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Exploring interdisciplinarity between physics and mathematics: the design of a linguistic and an epistemological tool for analysing texts about the parabolic motion

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Each time a woman stands up for herself, she stands up for all women.
Maya Angelou

Abstract

This thesis project fits into the research field of *Physics Education*. It is part of the IDENTITIES project, an Erasmus+ project whose main aim is to design and develop innovative and transferable interdisciplinary teaching modules for pre-service teachers, covering both curricular and STEM topics. The thesis aims to contribute to the project by developing linguistic and epistemological tools that future teachers can use to explore and analyse both the disciplinary identity of physics and mathematics and their intertwining that emerge from high school textbooks. In particular, it was decided to consider a topic which is well-known and already addressed by the IDENTITIES research group, namely the parabolic motion, present in every physics textbook.

In the thesis, the two analytical tools I contributed to develop are presented and applied to an entire chapter of a textbook, concerning motion in two dimensions. The linguistic tool derives from a re-elaboration of the typical techniques of linguistic analysis. The epistemological tool is built starting from the Reconceptualized Family resemblance approach for the Nature of science (RFN). The two tools allow us to highlight very different aspects. On one hand, the linguistic tool shows how the textual, syntactic and lexical choices convey a particular image of the two disciplines. On the other hand, the epistemological tool has led us to carry out the cognitive-epistemic features of the disciplinary knowledge involved in the chapter. By combining the two tools and the results obtained from their application, important considerations can be made about what aspects of physics are proposed by a high school textbook and what role mathematics plays in it.

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Introduction

This thesis project is framed in the research field of the *Physics Education* and aims to draw out what image of physics and mathematics emerges in high school.

In particular, it was decided to look at selected written texts: the textbook. Textbooks, indeed, play a central role in school science education in all countries: they help to "organize information, guide inquiry, present important scientific facts, improve problem solving skills, consolidate learning, illustrate abstractions, and develop reading skills" (Mcdonald & Abd-El-Khalick, 2017, p. 3); they are powerful resources, capable of influencing both students and teachers. For this reasons, it is crucial that their content, both explicit and implicit, is investigated from several points of view.

For this study, it was decided to consider a precise topic in physics: the parabolic motion. The choice is linked to the Erasmus+ project IDENTITIES, within which this thesis is developed. This project is coordinated by the University of Bologna, in which also the universities of Parma, Barcelona, Crete and Montpellier are collaborating. Its core aim is to develop interdisciplinary teaching modules for future teachers, covering both curricular and STEM topics.

This thesis is articulated in three chapters plus the conclusions.

Chapter 1 contextualizes the research, illustrating the relevance of interdisciplinarity in science education and describing the IDENTITIES project. In particular, it is presented the work of the Italian research group, who designed a module that shows how the topic of parabolic motion lends itself very well to an interdisciplinary analysis with mathematics, through a relevant historical and philosophical pathway. The thesis aim is presented, which is to design tools that can bring out the disciplinary and interdisciplinary structure of any scientific text. These tools will then be applied to a selected textbook, *Physics*, volume 1, by James S. Walker (2017), which devotes an entire chapter to the topic of two-dimensional motion.

To find out how physics and mathematics are treated in this section and how they interact with each other, it was decided to analyse the chapter from two different perspectives.

In Chapter 2, the text is studied from a linguistic point of view, namely by observing the textuality, syntax and lexicon. The first section is dedicated to the description of characteristics of scientific language and texts, and to present the analysis tool. Then, this tool is run in on a synthetic textbook, *I problemi della fisica. Meccanica e termodinamica* (Cutnell et al., 2015). Later it is applied to Walker's one. Finally, it is also applied to an extract from *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (tr. "Dialogues concerning two new sciences", Galilei, 1638), concerning the demonstration of the projectile motion, to present a comparison with a historical text.

In Chapter 3, the study looks at the epistemological characteristics of the text. In this case the tool taken into consideration, the Reconceptualized Family resemblance approach for the Nature of science (RFN), is a framework already known in the field of *Physics Education*. The first section is devoted to the description of this tool, as originally theorized by Irzik & Nola (2011, 2014) and then adapted for educational purposes by Erduran & Dagher (2014). In particular, a part of the framework, known as "cognitive-epistemic system", is described in detail, because is the one that is used for the analysis of *Physics*. The next section is dedicated to the presentation of the methodologies chosen to convert the theoretical framework into an analytical tool. Later, the results are presented: first all the sub-categories highlighted within the epistemic-cognitive system are shown, then some graphs illustrate their distribution within Walker's chapter. Finally, in the last section of the chapter, the results of the linguistic analysis will be taken up and integrated with those gained through the epistemological tool.

A brief discussion of the reasons for this thesis and a summary presentation of the achievements is provided in the Conclusions section.

Chapter 1

Research context

1.1 Interdisciplinarity in Science education

School curricula are usually organized in disciplines, because of the basic assumption that disciplinary knowledge offers many benefits and fosters the development of significant forms of reasoning. This form of knowledge organization, in spite of its potential, is nowadays being questioned. For example, it has been argued that disciplines reveal some weaknesses when the goal of knowledge is to find a way to address and manage real-world problems (Weingart & Stehr, 2000). In particular, being the societal problems intrinsically interdisciplinary, several social stakeholders advocate a shift from knowledge to skills (Lee et al., 1995): in this regard, schools are requested to support the development of skills that labor market needs, like strategic thinking and planning, modeling, etc., or the formation of civic competencies to become aware and responsible citizens, like critical thinking, managing uncertainty, etc. (Khan, 2019; Bromham et al., 2016). Also reports from OECD (2021) and EU stress the need to prepare the students to cope with the challenges of the 21st century (i.e. climate change, artificial intelligence, nanotechnologies, etc.).

Among other implications, this goal is urgent because it sharply impacts on their perception of the future: the worries arisen by the global problems lead to feel the future no longer as a promise but as a threat (Benasayag & Schmit, 2013) and the uncertainty which pervades our contemporary society does not allow young people to project themselves forward and to understand their role in the world (Rosa, 2013).

To fill the gap and face these challenges, there is a larger and larger consensus among stakeholders on the need to educate young people in the STEM field, a field that integrates Science, Technology, Engineering and Mathematics, and to highlight the important role of these subjects in economic growth, technological innovation and national health (Broggy et al., 2017).

STEM education has rapidly become a dominant discourse in political, economic and educational spheres, as a result of expected benefits in addressing many real-life problems, where a joint effort is required (Uddin et al., 2021). In fact, topics such as climatology, artificial intelligence, nanotechnologies, etc., have a very specific interdisciplinary nature, but the current teaching policies, organized into disciplines, are struggling in finding contexts to turn them into curricular themes (Broggy et al., 2017).

Research in the fields of mathematics and science education suggests that interdisciplinarity is not only fundamental in addressing emerging STEM topics with an intrinsically interdisciplinary nature, but it is also significant to deal with curricular themes and to comprehensively understand and interpret the nature of science and the intrinsic and authentic connections between the disciplines (Karam, 2015; Kalmark Andersen, 2017). Mathematics and physics, in spite of their historical co-evolution, are usually taught, at all levels of education, as separate subject matters (Tzanakis, 2016). This separation is rarely present in the history of the two disciplines, and can lead to a sort of impoverishment and trivialization, not only of the relationship between the two disciplines, but also to the disciplines per se: "mathematics is merely a tool to describe and calculate, whereas, [...] physics is only a possible context for applying mathematics previously conceived abstractly" (Tzanakis & Thomaidis, 2000, p. 49). The dichotomy between mathematics and physics creates learning problems, as students have difficulty in understanding where mathematical concepts come from and why physics has little to do with their experiential world (Karam, 2015). To address this problem, Tzanakis suggests that "mathematics and physics should be conceived (hence, taught and learnt) both as the *result* of intellectual enterprises and as the *procedures* leading to these results. Knowledge gained in their context has an evolutionary character; by its very nature, *historicity* is a deeply-rooted characteristic" (Tzanakis, 2016, p. 4).

Both the complex challenges of contemporary society and the didactic problems associated with curricular topics require the search for new forms of interaction between fields and disciplines, in order to explore novel ways and diverse points of view to address the current issues. To interpret these problems, it is increasingly necessary for the boundaries between disciplines to be crossed and, to do so, it is necessary to go to the heart of the disciplines and recover the sense of their flexible and evolutionary nature (Levrini et al., 2019).

The concept of interdisciplinarity embodies the idea of a relationship and reciprocity between the parties involved, that is a mutual exchange, while maintaining each discipline's own structure. The possibility of merging to create a new structure is supposed to be based on the foundational disciplines; interdisciplinarity represents a dialogue between the parties, an exchange of good practices (Frodeman et al., 2017). It "integrates information, data, methods, tools, concepts, and/or theories from two or more disciplines focused on a complex question, problem, topic, or theme. The key defining concept of interdisciplinarity is *integration*, a blending of diverse inputs that differs from and is more than the sum of the parts" (Peek & Guikema, 2021, p. 1048). It differs from multidisciplinarity, which represents the juxtaposition of different knowledge but does not imply reciprocity, and transdisciplinarity, which refers to something that goes beyond the structure of disciplines, but does not imply a comparison and exchange between the parties (Peek & Guikema, 2021).

Thompson Klein makes the point that interdisciplinary teaching can take many forms. Thus: "interdisciplinary initiatives are often described by the form or structure they take (e.g. team teaching), the motivation behind them (e.g. to serve societal or employment needs), how the disciplines will interrelate (e.g. math will be taught in the service of chemistry), or by labelling the level of integration (e.g. from borrowing to synthesis). [So that] the term interdisciplinarity is used variably as a concept, a methodology, a process, a way of knowing and even a philosophy", showing how teaching interdisciplinarity is a multi-faceted task (Thompson Klein, 1990, p. 55).

This notion of interdisciplinarity is at the basis of the IDENTITIES project, within which this thesis is situated and that is presented in the next Section.

1.2 The IDENTITIES project

This thesis is part of the research work on interdisciplinarity carried out by the group of *Physics Education and History of Physics* of the University of Bologna (Branchetti & Levrini, 2019; Branchetti et al., 2019; Levrini et al., 2019, 2020; Gombi, 2020; Bagaglini et al., 2021). In particular, it is situated within the framework of IDENTITIES (Integrate Disciplines to Elaborate Novel Teaching approaches to InTerdisciplinarity and Innovate pre-service teacher Education for STEM challenges), an Erasmus+ project coordinated by the University of Bologna, which involves further 4 partners. IDENTITIES project aims to design novel teaching approaches on interdisciplinarity, through innovative and transferable teaching modules and courses, to innovate pre-service teacher education for contemporary challenges. The central theme of the modules is the interdisciplinarity in STEM fields, with a focus on the links and interweaving between physics, mathematics and computer science. To this purpose, IDENTITIES stands out for doing research within three key-aspects:

1. To analyse two types of interdisciplinary topics, the advanced STEM topics, like climate change, artificial intelligence, quantum technologies, and the interdisciplinary curricula topics concerning "border problems", like the parabola and the parabolic motion.

2. To respect and value the identities of each discipline and save the potentialities of teaching and learning in the disciplines, since they offer the basis for successful approaches to interdisciplinarity, meant as an integration of disciplines.

3. To explore interdisciplinarity through the lenses of linguistics and epistemology, to recognize lexicon and epistemic aspects which belong to each discipline and the one which is on their boundary, identifying "an epistemology and a language for interdisciplinarity".

This thesis wishes to contribute to these key-points. It aims to provide two analytical tools to analyse scientific written texts, able to point out particularities of the disciplinary identities of physics and mathematics, as well as the relationships between them. The first tool is linguistic, and it derives from a re-elaboration of the typical tools of textual analysis, aimed at highlighting and bringing out what image of the disciplines is conveyed by a text. The second tool is epistemological, and has been built from the Reconceptualized Family resemblance approach for the Nature of science (RFN), a very well-known framework in science education that will be used, in this thesis, as a lens to analyse texts and to identify the cognitive-epistemic system of the disciplinary knowledge involved there.

In order to verify their reliability and generalizability, at the end of each implementation they are tested against selected texts.

1.3 The IDENTITIES module on the parabola and parabolic motion

The texts analysed in this thesis refer to the theme of *parabola and parabolic motion*, being this the theme of a module that the Italian group has designed within the IDENTITIES project, since it has a profoundly interdisciplinary history spanning over the centuries. This module has been tested the first time in a course for in-service teacher education, within the PLS (Piano Lauree Scientifiche) of Bologna.

The course was entitled *Strumenti di analisi e comprensione del testo scientifico per l'interdisciplinarità: un confronto tra fonti e manuali su temi di fisica e matematica* (tr. "Tools of scientific text analysis and comprehension for interdisciplinarity: a comparison of sources and textbooks on physics and mathematics topics"). It was organized in the period of November−December 2019, as follows:

- 1°lesson: Introduction (Olivia Levrini). Presentation of the topic and the selected texts; their disciplinary analysis (Laura Branchetti, Alessia Cattabriga, Paola Fantini).
- 2°lesson: Linguistic tools for text analysis (Veronica Bagaglini, Matteo Viale).
- 3° lesson: Epistemological tools for the argumentative analysis of texts (Sebastiano Moruzzi, Carlotta Capuccino; tutor: Enrico Liverani, Alessia Marchetti, Elena Tassoni, Luca Zanetti).
- Conclusive workshop, dedicated to a wider debate among the participants, on the possible contributions and implementations of the proposed analyses in the didactic dimension of the class (tutor: Eleonora Barelli, Enrico Liverani, Alessia Marchetti, Sara Satanassi, Elena Tassoni, Luca Zanetti).

A variety of texts on the topic were selected and analysed during the lessons with tools from physics and mathematics, but also through the lens of linguistics and epistemology, in order to bring out interdisciplinarity.

During the first meeting, the parabola in the history of mathematics from Euclid to Kepler was illustrated, showing the role of physics in the evolution of the conceptualization of conics in mathematics. Moreover, the role of the parabola in the establishment of physics as a discipline was discussed, in order to show how mathematization, and specifically the study of conics, led to the foundation of the scientific method.

