education. As some interviewees have mentioned and as one can see from the syllabus of the subject, there is a possibility to lose the richness of details in the narrative of the debate because not all the students are taking physics and mathematics classes and they could not be able to understand the evolution of concepts of analysis like the continuity or differentiability. Another big problem could be the lack of time. In the syllabus of the scientific education subject, the treatment of the debate is thought as an introductory theme to a much bigger chapter on the sound and music as information carriers. Nevertheless, some aspects of the nature of science, as the importance of debate in the scientific community or the collaboration between the scientists, can emerge from the treatment.

In Italian high schools, physics and mathematics can be taught by the same professor, and this is certainly an advantage for the treatment of the debate. The teacher has a possibility to show the richness of the contents because is free to decide how many lessons to dedicate, but since there is no compulsory program defined in detail as in France, the discussion of interdisciplinary topics is left to the goodwill of the teacher.

At the university level, the treatment of a vibrating string during courses of Mechanics would probably always be seen as an exercise but after obtaining the solution, the professor can with a few sentences remind students that it took years of debate to achieve that result. A more complete treatment of the debate, as proposed in this chapter of the thesis, is possible at the History of Physics/Mathematics or Didactics of Physics/Mathematics lectures. Another interesting activity could consist of going to read pieces of original articles and to look at the language used or to analysed the evolution of the mathematical concepts.

During the teachers' training, it can be also stressed the importance of the history of science for a better understanding of the nature of science. The use of the historical narrative makes the lecture more attractive for the students and help them to understand, how science worked/works. It is also important to stress that what is obvious nowadays could not have been centuries ago and a great worked was done to make some conceptual changes.

Talking about interdisciplinarity and the relation between the disciplines can open a debate on what a discipline is, showing how labile this concept is. What in the eighteen century could be understood as a topic of mathematics, nowadays could be not and vice versa. The disciplines are social constructs as would say Professor Hugon (Hugon, 2004), and their appearance, development and disappearance are dynamic processes. The study of the interdisciplinarity and the disciplinary contents can bring us to a greater awareness of the organization of our knowledge and the richness of epistemologies of the individual disciplines

FINAL DISCUSSION AND CONCLUSION

This thesis addressed the problem of teaching interdisciplinarity in science education in France, with a specific focus on the interaction between mathematics and physics. Although the work was not intended to compare the two school systems, where possible, an attempt was made to make a parallelism with the Italian one.

The first chapter analysed and presented the French high schools that underwent a recent reform of the final exam *Baccalauréat 2021*. The French Government introducing the choice of the optional and specific subjects, gave to the students the possibility to build their educational path, and in doing so to deepen also the study of scientific subjects. On the other hand, the content of the courses became more disciplinary.

The subject "scientific education", the interdisciplinary lectures par excellence compulsory for all students, would have the opportunity to deal with interdisciplinary topics, but having students with a various background in the classroom risks compromising this purpose, making only juxtapositions of the proposed topics. Another problem is the too large syllabus that does not leave to deepen the various themes covered. By not going into the depth of the issues, interdisciplinary reasoning can never really emerge.

In the second chapter, some theories on interdisciplinarity were discussed and Thompson Klein's taxonomy seemed to describe the best the interactions between the disciplines. The taxonomy was used in the third chapter to interpret and compare the different definitions given by the interviewed researches. After a debriefing activity and a discussion in the research group in didactic of physics of the University of Bologna, we reached the conclusion that the taxonomy had to be previously interpreted to fix some non-uniqueness of the categories description.

The analysis of six interviews in the third chapter showed that there is a great deal of interest in research on interdisciplinarity activities. The collaboration of different disciplinary teachers is meaningful for the development of interdisciplinary teaching because a significant interdisciplinary approach can be set up in the comparison between experts of different domains. The most appropriate case studies can be taken from the history of science. An adequate university teaching research on STEM-related problems should be promoted, developing a system for evaluating the effectiveness of educational actions.

In the last chapter, the vibrating string controversy was presented and it resulted a suitable case study of interdisciplinarity. It has been shown that the solution of the vibrating string problem reduced to a simple exercise in a course of mechanics loses all its richness in ideas, concepts, and methods. It was also discussed that a deepening in the historical debate, showing the different positions on how to solve the problem and on what kind of solutions are acceptable or not, can highlight the various epistemological positions of the protagonists. The mathematization of physics and the definition of new disciplinary fields are the results of the contamination of knowledge and methods during the debate. It was also shown that the vibrating string controversy can be treated at many levels, from the high school to the university, and several aspects can be brought out.

This thesis has tried to address the issue of interdisciplinarity in science education by illustrating the vastness and the richness of the problem, trying to reach a point of arrival, but much more can still be done.

ACKNOWLEDGEMENTS

The writing of a thesis is a special period when every student has an opportunity to look at what she/he has been doing and studying. It represents a great chance to face with some interesting study-topics and to do a real deepening, I would metaphorically say to grasp an issue or a problem that during the lectures was only gently caressed.

This thesis represents an end for my educational path at the University of Bologna, but it is just a starting point for my future teaching professional career. The reflections and the deepening that I made in these two years will continue to stimulate me in future practice. It was a wonderful period that had and still has a big impact on me. I would like to express my acknowledgements to some people who have been accompanied me on this journey or who helped me directly or indirectly in the various stages of my educational path and thesis writing.

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