Topics such as the parabolic motion, the geometric concept of parabola and the two-dimensional kinematics are crucial for high school students, taught in the early stages of mathematics and physics education, but they are also relatively simple, compared to topics that have been the subject of other interdisciplinary analysis, like the black body (Branchetti et al., 2019).

The module has been progressively refined and tested four times: two times within the course *Physics teaching: theoretical and experimental aspects* of the Bachelor and Master Degree in Physics, University of Bologna (October−November 2020 and 2021), one time within the course *Mathematics Education* of the Master Degree in Mathematics, University of Milan (November−December 2021) and within the first international Summer School of the IDENTITIES project (June 28th−July 2nd, 2021).

The entire module is articulated in six sub-modules. After an introduction to interdisciplinarity through the metaphor of the "crossing boundaries", two sub-modules concern parallel narratives of the history of parabola and parabolic motion, respectively, in mathematics and physics; the reconstruction has been carried out to stress the co-evolution of the two disciplines. Then, other two sub-modules focus on linguistic and epistemological analysis of texts on the theme "parabola and parabolic motion", aimed to reflect on both disciplinary identities and interdisciplinarity. Finally, the last sub-module is designed to rethink the initial opened questions.

During the implementations of the module, it emerged the need to build more and more operational teaching tools, to guide the teachers throughout the linguistic and epistemological analyses of textbooks—tools that could be used in the fourth and fifth sub-modules. This is the main goal of the thesis.

In the following, a summary of the contents of the sub-modules about the history of mathematics and physics is reported, in order to contextualize my research work, aimed to build the linguistic and epistemological tools for the analysis of the textbooks. Their design and application will be the main core of the thesis and will be described in Chapters 2 and 3, respectively.

1.3.1 The parabola – the theme from a mathematical point of view

In the sub-module about the history of mathematics, the term *parabola* has been introduced by stressing that it derives from the Greek word *parabolé*, a descendant of *parabállõ*, meaning "to flank, to parallel". This definition can be interpreted from a geometric perspective, considering the parabola as the conic section obtained by the intersection of a cone with a parallel plane passing through the directrix. This way of building the figure refers to one of the most influential works on the subject, *Conics*, by Apollonius of Perga (262−190 BC), which can be seen as a generalization of the results obtained by Euclid in the 11th book of his *Elements*. The peculiar approach Apollonius had within the *Conics* has several innovative features, such as the different construction of the cone (which now becomes a double cone, with a circular base) and the construction of the different conics. Unlike Euclid, who used different cones generated by different triangles, to define the different conics, for Apollonius the conics could be found by taking different sections of the same cone (Fig. 1.1). Following the argumentative structure of Euclid's *Elements*, Apollonius begins his *Conics* with two sets of definitions, using them to demonstrate the numerous series of propositions that follow. For example, in the first book, geometric objects are defined, such as the cone or the conic section. After this, sixty propositions and various corollaries follow: these propositions concern the different curves which are generated by a plane secant to the cone (i.e. the parabola, the hyperbola, the ellipse and the circumference), explaining their properties and often making use of figures to demonstrate the statements made. This kind of rigorous and organic construction of the dissertation was an important legacy from ancient Greek geometry, crucial to the historical development and expansion of both mathematical and physical knowledge.

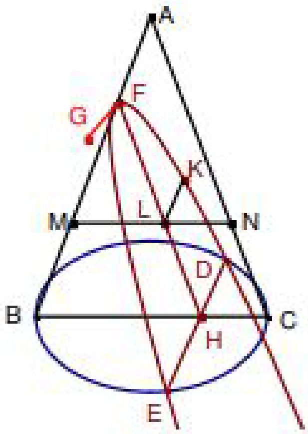


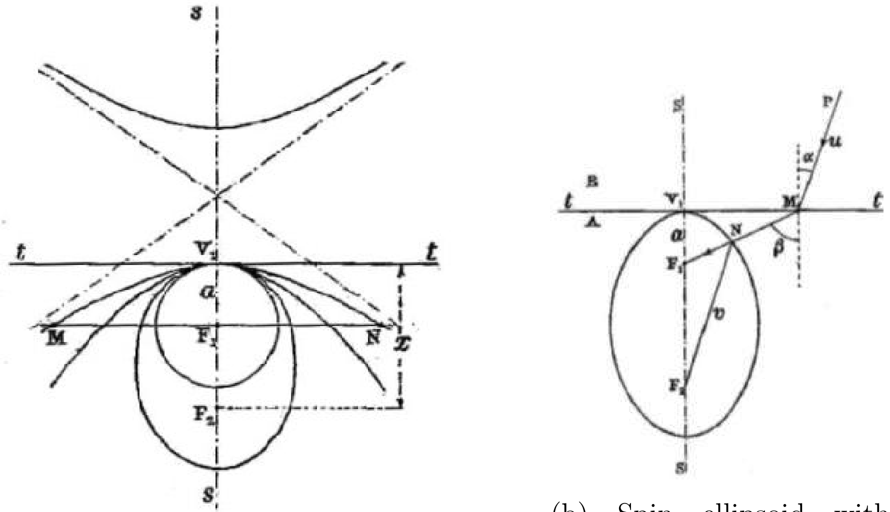
Fig. 1.1: The sign of the parabola.

Particular importance has been attached, in the sub-module, to reflect on the study of conics through their construction with mathematical machines: conic sections can be generated through intersections between a plane and a cone, but it is also very important to consider the construction of conics realized through these mathematical machines. The constructability of the conics through these machines is linked to the characterization of the conics in terms of the focal properties.

Among the construction of the conics there is, however, the special case of the parabola: in fact, despite conics' focal properties were part of the investigations, Apollonius himself excluded the parabola from the discussion on the focal properties of the conics. It is not a coincidence that, until Kepler, there was no wire machine that allowed its construction. The issue of why Apollonius treated the parabola differently when he concentrated on the focal properties was debated. It stimulated a reflection that made it possible to stress that Apollonius avoided including the parabola in the treatment based on focal properties because he had considered "the lines that, reflected, converge towards the focus" as parallel. Upstream, there was in fact a very important issue for the Ancient Greeks: the concept of *infinity*, a well-known epistemological obstacle.

To think of parallel lines as lines that can meet at a "point at infinity" requires a major step forward, not only conceptually, but also philosophically. Centuries passed, and it was necessary to break out the theoretical constraints of the discipline in order to take this important step forward. It was Kepler who would later significantly advance the study of the parabola in analogy to other conics, thanks to an excursion into the study of optical phenomena.

Step by step, the focus shifted towards physical topics. Around the 13th century, the study of conics was resumed with great interest, since there were numerous physical studies, in particular optical, concerning them. In this period for example, Witelo, in *Optics*, resumed the study of the focal properties of conics, since these properties are applied in burning mirrors (mirrors whose surfaces have the shape of a rotating paraboloid). Kepler really loved the analogies: he thought that phenomena in nature that might not seem to have any apparent relationship to each other, when studied in depth and detail, they can present surprising analogies (Fig. 1.2).



(a) Continuous plane deformation.

(b) Spin ellipsoid with analogy between reflection and refraction.

Fig. 1.2: Kepler's contribution to the theory of conics (from Viola).

Driven by this passion to use them, he studied reflection and refraction, looking for an analogy between the two phenomena through conics. Reasoning by analogy, he associated parallel lines with the particular case of incident lines meeting at infinity. Thanks to this study, Kepler was able to find a way to unify the study of all the conics from the point of view of their focal properties, also including the parabola.

1.3.2 Parabolic motion – the theme from a physical point of view

After highlighting the role played by physics in the characterization of conics in mathematics, introducing the concept of focus at infinity and the possibility of constructing a parabola with a mathematical machine, the following sub-module foresees the analysis of the role played by the parabola in the birth of modern physics (Renn et al., 2001). In fact, the parabolic motion has been treated for its role in changing the cosmological vision (from medieval to modern), in the process of unification of "the earth and the skies", and in the mathematization of the "sublunary world" (Renn et al., 2001; Koyré, 1939).

Particular emphasis is paid on showing to what extent the two basic motions – uniform rectilinear motion and uniformly accelerated motion – represented a profound ontological change in the description of the world. Their distinction overcame the medieval distinction between natural and violent motions and opened the way to a new conceptualization of the relationship between matter, space and time (see Gilbert & Zylbersztajn, 1985, for a summary of the historical episode for didactic purposes).

For many centuries, the Aristotelian paradigm formed the basis of the scientific and philosophical theories presented to society and influenced the work of many scientists. At the end of the XVI century, the trajectory of a projectile was still being investigated; the first steps were being taken towards the mathematization of phenomena "on Earth" and the construction of new interpretative lenses. The distinction between violent and natural motion was affected, and the interpretative schemes that were considered, according to tradition, were the straight line and the circular line (the strong Aristotelian influence saw the circumference and the straight line as the only two irreducible figures).

According to Tartaglia's theory, the trajectory of a projectile was a composition of three parts: there was initially a straight part, followed by a section of a circle, which ended in a straight vertical line (Fig. 1.3). The first part (AB) of the trajectory derived from the idea that initially there was a dominant violent motion, as well as the central curved part (BC), in which the violence of the motion decreased more and more; the last straight part (CD) was instead in accordance with the prevalence of the weight of the projectile over the violent motion and with the tendency of the bodies to reach the centre of the Earth (Renn et al., 2001).

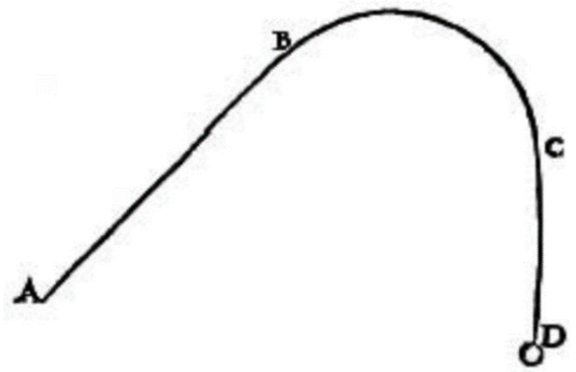


Fig. 1.3: Tartaglia's representation of the projectile motion in his *Nova Scientia* (1537).

After this contextualization, key historical texts from Guidobaldo del Monte and Galileo Galilei are analysed, to recognize some epistemological core elements of their writings, that paved the way to establish the specific method.

1.3.2.1 Guidobaldo del Monte

Specifically, the sub-module foresees, as a first step, an analysis of the following excerpt from Guidobaldo's notebook (1592), concerning the study of the motion of a projectile, which is accompanied by an explanatory drawing (Fig. 1.4):

> «If one throws a ball with a catapult or with artillery or by hand or by some other instrument above the horizontal line, it will take the same path in falling as in rising, and the shape is that which, when inverted under the horizon, a rope makes which is not pulled, both being composed of the natural and the forced, and it is a line which in appearance is similar to a parabola and hyperbola. And this can be seen better with a chain than with a rope, since [in the case of] the rope ABC, when AC are close to each other, the part B does not approach as it should because the rope remains hard in itself, while a chain or little chain does not behave in this way. The experiment of this movement can be made by taking a ball colored with ink,

and throwing it over a plane of a table which is almost perpendicular to the horizontal. Although the ball bounces along, yet it makes points as it goes, from which one can clearly see that as it rises so it descends, and it is reasonable this way, since the violence it has acquired in its ascent operates so that in falling it overcomes, in the same way, the natural movement in coming down so that the violence that overcame [the path] from B to C, conserving itself, operates so that from C to D [the path] is equal to CB, and the violence which is gradually lessening when descending operates so that from D to E [the path] is equal to BA, since there is no reason from C towards DE that shows that the violence is lost at all, which, although it lessens continually towards E, yet there remains a sufficient amount of it, which is the cause that the weight never travels in a straight line towards E.»[1] (from Renn et al., 2001, pp. 15−16)

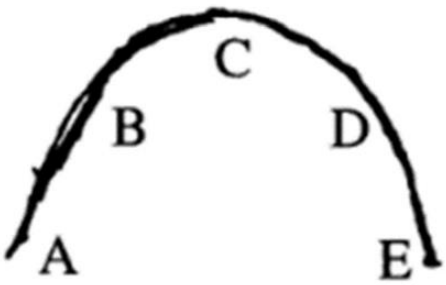


Fig. 1.4: Guidobaldo's representation of the projectile motion (1592).

The excerpt, despite its conciseness, is particularly interesting because it highlights the arguments used to describe and interpret a phenomenon that requires radical paradigm shifts from the medieval ones. Analysis of this passage shows how the study of parabolic motion and its mathematical interpretation played a leading role in the steps that led to the birth of modern physics and what characterizes it as a discipline, namely the scientific method based on experiment, understood as

[1] Tr. «Se si tira una palla o con una balestra o con artiglieria, o con la mano, o con altro instrumento, sopra la linea dell'horizonte, il medesimo viaggio fa nel callar che nel montare e la figura è quella che rivoltata sotto la linea horizontale fa una corda che non stia tirata, essendo l'un e l'altro composto di naturale e di violento et è una linea in vista simile alla parabola et hyperbole e questo si vide meglio con una catena che con una corda, poiché la corda ABC, quando AC sono vicini la parte B non si accosta come dovrebbe perché la corda resta in sé dura. Che non fa così una catena, o catenina. La esperienza di questo moto si po' far pigliando una palla tinta d'inchiostro, e tirandola sopra un piano di una tavola, il qual stia quasi perpendicolare all'horizonte, che se ben la palla va saltando, va però facendo li punti, dalli quali si vede chiaro che sicome ella ascende così anco descende ... la violentia che ella ha acquistato nell'andar sù, fà che nel callar vada medesimamente: superando il moto naturale nel venir giù ... essendo che non ci è ragione che dal C verso DE mostri che si perda a fatto la violentia ... La violenza che ha superato [il percorso] da B a C, conservandosi, opera in modo che [il percorso] da C a D sia uguale a CB, e la violenza che diminuisce gradualmente quando si discende opera in modo che [il percorso] da D a E sia uguale a BA, poiché non c'è motivo da C verso DE che dimostri che la violenza sia completamente persa, ma che, sebbene diminuisca continuamente verso E, tuttavia ne rimanga una quantità sufficiente, che è la causa per cui il peso non viaggia mai in linea retta verso E.» (from Cerreta, 2019)

a method for producing new knowledge, and the use of mathematical language to investigate nature.

This is an important step because it implies a paradigm shift from the medieval one, in which natural motion and violent motion were always distinct and the trajectory could not be symmetrical, but a juxtaposition of rectilinear and circular parts; the recognition of the symmetry of the curve helped revising the concept of motion.

From this observation, one then looks for the mathematical curve that could best represent this motion and design an experiment to support the hypotheses made. Although a similarity with the parabola and the hyperbola is evident, Guidobaldo concludes that the catenary was the most appropriate candidate to describe the trajectory, since it could be explained in terms of combination of violent and natural motions.

The analysis of this passage was carried out to show, in this delicate epistemological transition, how a discourse was being articulated, that we now recognize as typical of physics. It can be divided into two parts:

1. First, there is a simple and very "concrete" description of the process, made of direct experience, search for regularities from the observation and for the answer to the question about the shape of the trajectory (without yet assuming interpretative hypotheses). A method for producing knowledge is emerging, which consists of understanding how to visualize the phenomenon, how and what to observe and how to "give reasons" for the observation.

2. Then, the explanatory part is constructed, so that we move from experience to experiment, from description to interpretation of observations. The method becomes more sophisticated. Interestingly, the assumptions made about the mathematical nature of the trajectory are then translated into the "physicality" of the trajectory, the chain.

1.3.2.2 Galileo Galilei

The experiment described by Guidobaldo in his notes and the process of mathematizing movement continues in Galilei (1638)'s *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (tr. "Dialogues concerning two new sciences" by Crew & De Salvio (1914)). The analysis of this text is the second step foreseen by the sub-module, so a summary of its content is presented in Table 1.1.

Table 1.1: Brief description of the content of *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Galilei, 1638).

DAY	PLOT
First day. *First new science, treating of the resistance which solid bodies offer to fracture.*	It covers the tensile strength of strings; the shear strength of nails; the compressive strength of solids; the chemical structure of materials; the existence of a vacuum; comparisons between bodies made up of indivisible atoms and continuous bodies; the buoyancy of bodies; optics and parabolic mirrors; the speed of light; falling bodies of different weights; falling bodies in a vacuum and in air; pendulum oscillations; acoustics and musical harmony.
Second day. *Concerning the cause of the cohesion.*	It is the first rational treatise on the science of construction that goes beyond the empirical and approximate criteria of Renaissance treatises on architecture. It demonstrates the rule of resistance of beams or the law of right bending. Engineers in the 18th and 19th centuries based their theory of the beam on Galilei's research. Galilei's static law of the beam is the most important discovery of building resistance at the basis of modern engineering.
Third day. *Second new science, treating of motion.*	The discovery of the principles of dynamics is illustrated, including the principle of inertia and the principle of constant accelerations in falling bodies. The chapter shows the principles of dynamics for uniform rectilinear motion and uniformly accelerated motion, from which the equations of motion for falling bodies and mechanical vibrations are deduced, and the principle of isochronism of the pendulum (important for the measurement of time). Galilei's first experiments on the fall of small metal balls on an inclined plane are illustrated and the related geometric-mechanical demonstrations.
Fourth Day. *Violent motions. Projectile.*	First scientific theory of body movements in two dimensions, applied to projectile launching. Galilei shows that the projectile travels along a parabolic orbit and demonstrates the principle of composition of movements. At the end of the chapter, the geometric theory of projectile launching is summarized in ballistic tables, to be used to calculate the range of the projectile, as the angle of the inclination of the cannon on the ground varies.

At the end of the second day, a "wonderful way" of drawing the parabola is presented, with some easy and fast rules that do not exploit the mathematical properties of the parabola, but the trace left by the movement of a round bronze ball thrown on an inclined metal mirror:

> «There are many ways of tracing these curves; I will mention merely the two which are the quickest of all. One of this is really remarkable; because by it I can trace thirty or forty parabolic curves with no less neatness and precision, and in a shorter time than another man can, by the aid of a compass, neatly draw four or six circles of different sizes upon paper. I take a perfectly round brass ball about the size of a walnut and project it along the surface of a metallic mirror held in a nearly upright position, so that the ball in its motion will press slightly upon the mirror and trace out a fine sharp parabolic line; this parabola will grow longer and narrower as the angle of elevation increases. The above experiment furnishes clear and tangible evidence that the path of a projectile is a parabola [. . .]. In the execution of this method, it is advisable to slightly heat and moisten the ball by rolling in the hand in order that its trace upon the mirror may be more distinct.»[2] (from Crew & De Salvio, 1914, pp. 148−149)

Then, at the beginning of the third day, Galilei explains in great detail the "physical procedure" and the "experimental specifications" for drawing the curve; he based his reasoning on a rigorous mathematical proof, claiming that the two motions to be composed are the equable one (uniform rectilinear) on the horizontal and the uniformly accelerated one on the vertical, removing once and for all the idea of natural and violent motions. This part of the text will be illustrated and deeply analysed under the linguistic point of view in Section 2.4: on pages 58−59, it is possible to read the extract in question and see the drawing made by Galilei (Fig. 2.12).

What emerges from the analysis of Galilei's proof is that mathematics is used with a *structural role* in the construction of physics as a discipline. This is in contrast to the approach of many modern textbooks, in which parabolic motion is obtained through algebraic steps, thus using mathematics as a mere calculation tool (Uhden et al., 2012). This passage is also analysed by observing that a very sophisticated discursive practice is highlighted, in which the discourse is articulated on different levels: the mathematical, the "physical observational" and the more properly experimental. Methodologically, some mathematical properties of the parabola are

[2]Tr. «Modi di disegnar tali linee ce ne son molti, ma due sopra tutti gli altri speditissimi glie ne dirò io: uno de i quali è veramente maraviglioso, poiché con esso, in manco tempo che col compasso altri disegnerà sottilmente sopra una carta quattro o sei cerchi di differenti grandezze, io posso disegnare trenta e quaranta linee paraboliche, non men giuste sottili e pulite delle circonferenze di essi cerchi. Io ho una palla di bronzo esquisitamente rotonda, non più grande d'una noce; questa, tirata sopra uno specchio di metallo, tenuto non eretto all'orizonte, ma alquanto inchinato, sì che la palla nel moto vi possa camminar sopra, calcandolo leggiermente nel muoversi, lascia una linea parabolica sottilissimamente e pulitissimamente descritta, e più larga e più stretta secondo che la proiezzione si sarà più o meno elevata. Dove anco abbiamo chiara e sensata esperienza, il moto de i proietti farsi per linee paraboliche [...]. La palla poi, per descrivere al modo detto le parabole, bisogna, con maneggiarla alquanto con la mano, scaldarla ed alquanto inumidirla, ch´e così lascerà più apparenti sopra lo specchio i suoi vestigii.» (from Galilei, 1638, pp. 145−146)

highlighted and clarified, but the message is also conveyed that the trajectory is a parabola, understood as a mathematical object. It is also interesting to note that reflections on the role of mathematics in the construction of the science of reality can be opened up.

Fig. 1.5 summarizes the historical path of the parabolic trajectory of projectile motion. With Galilei and Guidobaldo, geometric laws seemed to take on real value and dominate terrestrial physics; for this reason, the language which nature must be questioned with becomes a mathematical language.

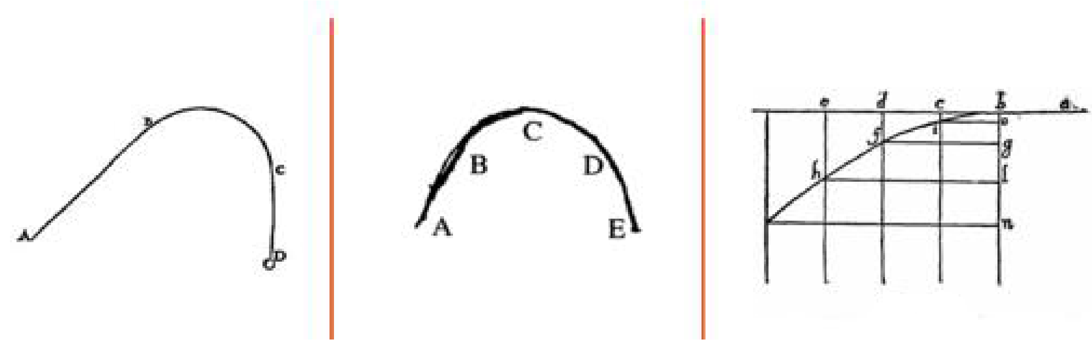


Fig. 1.5: Comparison of the three trajectories drawn by the scientists. From left: Tartaglia's representation; Guidobaldo's representation; Galilei's representation.

These studies show that both symmetry and proof played an important role in the modern view of physics, as they were concepts that triggered new ways of looking at earth phenomena. This new way of seeing radically changed the relationship between knowledge and reality, and was only possible through a close collaboration and integration between physics and mathematics.

The discourse related to the mathematization of terrestrial phenomena ends with a brief discussion of the "return from earth to sky" through Newton's *Principia*, in which attention is paid to the initial conditions of motion and speed, which determine the characteristics of the curve. Finally, he concludes that curves are not only parabolas, but more generally conics characterized by an eccentricity that depends on the initial physical conditions.

1.4 The choice of the texts and the content

What has been presented so far represents the context of this thesis, and from this the research questions emerge. The purpose, as anticipated in Section 1.3, is to create tools that can contribute to the implementation of sub-modules 4 and 5. In the previous experimentation, the tools used for the linguistic and epistemological analysis were drafted and needed to be refined and made more and more operational.

The tools used in this thesis have been developed contextually to their application to the analysis of a chapter of the high school textbook *Physics*, volume 1, by James S. Walker (2017). This text was already analysed by Gombi (2020) in his Master thesis, with the aim of designing a didactic activity. According to his idea, the main steps should guide through the evolution of physical thought under the epistemological point of view, from Tartaglia's theory of projectile motion to Galilei's demonstration of the parabolic trajectory of projectiles. To do so, he elaborated the course materials of PLS in order to construct lenses for textbook analysis, and applied them to the chapter of motion in two dimensions.

However, Walker's textbook is not the only text taken into consideration. In Chapter 2, where the linguistic tool is designed and applied, the analysis is also made on two other texts: the first is chapter 0 of *I problemi della fisica. Meccanica e termodinamica* (Cutnell et al., 2015), a textbook that serves as a synthesis; the other is an extract from *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Galilei, 1638), a historical text whose content has already been illustrated in Table 1.1.

The reason why we chose to consider two other texts in addition to the one selected by Gombi (2020) is that we needed a further text to calibrate the linguistic tool. Therefore, it was decided to first run it on Cutnell's textbook, whose results were partially predictable even without the application of the tool, thanks to its brevity; then it was applied on Galilei's extract, on which other researchers had already carried out analyses and drawn conclusions, in order to be able to compare the results obtained.

Below is a detailed description of the topics covered in *Physics*. In Walker's textbook, the chapter *Two-Dimensional Kinematics* is the fourth and it comes after chapter 2, dedicated to the one-dimensional kinematics, and chapter 3, about vectors in physics. Chapter 4 consists of 20 pages, articulated in five paragraphs, plus 9 pages made of review, conceptual questions and problems without solutions.

Introductory page

The topic is introduced with the following sentence on the first page of the chapter:

> «The main idea in this chapter is quite simple: horizontal and vertical motions are independent. That's it. This chapter develops and applies the idea of independence of motion to many common physical systems.» (page 88)

§ 4.1 – Motion in Two Dimensions

At the beginning, the motion in a two-dimensional space is exploited in order to introduce the reader to the independence of motions. To do so, it is observed the analogy between two procedures: first calculating the distance traveled by a turtle as $d = v_0 t$ along its straight line and, then, calculating the components of the distance along x and y, by projecting the distance vector on the axis. It is shown that the same result is obtained by projecting the velocity vector on the two axes first, and then calculating the distances traveled along x and y. Starting from this alternative way to solve the problem and the equations developed in the chapter about one-dimensional motion, this paragraph is devoted to obtain the equations of motion in two-dimensions, both for constant velocity and acceleration, in order to have all the instruments to obtain further results. These equations are listed in Table 1.2.

Table 1.2: Constant-acceleration equations of motion.

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$v_x = v_{0x} + a_x t$	$v_x^2 = v_{0x}^2 + 2a_x \Delta x$
$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$	$v_y = v_{0y} + a_y t$	$v_y^2 = v_{0y}^2 + 2a_y \Delta y$

§ 4.2 – Projectile Motion: Basic Equations

Here it is developed the physical model of the projectile, as an application of the independence of motion. The idea of what a projectile is in physics is spelt out in this section, as follows:

> « [...] a **projectile** is an object that is thrown, kicked, batted, or otherwise lunched into motion and then allowed to follow a path determined solely by the influence of gravity.» (page 92)

Then, the necessary assumptions are established, commenting on the gravitational acceleration of the objects and the possibility to neglect the Earth rotation and the air resistance. These assumptions are now incorporated in the general equations of motion, so that the equations for the case of the projectile motion are obtained in Table 1.3, by placing $a_x = 0$ and $a_y = -g$:

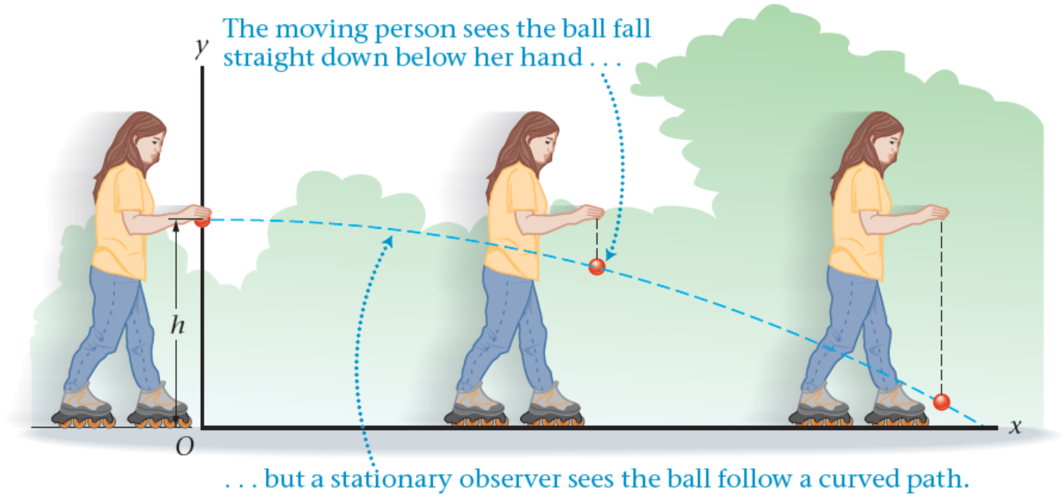
Table 1.3: Projectile motion.

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = x_0 + v_{0x}t$	$v_x = v_{0x}$	$v_x^2 = v_{0x}^2$
$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$	$v_y = v_{0y} - gt$	$v_y^2 = v_{0y}^2 - 2g\Delta y$

Before ending, it is described a situation that the student has to imagine, which leads to visualize the independence of motions in a real-life situation (Fig. 1.6). It is the "concrete demonstration" that horizontal and vertical motions are independent.

§ 4.3 – Zero Launch Angle

This section is devoted to discuss the particular case of a fully horizontal launch of the projectile, so that the angle between the initial velocity and the horizontal plane is $\theta = 0$. Considering that the x and y components of the initial velocity are $v_{0x} = v_0 cos0 = v_0$ and $v_{0y} = v_0 sin0 = 0$, the Table 1.4 shows how the equations of projectile motion are simplified, also setting $x_0 = 0$ and $y_0 = h$.



▲ **FIGURE 4-3 Independence of vertical and horizontal motions** When you drop a ball while walking, running, or skating with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.

Fig. 1.6: Demonstrating independence of motion (from Walker, 2017, p. 93).

Table 1.4: Equations for zero launch angle.

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = v_{0x}t$	$v_x = v_0 = constant$	$v_x^2 = v_0^2 = constant$
$y = h - \frac{1}{2}gt^2$	$v_y = -gt$	$v_y^2 = -2g\Delta y$

Moreover, these results are used to algebraically demonstrate that the trajectory $y = h - \frac{1}{2}gt^2 = h - \frac{1}{2}g(\frac{x}{v_0})^2 = h - (\frac{g}{2v_0^2})x^2$ has the form of $y = a + bx^2$, a parabola.

The last section is dedicated to obtaining the mathematical expression of the landing point.

§ 4.4 – General Launch Angle

This paragraph is an extension of the previous one, as it is considered the overall situation where the launch angle can be any value. In this way, the general equations of motion for the projectile are deduced and shown in Table 1.5.

Table 1.5: Equations for nonzero launch angles.

Position as a function of time	Velocity as a function of time	Velocity as a function of position
$x = (v_0 cos\theta)t$	$v_x = v_0 cos\theta$	$v_x^2 = v_0^2 cos^2\theta$
$y = (v_0 sin\theta)t - \frac{1}{2}gt^2$	$v_y = v_0 sin\theta - gt$	$v_y^2 = v_0^2 sin^2\theta - 2g\Delta y$

§ 4.5 – Projectile Motion: Key Characteristics

Finally, the mathematical expressions for the range and maximum range are deduced.

Than, it is explained that the projectile motion has several symmetries, first of all the symmetry of the parabola itself: Fig. 1.7 shows that the projectile lands with the same speed it had when it was launched, even if the velocities are different, because of the directions of motion.

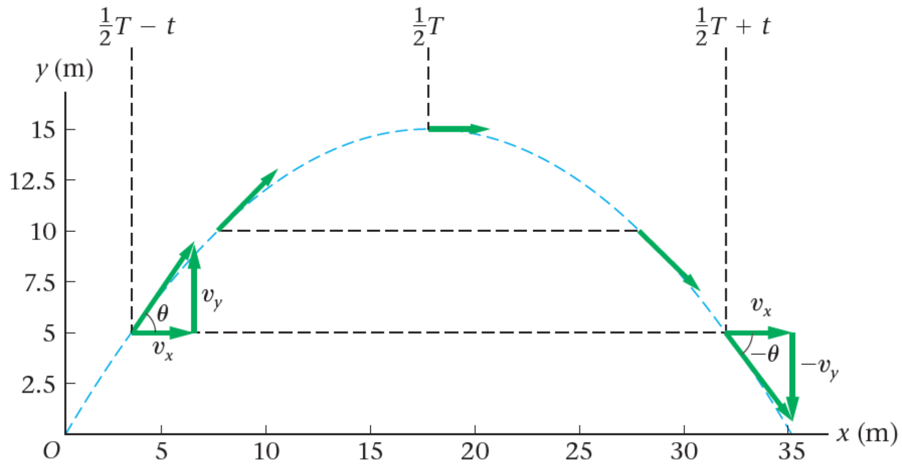


Fig. 1.7: Velocity vectors for a projectile launched at the origin (from Walker, 2017, p. 105).

To conclude the chapter, the author presents the properties of symmetries concerning the time of flight, the velocity vectors at a given height and the range of the projectile.

Chapter 2

The linguistic tool

Physics, like all the other disciplines, has its own object of study and its own ways of organizing and expressing its knowledge. It shares many of the characteristics of the other sciences, but also maintains its own distinctive features—for example the large emphasis on mathematics throughout almost all contexts (Doran, 2017). Natural language, as well as mathematics, allows physics to convey the complex nature of phenomena.

For this reason, language is one of the necessary components for understanding the scientific discourse, although it is often overlooked: to be literate in science means also to be able to understand the technical language that is used (Halliday & Martin, 1993). Scientists think about the world differently from common people and, as a result, science has its own specific language, which makes it not easily intelligible. Therefore, knowing the rules of this language and being able to identify them in a text are the first steps towards a correct interpretation of scientific discourse.

This chapter is devoted to describe the design and the application of a linguistic tool, elaborated in order to perform a linguistic analysis and to bring out the structure of a scientific written text, also from an interdisciplinary perspective. The idea of a linguistic analysis refers to the third point of innovation presented in the IDENTITIES project framework, described in Section 1.2. It aims to look at the two disciplines and their intertwining with different techniques and tools than those sciences usually involve. To do so, it will be necessary to go through the study of the special language of science.

In Section 2.1, I will illustrate the features of scientific language; in particular, since it has been chosen to apply the analysis to high school textbooks, the properties of this particular text type will be observed, and I will describe the grid that allowed to achieve certain results. Then, in Section 2.2 and 2.3 I will focus on the two textbooks selected for the analysis, *I problemi della fisica. Meccanica e termodinamica* (Cutnell et al., 2015) and *Physics* (Walker, 2017), looking, in particular, at the representation of the relationship between physics and mathematics. Finally, in Section 2.4 the grid will also be applied to a historical text, *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* (Galilei, 1638, tr. "Dialogues concerning two new sciences" by Crew & De Salvio (1914)), to test its effectiveness on a different text type and to observe the differences between modern texts and those of the 17th century. The comparison of the results will be presented in Section 2.5.

2.1 Text features and tool design for the linguistic analysis

Making a linguistic analysis of a text means studying how it is characterized by certain lexical choices and morphological and syntactic solutions. From the 1970s onwards, scholars began to observe the text as a unit, moving from a sentence's perspective (which was the main object of study until then), to study the ways in which it works, identifying how syntactic and semantic structures operate in its construction (De Beaugrande et al., 1994).

Both spoken and written texts can be defined, according to different parameters, into various typologies. The notion of "text", originally elaborated by textual linguistics but now widely accepted also in school grammars, refers to the metaphor of the "fabric", the weft of single threads which gives life to an organic whole (in Latin *textus* is the participle of the verb *texere*, "to weave").

The conditions for having a linguistic production (oral or written) as a text are seven: cohesion, coherence, intentionality of the sender, acceptability of the receiver, informativity, situationality and intertextuality. Cohesion and coherence are considered the most important, because they directly regard the text. Cohesion consists in respecting the grammatical relationships (number concordance between subject and predicate, gender concordance, a proper word order) and the syntactic connection among the various parts. Coherence, on the other hand, concerns the meaning of the text, and is therefore linked to the reaction of the recipient (Serianni, 2012, pp. 25−39). The others regard the relationship between sender and recipient, and the context in which the text is built. In a scientific community, the intention can be to inform about, to analyse some data or to demonstrate a hypothesis or a theory. The acceptability of the text is valued by the scientific community, who can recognise the features requested to be a scientific text. The informativity depends on the information value in a field: it has to convey new information about a topic, and it has to suit the situation and recall some other texts (the literature of a specific field). Further, linguists distinguish three regulative principles that must be fulfilled in order to speak about a text: effectiveness, efficiency, appropriateness.

In a scientific text, thus, sender and receiver are part of a specific community, in which the members use a particular variety of language, the specialized language for the scientific communication. Then, their texts have specific characteristics, that differentiate the common texts to the scientific texts, on different levels: morphological, lexical and textual levels. However, the lexicon provides the distinctive elements that identify a special language both with respect to other specialised languages and with respect to the common language (Cortelazzo, 1994).

2.1.1 The specialized language of science

To say what a specialized language is, we resume a definition suggested by a linguist, Cortelazzo (1994), who quotes Berruto (1974, p. 68):

> «a special language is a functional variety of a natural language, dependent on a specialized field of knowledge or sphere of activity, used, in its entirety, by a group of speakers that is smaller than the

> totality of the speakers of the language of which the special language is a variety, in order to satisfy the communicative needs (primarily referential needs) of that specialized field; the special language is constituted at the lexical level by a series of additional correspondences with respect to the general and common ones of the language and at the morphosyntactic level by a set of selections, recurring with regularity, within the inventory of forms available in the language.»[3]

According to this definition, the specialized language represents the variety of a natural language used by expertise, so it is used by a limited number of speakers, and it is meant to satisfy the communication needs of a certain community.

On the linguistic level, a specialized language is mainly marked by a specialised lexicon and the preference for certain syntactic structures though all allowed by the grammar of a language. The most common examples are the passive voice, the the agentless constructions[4], and the nominalization[5], aimed to represent the information in an objective way.

Regarding the syntax, there are also features which are sufficiently extensive in respect to common texts and scientific texts in history. Indeed, the history of the written scientific language is marked by the accentuation and consolidation of at least two important evolutionary tendencies:

- The strong development of the noun compared to the verb. In all specialized languages, the most informative terms tend to be nouns; verbs rather play a linking role and have a generic semantic content (Cortelazzo, 1994).
- The simplification of periods. In the course of time, the length of the period in scientific prose is progressively reduced: in the seventeenth century, a period contained an average of 63 words, 53 in the eighteenth century, 37 in the nineteenth, 27 in the twentieth (Viale, 2019).

Thus, the period contracts, as it tends to consist of a few words, and the noun increases its relevance, as it becomes information: this means that the semantic density of each sentence increases. Although this double tendency is generalized in the scientific field, it has a significantly different impact, depending on the type of text. On average, popular texts are more syntactically complex and less semantically and lexically dense than specialist texts (Gualdo & Telve, 2011, p. 240).

[3]Tr. «per lingua speciale si intende una varietà funzionale di una lingua naturale, dipendente da un settore di conoscenze o da una sfera di attività specialistici, utilizzata, nella sua interezza, da un gruppo di parlanti più ristretto della totalità dei parlanti la lingua di cui quella speciale è una varietà, per soddisfare i bisogni comunicativi (in primo luogo quelli referenziali) di quel settore specialistico; la lingua speciale è costituita a livello lessicale da una serie di corrispondenze aggiuntive rispetto a quelle generali e comuni della lingua e a quello morfosintattico da un insieme di selezioni, ricorrenti con regolarità, all'interno dell'inventario di forme disponibili nella lingua.» (Cortelazzo, 1994, p. 8)

[4]The scientific text focuses on objects, events and processes, especially in their abstractness, generalisability and timelessness, and not on the agent; this transforms the events described into processes. (Treccani)

[5]It is the transformation of a predicative sentence, which contains a verb, into a nominal sentence, where the verb is deleted and its functions are taken over by the noun. For example: "we locate the particles" becomes "the localization of particles" (Gualdo & Telve, 2011, p. 118).

The information density is given both by the specialised, but also by the construction of particularly complex syntagma, which can be formed by, for example, constructions as the following: noun + adverb + adjective (i.e. "algebraically closed field"), or several adjectives (i.e. "stable charged subatomic particles"), or even chains of complements around the verb. The result is that the information load is no longer on the verb, which loses importance both semantically (use of relation and status verbs) and morphologically (use of the present tense), but on the noun.

In particular, the lexicon, with its specific terms, is quite difficult, impenetrable for a common user of a language, an inexpert, because it conveys the concepts, notions, and defines tools of a particular field (Serianni, 2012, p. 91). For example, scientific terms like "eclampsia" or "eviction" are known only by sector professionals; on the contrary, some other terms used by specialists are widely known to the general public, because they are becoming part of the everyday language, like "stomatitis" and "indult", that are terms used by doctors and lawyers' language respectively.

The first two terms do not give rise to ambiguity, because they can be used only in their technical meanings. In many other cases, however, specialized languages use the *redetermination*, which consists in assigning a technical meaning to common words. This can cause possible misunderstandings. In physics, for example, terms such as "momentum", "force", "work", etc. have both a common and technical meanings: the latter one can be unintelligible for a speaker who is not part of the scientific community.

Further, the words are used just with a denotative meaning: for example, the word "oxygen" in chemistry only indicates the element of the periodic table, labelled with an "O" and characterized by certain features, whereas it never takes the meaning of "relief", as it can happen in common language (i.e. "That loan will get me a breath of oxygen"). This "emotional neutrality" is another feature clearly distinguishes specialized languages from common language, according to the need to represent the information in an objective way, as we have already seen in the syntactic level (Serianni, 2012, pp. 90−92).

On a textual level, we see that scientific texts present a combination of coding, which involves mathematics, images, symbolism, demonstrations and many other resources, that each bring their own functionality to the communication.

This information provides a detailed picture of the important aspects that need to be considered when examining a scientific discourse; therefore, an analysis grid will be proposed on the basis of what has just been described.

2.1.2 Presentation of the analysis tool

Summing up what has been described, it is now possible to build up an investigative tool, capable of pursuing the following goals:

- pointing out the structure of the scientific discourse;
- recognizing the characteristics of physics and mathematics that are represented in the text;
- providing an image of both disciplines as they are represented by the senders.

The result of these studies is the grid represented in Table 2.1.

According to the theoretical framework, the grid for the linguistic analysis of a specialized text is articulated in three parts (textual, syntactic and lexical level). The structure is based on the following features:

1. *Textual level.* It is whereby is possible to observe more general structure of the text and the thematic progression, and how it describes the interweaving of mathematics and physics. We have looked at the thematic progression (whether is linear or not), at the use of connectives, repetition, implicit content, etc. This can highlight how different parts of the text communicate with each other to deliver the message as intended by the sender.

2. *Syntactic level.* It is that reveals the prevalence of coordinates or subordinates and, hence, points out the complexity of the arguments proposed, the logical links between different portions of sentences and text.

3. *Lexical level.* It is constituted by the particular terminology used in the text, is fundamental to observe the scientific lexicon and its belonging to different disciplines. This can tell, in a first approximation, how much the lexicon of different disciplines is present and which these disciplines are; also this analysis allows to deeply consider which and whether terms can belong to one or more disciplines. Particular attention has been addressed to the type of the verbs used and how they convey a certain image of the disciplines.

Table 2.1 represents the analysis The first column describes the three linguistic levels: textuality, syntax, and lexicon; the second proposes some questions aimed to address the analysis; the third proposes the analytic focus to find the answer (what one has to focus in the text to search for the answers); finally, the last one describes the goals of the analysis.

Table 2.1: Grid for the linguistic analysis of scientific texts.

Levels	Questions	Analytic focus	Goals
Textual	- How does the information develop? - What codes, in addition to the natural language, are present? What functions do they have? How do they dialogue with the main written text? - Is there information that remains implicit?	- Observation of the text in its information blocks (i.e. paragraphs); - Observation of how the various codes interact with each other.	- Check what images of the disciplines emerge.
Syntactic	- Does the paratactic or hypotactic structure prevail? - What types of subordinates are present in the text? - What relationships are established between the various clauses of information?	- Analysis of grammar and syntax of clauses.	- Point out the structure of information and its richness.
Lexical	- Which physics' words are technical terms? Which are mathematical terms? - Which highlighted technical terms do occur most frequently? - What are the most frequent verbs? What kind of verbs are they? What meanings do they convey?	- Use of a dictionary, to check all the potential meanings of a term and also its fields; - Count words and verbs and analyse their meaning and frequency; - Analysis of portions of text containing technical terms from different fields.	- Cluster and quantify the presence of technical terms relating to physics, mathematics, or positioned in their boundaries; - Observe what images of the disciplines emerge from the used verbs.

This tool is fundamental if one wants to approach any text and extract significant information regarding the idea of the relationship between physics and mathematics that emerges from it. In order to test the just developed grid, it was decided to apply it to a particular kind of scientific text: the high school textbook.

2.1.3 A context for application: the scientific textbook

There are many classification models for texts, but the most traditional one separates texts on the basis of a functional criteria, depending on the aim of the text: to describe, to report, to instruct, etc. (see Werlich, 1982, pp. 39−41, for further details). Usually, no text converges in one category only, since every part of the text has a specific function (for example, a text aimed at argument will have part dedicated to describe data); nevertheless, the categorization is based on the dominance of a function on others (Lala, 2011).

School textbooks have always been the main instrument of dissemination, and they are very powerful resources in the decisive phase of learning a discipline; but, above all, they convey a study method (Serianni, 2012, p. 177). It falls into the category of the informative/expository texts, as it has the aim of enriching the receiver's knowledge about a certain subject. It may also contain, to a certain extent, parts of narrative texts (historical parts, telling of anecdotes), descriptive texts (descriptions of phenomena, objects), regulatory texts (how to make experiments) and argumentative texts (convincing of the validity of a certain hypothesis).

The scientific textbook is characterised by the presence of well-organized contents through a clear articulation into blocks, often composed by a main text, supplementary texts and paratexts (tables, boxes, graphs, etc.), which usually show a gradual increase of information according to a logical criterion. The content of the textbook should supply all the information needed to build the knowledge, including especially what the receiver is not required to know before reading that textbook. It is usually designed with an introduction, where the main argument is defined, a main part, and a conclusion, where there is a summary of the results. As a consequence of this structure, the text results particularly rigid in its linguistic structural choices, in the sense that it has to describe phenomena as exactly as possible, using abstract constructs and rigorous ways to illustrate experiments or report demonstrations. In most cases, the authors tend to leave little room, if any, to interpretation (Gualdo & Telve, 2011).

In the last 40 years, school textbooks have been significantly renewed: in particular, they have change how they address the reader, adopting linguistic solutions that imitate the linguistic features of media dissemination (especially on television). It is a trend that can be observed in different ways at all school levels (Gualdo & Telve, 2011, pp. 196−200).

Further, the titles of subsidiaries do not evoke the content (i.e. *Sussiblu*, *Domino*, *TG school*), as they used to in the past, and so do secondary school textbooks, which during the years have replaced those title with aseptic and referential expressions, like *Introduction to...* or *Elements of...*, with more promotional and vivacious ones, like *Physics: ideas and experiments. From pendulum to quark* (Gualdo & Telve, 2011, p. 197).

The *discursiveness* is precisely the new dominant feature of textbooks style, where there are several rhetoric and formalist expedients (Gualdo & Telve, 2011, p. 197):

- the arguments are announced ("In the following sections it will be discussed...");
- the discourse is addressed to the reader through the first and second plural person ("Have you ever seen this equation?", "As we found out in the previous chapter...") rather than a third-person;
- the suspension points are used to create suspense ("If you think about it, after all...");
- the author use expressions that are typical of spoken language and she/he refers to daily life, both in lexicon ("a pinch of sulfur") and metaphors ("ATP is like a loaded spring");
- the text is enriched with iconic apparatus (graphs, tables, figures, etc.);
- the discourse is interspersed with questions which aim to draw students' attention, proposing a new topic ("What to measure, and how?") or sharing a reflection with them ("Are we really sure?").

As can be noticed, some of these measures are not new in dissemination discourse, but are shared with the most attractive tool belonging to the TV documentary. It takes advantage of dialogic and emphatic mechanisms to achieve the maximum involvement from the public. Actually, the presentation of the topic is present in TV documentaries; as well, the use of the including first plural person; the continuous search of contact with the audience; the use of similes and metaphors, in particular the one of the journey which pushes on the borders of rationality, to touch legend and fantastic; the use of mechanisms of suspense (i.e. "We are going to talk about a topic which has always divided scientists"); etc. There are also several technicalities, but are mainly collected within the episode, not in the title, and are paraphrased or presented with synonyms.

Some formal contacts between the didactic tool par excellence (the textbook) and the luckiest informative tool (the documentary) may have originated from an emulative attempt of the first towards the second. Even if the interplay could be unhealthy for the didactic activity, it has been noted that a more reasoned relationship between these communication methods may result in an upgrading of both, to eliminate the deficiencies (Gualdo & Telve, 2011, p. 198).

For what concerns the interdisciplinary, the analysis conducted on textbooks reveal unwillingness to it, and even if the quality of content and method could result appropriately and reasonably, the part relating to the scientific explanation and experiment turns out to be lacking (in contrast to the common sense). The historical and social dimension of science seem to be marginal or ignored. This may contribute to building an abstract idea of science (deprived of references to people, places, etc.). As a result, scientific theories tend to be presented as information of absolute value, because the idea of inquiry and research is left behind, and so the critical thinking: this leads, sometimes, readers to doubt the same scientific method and it pushes the students to memorize information, descriptions, statements, etc. (Gualdo & Telve, 2011, pp. 196−197).

As announced, the grid developed in Section 2.1.2 will be tested on textbooks, so the characteristics that distinguish them must be taken into account in order to make an analysis as accurate as possible.

2.2 First application of the tool to a textbook

As previously stated, I applied the grid to two high school physics textbooks; in particular in this section I analyse the chapter 0 of *I problemi della fisica. Meccanica e Termodinamica* (Cutnell et al., 2015). The version of the chapter I chose is in Italian, so it was subjected to a translation from English which, although minimal, modified the original version; however, this factor will not be taken into account, as it does not influence the result.

The chapter 0 is entitled *Richiami di cinematica* and offers a review of the main definitions and formulas resulting from the study of rectilinear, accelerated, parabolic and circular motions. Therefore, it is not suitable for those approaching these topics for the first time, as it does not argue the concepts proposed nor demonstrate the results obtained, but presents a synthesis that aims to summarize and collect the main information.

I decided to apply the grid on this chapter to test its validity, before using it on a more complex textbook. Being a summary, the chapter carries an imagine of physics and mathematics already defined, without any explicit explanation, and it results quite evident to the reader. The use of the grid help to observe which linguistic elements (words, construction, etc.) contribute to the formation of these images.

2.2.1 The linguistic analysis of the chapter

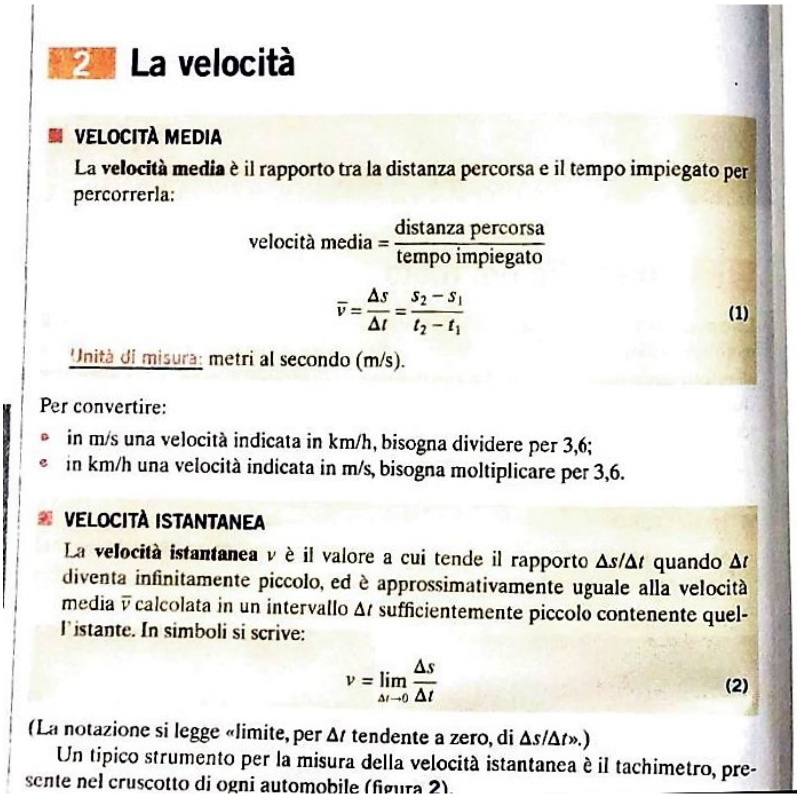
The chapter 0 consists of 12 paragraphs, which follow the typical progression of information presented by all textbooks (Gombi, 2020, p. 35):

- the first four paragraphs deal with the uniform rectilinear motion, the concepts of rectilinear motion, average speed and instantaneous speed, space-time graph;
- from paragraph 5 to 9 are presented the notions of uniformly accelerated rectilinear motion, which define the concepts of mean acceleration and instantaneous acceleration, the space-time and space-speed graphs, and free-fall motion;
- the following paragraphs 10 and 11 explain the composition of motions and describe the projectile motion and trajectory, and its characteristics;
- the chapter is concluded with the circular motion.

The text leaves most of the reasoning as implicit content and, at the same time, the explicit content is not enough to support reasoning: most of the paragraphs do not exceed 17 lines in length (with a minimum of 8 and a maximum of 38), which means an average of 6.5 sentences per paragraph (with a minimum of 2 and a maximum of 15).

Each paragraph contains one or more definitions, expressed either in language or in mathematical formulas; a few supplementary lines, for the purpose of solving the exercises; a series of exercises, guided or not, with a graphic apparatus or images that explain the requirements. An example is provided by Fig. 2.1 and Fig. 2.2.

The given information is structured to be efficient, in the sense that the minimum necessary is provided to recall the main results to the student's memory.



2 La velocità

VELOCITÀ MEDIA

La **velocità media** è il rapporto tra la distanza percorsa e il tempo impiegato per percorrerla:

$$\text{velocità media} = \frac{\text{distanza percorsa}}{\text{tempo impiegato}}$$

$$\bar{v} = \frac{\Delta s}{\Delta t} = \frac{s_2 - s_1}{t_2 - t_1} \quad (1)$$

Unità di misura: metri al secondo (m/s).

Per convertire:

- in m/s una velocità indicata in km/h, bisogna dividere per 3,6;
- in km/h una velocità indicata in m/s, bisogna moltiplicare per 3,6.

VELOCITÀ ISTANTANEA

La **velocità istantanea** v è il valore a cui tende il rapporto $\Delta s/\Delta t$ quando Δt diventa infinitamente piccolo, ed è approssimativamente uguale alla velocità media $\bar{v}$ calcolata in un intervallo Δt sufficientemente piccolo contenente quel-l'istante. In simboli si scrive:

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \quad (2)$$

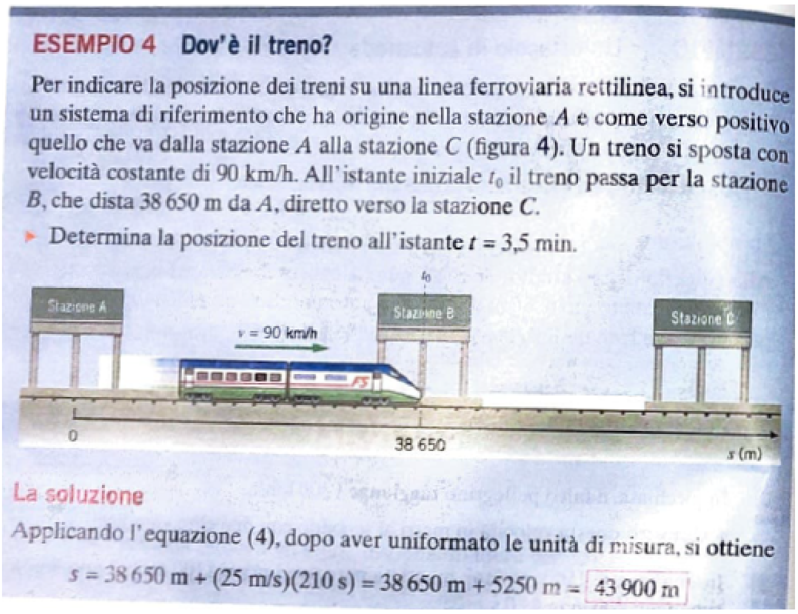
(La notazione si legge «limite, per Δt tendente a zero, di $\Delta s/\Delta t$».)

Un tipico strumento per la misura della velocità istantanea è il tachimetro, presente nel cruscotto di ogni automobile (figura 2).

Fig. 2.1: Example of the structure of a paragraph (from Cutnell et al., 2015, p. 12).

From the syntactic point of view, the chapter is characterized by the brevity of the period, which in large part coincide with the sentence delimited by the point: each sentence contains on average 21.5 words (ranging from a minimum of 15 to a maximum of 30). As a consequence, parataxis prevails: in each sentence, only one concept is expressed, rarely contextualized or deepened, so that one has the feeling that the content is nothing more than a list of information. Indeed, there are neither argumentation nor demonstration to support the data or the results shown in the text. Sometimes, we can find some sentences introduced by "cioè" (tr. "which means"), which should introduce an explanation of what precedes; however, instead of explaining the concept just stated, the clause introduced by "cioè", usually, transcribe the content conveyed by natural language to mathematical formulas and vice versa. Some conditional periods are present in the typical mathematical writing as "if... then...", in particular, when the initial conditions are presented in order to write the laws of motion.

The lexicon in this textbook presents a very high presence of technical terms in relation to the number of nouns: out of a total of 896 words (excluding articles and conjunctions) there are 349 technical terms, which is equivalent to more than 39% of the total considered. This fact reveals a very high semantic density: if each sentence has on average 13 words (excluding articles and conjunctions), this means that 5 will be technical terms belonging to the two disciplines considered.



ESEMPIO 4 Dov'è il treno?

Per indicare la posizione dei treni su una linea ferroviaria rettilinea, si introduce un sistema di riferimento che ha origine nella stazione A e come verso positivo quello che va dalla stazione A alla stazione C (figura **4**). Un treno si sposta con velocità costante di 90 km/h. All'istante iniziale t_0 il treno passa per la stazione B, che dista 38 650 m da A, diretto verso la stazione C.

▸ Determina la posizione del treno all'istante t = 3,5 min.

La soluzione

Applicando l'equazione (4), dopo aver uniformato le unità di misura, si ottiene

$$s = 38\,650 \text{ m} + (25 \text{ m/s})(210 \text{ s}) = 38\,650 \text{ m} + 5250 \text{ m} = \boxed{43\,900 \text{ m}}$$

Fig. 2.2: Example of a guided exercise (from Cutnell et al., 2015, p. 14).

This is also demonstrated by another fact: out of a total of 1685 words (including articles and conjunctions) only 151 verbs appear, namely the 9% of the total words. The technique of *nominalization*, presented in Section 2.1.1, can be observed here in its maximum expression. In particular, most of these verbs are the verb "to be" in copula function; this fact is in line with the idea of the formular to provide mostly definitions and list formulas. Procedural or thinking verbs are practically absent.

The large presence of coordinated sentences, the lack of argumentations and explanation, and the high density of technical and scientific terms show that physics is represented mostly as a list of definitions and mathematical formulas. Consequently, mathematics results to assume an instrumental role, used as an algebraic tool for computation. Indeed, an informative text, aimed to explain the reasoning and to demonstrate the results, should have a higher syntactic complexity and a lower semantic and lexical density (Gualdo & Telve, 2011, p. 240).

These choices are adequate and certainly in line with the purpose of the chapter, which is to summarise the notions already explained in previous volumes of the textbooks. The grid has verified what we expected from a chapter dedicated to summarising the main concepts on the motion: the central part of the chapter aims to recall notions that are considered already known to the reader by the author.

2.3 Application of the tool to the textbook *Physics*

As said in Section 1.4, the main text considered for the analysis is the fourth chapter of the textbook *Physics* (Walker, 2017), that deals with parabola and the parabolic motion. After having tested the grid, we are now ready to apply it to a more complex chapter of a textbook, in order to bring out the linguistic features useful to describe how the image of physics and mathematics are conveyed.

Before effectively starting the analysis, one idea must be kept in mind: when an entity is mentioned in the text for the first time, it begins to exist, to be part of the textual world, of the representation, whether it exists in the extra-textual reality, in the external world, or no. A distinction can therefore be made between objects of reality and objects of the text or the universe of discourse (Andorno, 2003, pp. 27−39). For example:

> «In this section we develop equations of motion to describe objects moving in two dimensions.» (page 89)

> «To study motion with constant acceleration in two dimensions [...] » (page 91)

The entities underlined in these sentences begin to form in the reader's mind as soon as they are named. Properties and qualities can thus be attributed to these entities in the text, to the conceptual image, which may or may not refer to extra-textual reality.

Thus, a scientific textbook also constructs and conveys to the reader a textual world. Reading a physics textbook, therefore, also means getting an image of physics, of its objects and of the ways in which they are investigated, beyond the notions and definitions. In this regard, the linguistic choices made by the authors of textbooks determine the representation that the reader will have of a certain discipline. We will look at a textbook from a linguistic point of view and try to understand how the message is conveyed, in the following section.

2.3.1 Textual level

The chapter consists of 20 pages, articulated in an introductory page and 5 paragraphs; 9 pages review the previous content, arising conceptual questions and proposing problems without solutions.

The first page presents the title of the chapter, *Two-dimensional kinematics*, and a figure that is supposed to represent parabolic motion. On the right side, there is a first box with paratext, the *Big Ideas*, in which are listed three concepts, numbered with the purpose to show the sequence of the conceptual network of the content, like a summary. The first one is the following:

> «1. Two-dimensional motion consists of independent horizontal and vertical motions.» (page 88)

It represents the most general and important concept, because it recalls the title and it is defined as the main idea of the chapter.

The second and third ideas are the following:

> «2. Objects in free fall move under the influence of gravity alone.» (page 88)

> «3. Objects in free fall follow parabolic paths.» (page 88)

The latter can be considered as "secondary" concepts, because they are the application of the main idea: the second idea anticipates what happens when dealing with an object in free fall, that is $a_x = 0$ e $a_y = -g$; the third one anticipates that the trajectory drawn by this object has the shape of a parabola. They are briefly introduced in the first page and recalled singularly right next to the text they are talking about, mildly adding further specifications.

Few lines are dedicated to the enunciation of the main idea, which is proposed as the guide idea of the whole chapter:

> «The main idea in this chapter is quite simple: horizontal and vertical motions are independent. That's it. This chapter develops and applies the idea of independence of motion to many common physical systems.» (page 88)

Differently to the scientific language of science, which tends to avoid connotations, in the textbook, the main idea is described through the connotative adjective "simple": this seems to be an expedient to reassure the reader about the simplicity of the concept, making her/him continue reading without any fear of the new physical topic.

Then, it is important to note that they have a didactic introduction just after their title, in which the topic are defined:

> 1. «In this section we develop equations of motion to describe objects moving in two dimensions. First, we consider motion with constant velocity, determining x and y as functions of time. Later, we investigate motion with constant acceleration.» (page 89)
>
> 2. «We now apply the independence of horizontal and vertical motions to projectiles.» (page 92)
>
> 3. «A special case of some interest is a projectile launched horizontally, so that the angle between the initial velocity and the horizontal is $\theta = 0$. We devote this section to a brief look at this type of motion.» (page 94)
>
> 4. «We now consider the case of a projectile launched at an arbitrary angle with respect to the horizontal.» (page 99)
>
> 5. «We conclude this chapter with a brief look at some additional characteristics of projectile motion that are both interesting and useful. In all cases, our results follow as a direct consequence of the fundamental kinematic equations (Equations 4-10) describing projectile motion.» (page 103)

These anticipations seem to be inscribed in the tendency to assume stylistic features typical of mass media dissemination, discussed in Section 2.1.3, and to reflect the intention of the sender (the author of the textbook) to guide the reader in the structure of the text or, at least, to make her/him aware of the thematic progression of the discourse.

In order to give a complete but schematic picture of the progression of the topic, I have made up a concept map.

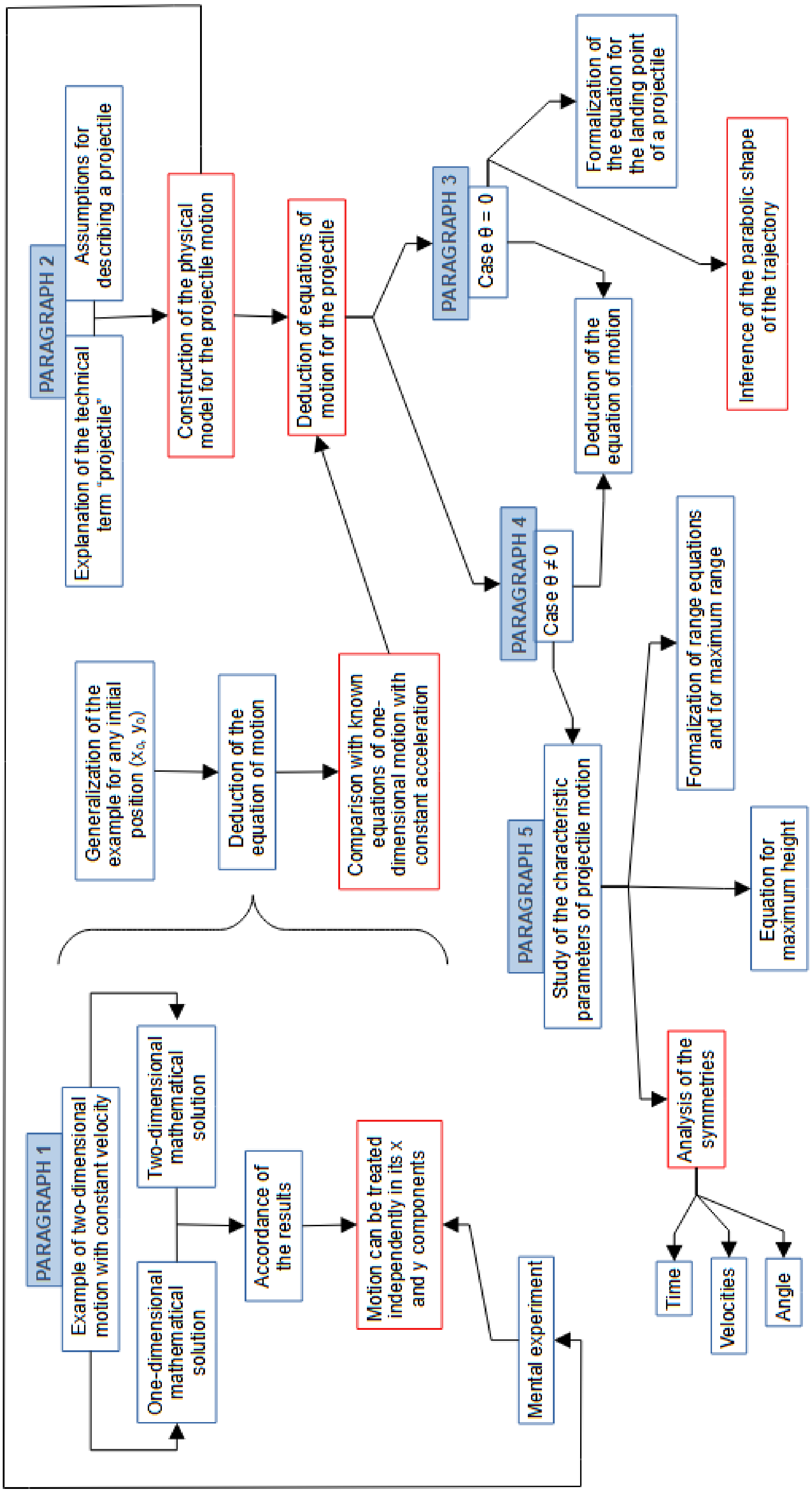


Fig. 2.3: Concept map summarizing the chapter four of the Walker's textbook.

As can be seen from the diagram in Fig. 2.3, paragraph 4.1 deals with the description of motion in two dimensions from the point of view of its fundamental equations. It shows two different techniques for solving the proposed exercise which lead to the same correct result, demonstrating that motion can be treated independently in its x and y components, and the general equations are stated.

The following paragraph 4.2 is devoted to the definition and assumptions about the projectile motion and to the rewriting of the equations of motion for the particular case of projectile motion; this part concludes with an experiment showing what is the shape of its trajectory.

Paragraphs 4.3 and 4.4 are devoted to the study of two-dimensional motion in the cases in which the angle between the initial velocity and the horizontal plane is zero or non-zero; in particular, it is demonstrated that the curved trajectory of the projectile motion is mathematically a parabola.

The chapter ends with paragraph 4.5, where the main physical features involving the projectile motion are shown, as range, symmetry, etc.

Thus, the content progresses by, adding gradually more information about the main topic stated at the beginning of the chapter, the motion in two dimensions. In this way, the textbook begins from the definition of motion in two dimensions, it applies it to the particular case in which a body can be treated as a projectile, and it derives equations for every possible launching situation. This progression is explicit in the first few lines of all the paragraphs, so that the student is always involved in the general plan of the discourse and the reasoning that the textbook conveys.

It has to be highlighted that, in some paragraphs, the information is organised as blocks of knowledge about a particular activity to do, in order to describe the procedures to obtain understanding of the topic or formulas. For example:

> «To begin, consider the simple situation shown in FIGURE 4-1. [...] First, we determine the speed of the turtle in each direction. [...] Next, we find the distance traveled by the turtle in the x and y directions [...] » (page 89)

The expressions “to begin”, “first”, “next” convey the idea of a knowledge structured as a to-do list.

For the purposes of interdisciplinary analysis, it is interesting to note that, usually, physics’ community organises the knowledge in its papers, textbooks, etc. mixing different codes: natural language, mathematics, graphs and images (Doran, 2017); indeed, Walker’s textbook adopts linguistic, mathematical and iconographic languages, which interact with each other.

Mathematical language appears, first of all, in the form of formulas, which are mainly used in two ways:

1. repeating a concept previously expressed through words, as in the following sentence:

   > « [...] the turtle moves in a straight line a distance given by speed multiplied by time: $d = v_0 t$» (page 89)

2. presenting sets of equations, necessary to carry out the reasoning and to solve the exercises. Indeed, the equations of motions are derived; then, they are

adapted and applied to the particular case of the projectile motion; later, they are rewritten as a function of the launch angle; finally, the formula for the range of the projectile in relation to this angle is determined.

Furthermore, mathematics dialogues very actively with images: figures and graphs are very common in the chapter, which, in turn, constantly interact with the main text. These figures (i.e. Fig. 2.4) do not simply reproduce real situations, as an exemplification, but are always inserted in a two-dimensional reference system. Often, the initial conditions of the situation described in the images are specified (i.e. position, initial velocity, etc.), the trajectories of the bodies are traced, and anything that may be useful to observe is specified.

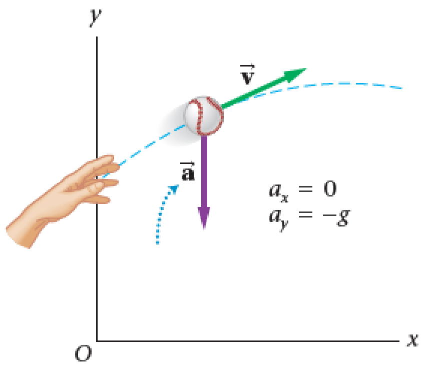


Fig. 2.4: Example of figure (from Walker, 2017, p. 93).

The main text is often interspersed with guided exercises, as shown in Fig. 2.5. The various codes are mixed, with the aim of producing an exercise clear and understandable: in fact, the text which introduces the problem is always supported by an image which illustrates it, flanked by an explanation on the left and right sides of the main text. Also, there are a part in which the strategy to be undertaken is explained, recalling the necessary equations, and a part where the solution of the problem is supplied, in each passage, expressed both linguistically and mathematically.

Within the main text, small sections are devoted to an in-depth study: they are called *Real World Physics* and show concrete applications of the topic discussed in the respective paragraphs. The first of these sections is found in the first paragraph and illustrates a possible application of the mathematical description of motion in three dimensions: the Traffic Collision Avoidance System. The second is in the third paragraph and shows figures of parabolas produced by real-world projectiles, like lava bombs and fountain jets. The last one is in the fifth paragraph and describes a golf ball being dropped on the moon: the example is aimed to highlight how range and gravity are inversely proportional.

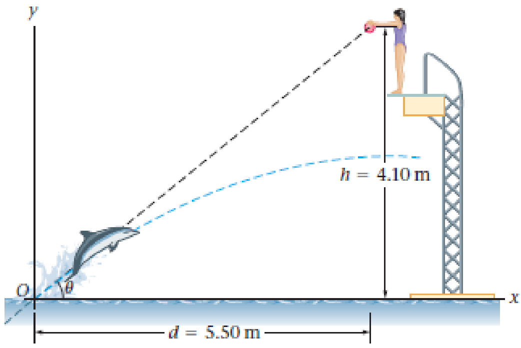
The presence of this kind of content may encourage students to consider the concrete applications of physics, and not just see it as a theoretical discipline that remains in the textbook. In this regard, it is also interesting to note the presence of *QR codes*, that link to videos of real experiments, sometimes proper laboratory equipment, to allow students to appreciate the results of physics in ideal settings.

Already on the first page of the chapter (but in general in all the paragraphs), it has been noted that the modelling process is not explicit, but it is taken for granted:

A trained dolphin leaps from the water with an initial speed of 12.0 m/s. It jumps directly toward a ball held by the trainer a horizontal distance of 5.50 m away and a vertical distance of 4.10 m above the water. In the absence of gravity, the dolphin would move in a straight line to the ball and catch it, but because of gravity the dolphin follows a parabolic path well below the ball's initial position, as shown in the sketch. If the trainer releases the ball the instant the dolphin leaves the water, show that the dolphin and the falling ball meet.

PICTURE THE PROBLEM

In our sketch we have the dolphin leaping from the water at the origin $x_0 = y_0 = 0$ with an angle above the horizontal given by $\theta = \tan^{-1}(h/d)$. The initial position of the ball is $x_0 = d = 5.50$ m and $y_0 = h = 4.10$ m, and its initial velocity is zero. The ball drops straight down with the acceleration due to gravity, $a_y = -g$.



REASONING AND STRATEGY

We want to show that when the dolphin is at $x = d$, its height above the water is the same as the height of the ball above the water. To do this we first find the time when the dolphin is at $x = d$, then calculate y for the dolphin at this time. Next, we calculate y of the ball at the same time and then check to see if they are equal. Because the ball drops from rest from a height h, its y equation of motion is $y = h - \frac{1}{2}gt^2$, as in Equations 4-7 in Section 4-3.

Known Initial speed, $v_0 = 12.0$ m/s; initial horizontal distance to ball, $d = 5.50$ m; initial vertical distance to ball, $h = 4.10$ m; acceleration of ball and dolphin, $a_y = -g$.

Unknown Height at which dolphin and ball meet, $y_d = y_b = ?$

SOLUTION

1. Calculate the angle at which the dolphin leaves the water:

$$\theta = \tan^{-1}\left(\frac{h}{d}\right) = \tan^{-1}\left(\frac{4.10\text{ m}}{5.50\text{ m}}\right) = 36.7°$$

2. Use this angle and the initial speed to find the time t when the x position of the dolphin, x_d, is equal to 5.50 m. The x equation of motion is $x_d = (v_0 \cos\theta)t$:

$$x_d = (v_0 \cos\theta)t = [(12.0\text{ m/s})\cos 36.7°]t = (9.62\text{ m/s})t = 5.50\text{ m}$$

$$t = \frac{5.50\text{ m}}{9.62\text{ m/s}} = 0.572\text{ s}$$

3. Evaluate the y position of the dolphin, y_d, at $t = 0.572$ s. The y equation of motion is $y_d = (v_0 \sin\theta)t - \frac{1}{2}gt^2$:

$$y_d = (v_0 \sin\theta)t - \tfrac{1}{2}gt^2 = [(12.0\text{ m/s})\sin 36.7°](0.572\text{ s}) - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2 = 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

4. Evaluate the y position of the ball, y_b, at $t = 0.572$ s. The ball's equation of motion is $y_b = h - \frac{1}{2}gt^2$:

$$y_b = h - \tfrac{1}{2}gt^2 = 4.10\text{ m} - \tfrac{1}{2}(9.81\text{ m/s}^2)(0.572\text{ s})^2 = 4.10\text{ m} - 1.60\text{ m} = 2.50\text{ m}$$

Fig. 2.5: Example of guided exercise (from Walker, 2017, p. 102).

immediately after the explication of the main idea («horizontal and vertical motion are independent»), there is the anticipation of what will take place in the chapter, namely the application of the independence of motions "to many common physical systems". Going from the main idea to the anticipation, a very important passage is left as implicit content, which non-expert readers would hardly notice by themselves, and it is the following: the fact that independence of motion will be applied to a modelling of reality, the physical systems, and not to reality itself. Studies in *Physics Education* show that this lack could create many difficulties in understanding the arguments, since the concept of approximation, on which the laws of physics are based, is not emphasised.

Moreover, there is no trace of a cultural and historical contextualization, neither in the introductory part nor in the sections that follow: the author immediately concentrates the attention of the reader on the description of the main scientific content, without contextualising it, without any information about the framework in which the topic of the parabola and parabolic motion was born and developed. This is a very common choice of modern textbooks, maybe also because of space limitations. In any case, they tend to take out this part as it is seen as unnecessary, superfluous, useless for understanding the main scientific notions (from Gualdo & Telve, 2011, pp. 196−197). This may cause students to miss the questions from which the physic investigation began and the context where the scientific concepts have been constructed. In particular, as shown in Section 1.3, the topic of parabola and parabolic motion is very rich and can easily be approached from an interdisciplinary perspective, precisely because of the history behind it. Nevertheless, by omitting it, an opportunity is missed to bring it to the fore.

In textbooks, the definitions of the scientific terms should be frequent. Nevertheless, the definition of *projectile*[6] is often taken for granted in many textbooks (as shown in Bagaglini et al., 2021, p. 38. For example, there is no such definition in Cutnell, as well as in other textbooks). On the contrary, this clarification of projectile's the meaning is not left out by Walker. It is mentioned on the introductory page of the chapter, where it is said:

> «When you hear the word projectile, you probably think of an artillery shell or a home run into the upper deck. But the term projectile applies to any object moving under the influence of gravity alone. For example, a juggling ball undergoes projectile motion—and follows a parabolic path—as it moves from one hand to the other. In this chapter we explore the physical laws that govern projectile motion.» (page 88)

And then, in paragraph 4.2, the one concerning the description of its motion, a

[6] In the English language, *projectile* is a strong technical term: Collins dictionary defines it in as "an object or body thrown forward" as the first meaning. So, in English language, there is no ambiguity about the reference, since we are already in the field of mechanics. On the contrary, in Italian, the term can be confused with the object that is inserted into a weapon in order to fire it (in English, it is a "bullet"). Indeed, also in Italian dictionaries the first definition of "projectile" is linked to physics: as "any body that can be thrown or is thrown" (Tr. Treccani: "ogni corpo che possa essere o sia lanciato"). However, the word has also a common meaning, referring to the military and ballistics field, as *bullet*. Consequently, in Italian, it is essential to define it within the sphere of physics.

proper definition is given:

> «Well, a **projectile** is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity.» (page 92)

Furthermore, the assumptions needed when considering an object as a projectile and studying its motion are made explicit. The fact that the term *projectile* is emphasized by the use of bold type (which is not common in the rest of the chapter) underlines again the importance that the author gives to the clearness of the concept, in order to understanding the scientific meaning of the term and, thus, the scientific concept.

Moreover, as we have already noted above, the textbook adds some boxes, called *Physics in context*, where the students can find both the references to previous chapters and also anticipation of what will be covered in following ones. For example:

> «The equations of one-dimensional kinematics derived in Chapter 2 are used again in this chapter [...] » (page 91)

> «The basic idea behind projectile motion is used again in Chapter 12, when we consider orbital motion.» (page 93)

2.3.2 Syntactic level

The small length of the sentences, illustrated in Section 2.2 for Cutnell's textbook, is also a characteristic of Walker's one. On the contrary, a feature that distinguishes the syntactic choices of Walker's textbook, unlike most of the modern textbooks, which tend to prefer parataxis, is the presence of coordinates and subordinates clauses.

In particular, the completive subordinate clauses are objects of perception and psychological verbs, concerning the request to the reader for observation and reasoning:

> «First, <u>notice that</u> the turtle moves in a straight line a distance given by speed multiplied by time [...] » (page 89)

In addition to the completive clauses, numerous implicit and explicit relative clauses have been found, which give more information about the phenomena mentioned in the text:

> «Compare these equations with Equation 2-11, $x = x_0 + v_0 t + \frac{1}{2}at^2$, <u>which gives</u> position as a function of time in one dimension.» (page 90)

> «Little error is made <u>in ignoring</u> the variation of g or the rotation of the Earth [...] » (page 93)

There are also other circumstantial subordinate clauses, such as modal and final, mostly implicit, which describe the manner and purpose of the various operations or procedures performed:

«Replacing v_0 with the x component of the velocity, v_{0x}, yields Equation 4-1.» (page 90)

«To study motion with constant acceleration in two dimensions we repeat what was done in one dimension in Chapter 2.» (page 91)

It is also possible to find conditional sentences, which explain the conditions for which certain formulas apply:

«If we choose ground level to be $y = 0$ [...] the initial position of the ball is given by $x_0 = 0$ [...] » (page 94)

Temporal subordinates are also inserted with the same intention:

«In addition, the rotation of the Earth can be significant when we consider projectiles that cover great distances.» (page 93)

Finally, in paragraphs 4.3 and 4.4 we also find causal clauses, which explain the reason for exceptions and the applications of various formulas:

«We've chosen the positive sign for the square root because the projectile was launched in the positive x direction, and hence it lands at a positive value of x.» (page 97)

«Since the projectile starts at the origin, the initial x and y positions are zero [...] » (page 99)

Usually, the fact that subordinate clauses are not used extensively reveals the absence of an explicit complex reasoning between the different parts of the discourse, in particular, the reasoning behind some passages of the formulas tends to be implicit. Therefore, these are left to infer to the students. This was clearly observed in the analysis of Cutnell's textbook of Section 2.2, but it is not completely true in Walker's textbook, which has a large number of subordinate clauses which allow the reader to follow the logical connection among information and, doing so, the reasoning underlying the explanation.

As previously mentioned, the presence of certain types of periods reveals the role that mathematics plays in the construction of the discourse: very often, in modern textbooks, there is a tendency to convey an image of the discipline that is mainly instrumental, leading students to get a limited idea of the relationship between physics and mathematics.

It was precisely for this reason that it was interesting to analyse Walker's textbook, as several roles assumed by mathematics within the physical world emerge. The presence of many subordinate clauses reveals a tendency to deepen, to avoid implicit forms, to logically connect the various parts of the discourse. Nevertheless, it is the lexicon, more than any other, that will allow to establish what role mathematics plays in each portion of the text.

2.3.3 Lexical level

The analysis of lexicon is especially focused on nouns and verbs: they both play a fundamental role in determining the image of physics, mathematics and their intertwining emerging from the text.

2.3.3.1 The nouns

The first step of the investigation on lexicon is looking at each word of the text, to establish whether they could be considered as belonging to the vocabulary of physics or mathematics. To do so, the Collins Online Dictionary and Wikipedia has been consulted; the choice of such instruments was driven by the need to find an objective support to categorize the words as part of physicist or mathematical vocabulary, without letting subjective representations and ideas interfere. During the analysis, the words have been coloured in three different colours: blue for physics, red for mathematics and green for the ones detected as parts of both contexts. Fig. 2.6 shows how the text looks like after this work.

This is exactly the opposite of the y component of the velocity when it was launched. Since the x component of velocity is always the same, it follows that the projectile lands with the same speed it had when it was launched. The velocities are different, however, because the direction of motion is different at launch and landing. In fact, there's a nice symmetry with the direction of motion: if the initial velocity is above the horizontal by the angle θ, the landing velocity is below the horizontal by the same angle θ. So far, these results have referred to launching and landing, which both occur at y = 0. The same symmetry occurs at any level, though. That is, at a given height the speed of a projectile is the same on the way up as on the way down. In addition, the angle of the velocity above the horizontal on the way up is the same as the angle below the horizontal on the way down. This is illustrated in Figure 4-15 and in the next Conceptual Example.

Fig. 2.6: Excerpt which shows how the chapter looks like after the categorization and coloring process (from Walker, 2017, p. 105).

The words linked to a word already coloured has been considered part of the same field:

> «A turtle starts at the origin at $t = 0$ and moves with a constant speed $v_0 = 0.26$ m/s in a direction 25° above the x axis. How far has the turtle moved in the x and y directions after 5.0 seconds?» (page 89)

Actually "moves" and "moved" are part of the semantic field of the word "motion", a typical concept of physics.

The same has been made for the pronouns which refer to coloured words, since these elements recall the same concept:

> « [...] what we are actually doing is simply writing these same equations again, but with different specific values substituted for the constants that appear in them.» (page 91)

> « Air resistance can be significant if a projectile moves with relatively high speed or if it encounters a strong wind.» (page 93)

The attention was addressed also to symbols, when these are used as synonyms of a technical term, and not a repetition of them:

> « Little error is made in ignoring the variation of g or the rotation of the Earth [...] » (page 93)

However, after the categorization process, it was decided to take into account only the nouns, as only these represent the technical terms of the two considered disciplines, and the verbs, because they are important in transmitting relationships between concepts.

Starting with the nouns, it was possible to quantify statistically their frequency, allowing to have visual representations about the distribution of the different lexicon throughout the chapter.

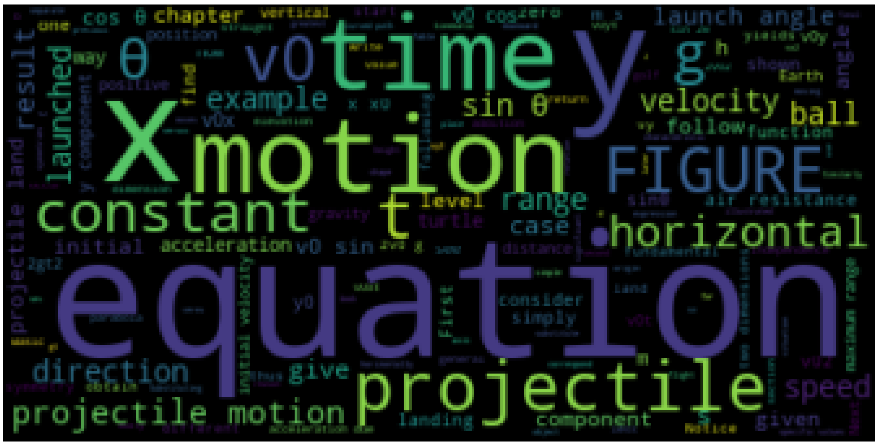


Fig. 2.7: Wordcloud of the whole text of chapter four in Walker's textbook.

Fig. 2.7 shows that "equation" seems to be the most commonly repeated word, followed by "motion", "x and y", "time" and "projectile". A more accurate counting is given by Fig. 2.8.

Observing instead the presence of all the technical terms, the distribution obtained is represented in Fig. 2.9.

Mathematics permeates many scientific disciplines, often underpinning theoretical and descriptive architecture; this happens especially in physics (Doran, 2017), as the results of this study confirms. Beyond the numerical count, it is interesting to observe how the role of mathematics changes within the chapter, in the paragraphs.

It has already been said, in Section 1.4, that the first paragraph is aimed at presenting the equations of two-dimensional motion, recalling also the equations of one-dimensional motion: in fact, as Fig. 2.9 shows, the paragraph 4.1 contains the highest number of words considered as part of the mathematical vocabulary: 101 words out of 170 technical terms (59%).

In the second paragraph, physics vocabulary prevails over mathematics one (49 words against 32), although it only accounts for 50% of the total. Here, mathematics provides the criteria to shape the model of the phenomenon, starting from physics hypotheses: actually, the projectile motion in two dimensions is studied and the mathematical equations are derived, although most of the passages are implicit.

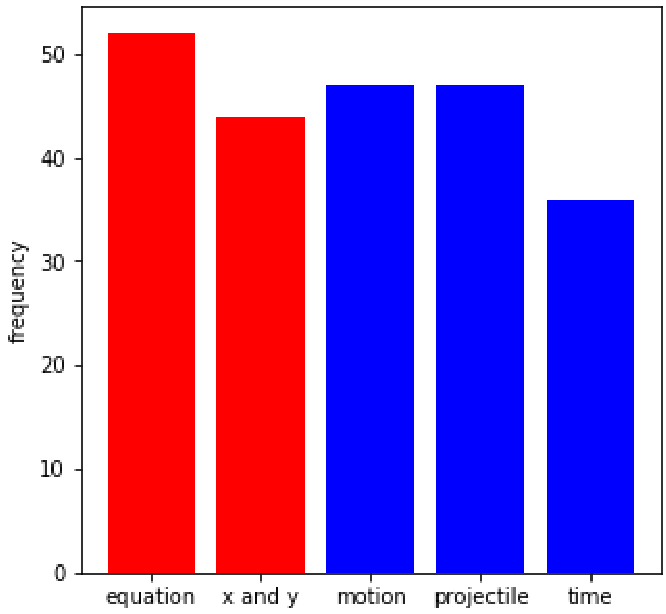


Fig. 2.8: Histogram of the 5 most frequent technical terms in the chapter.

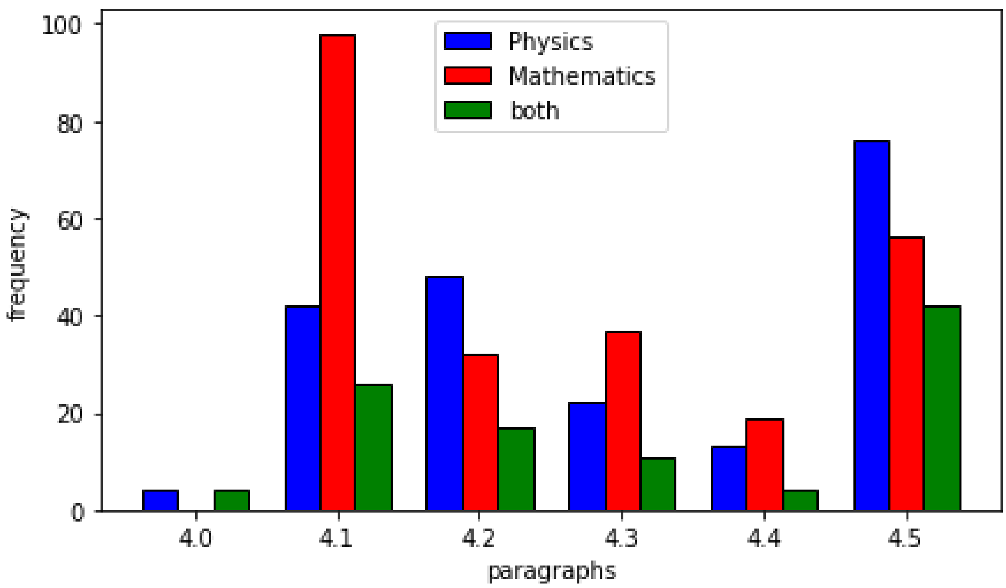


Fig. 2.9: Distribution of physics and mathematics technicalities within the chapter.

Unlike the paragraph 4.3 (as we see later), the passage entitled *Demonstrating Independence of Motion* cannot be regarded as a formal demonstration, because the argument does not follow a logical structure. The author is showing, "demonstrating", how independence of the motions can be assumed by observing the word.

In paragraphs 4.3 and 4.4, in which the projectile equations are reformulated as a function of launch angles, mathematics prevails, respectively with 51% and 53%. The third paragraph describes the shape of the projectile's trajectory, defining it starting from the equations of motion for a horizontal launch, through the application of strictly algebraic inferences:

> «Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives:
>
> $$t = \frac{x}{v_0}$$
>
> Next, substitute this result into the y equation to eliminate t:
>
> $$y = h - \frac{1}{2}g(\frac{x}{v_0})^2 = h - (\frac{g}{v_0^2})x^2$$
>
> It follows that y has the form:
>
> $$y = a + bx^2$$
>
> In this expression, $a = h = constant$ and $b = -\frac{g}{2v_0^2} = constant$. This is the equation of a parabola that curves downward, a characteristic shape in projectile motion.» (page 96)

The parabolic shape of the trajectory is deduced in a very typical way for physics textbooks, that is by means of algebraic, which is a very effective use of mathematics as it gives all the elements for exercises (Gombi, 2020, p. 35). Moreover, Gombi (2020) examined in detail which are the conditions for having a proof, and concluded by asserting that the extract just proposed can be considered a proof, despite the fact that the entire structure remains implicit and that there is not even a title or a reference to the fact that a proof is provided. For this reason, mathematics plays an argumentative role in this paragraph, because it allows the reasoning to be developed and conclusions to be inferred.

In the last paragraph, although the amount of physics terminology exceeds the other two, it has not reached the absolute majority. It is interesting to note, however, that this paragraph is the richest in technical terms (143 words and 42 boundary terms). In this case, mathematics can boost epistemological considerations about the phenomenon, stimulating a deeper comprehension: in fact, this section mainly studies the symmetry of the parabola, which originates as an object of geometry but crosses the border and enters the world of physics, providing properties to the trajectory of the projectile.

2.3.3.2 The verbs

The second step of the investigation on lexicon was looking at each verb: this analysis showed that the majority of verbs is related to the semantic field of *procedure*. This leads the reader to imagine the knowledge of physics as a list of actions to engage, in order to solve problems, and mathematics as the tool to do so. As we have noted before, the text structure sustains this image since it organizes the sub-paragraphs as blocks of information about the different actions to complete a procedure. Fig. 2.10 shows how verbs can be divided mainly into three categories, and how they are distributed within the chapter; Fig. 2.11 shows their total occurrence in percentage.

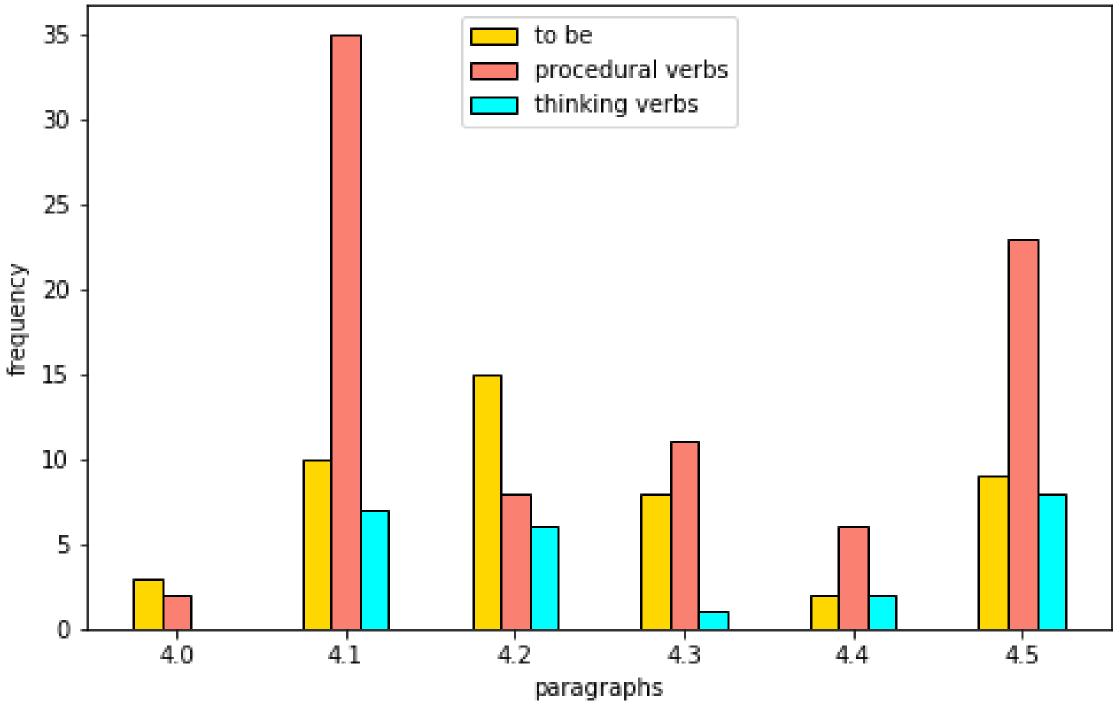


Fig. 2.10: Trend in verbs frequency within the chapter.

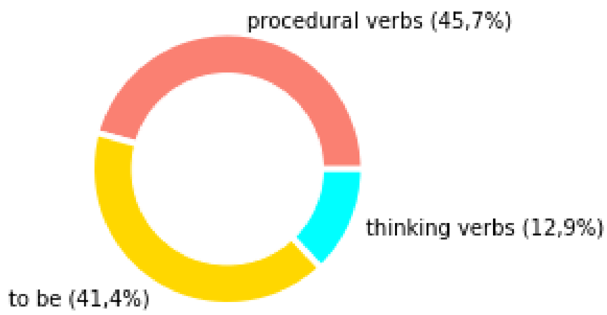


Fig. 2.11: Distribution of verbs in the three identified types.

In the chapter we detect a higher presence of *procedural verbs*, like “to obtain”, “to determine”, “to find”, “to apply”, etc., as can be seen in the following examples:

> «Next, we find the distance traveled by the turtle [...] » (page 89)
>
> «To obtain y as a function of time, we write y in place of x [...] » (page 91)
>
> «Substituting these specific values into our fundamental equations for projectile motion (Equations 4-6) gives the following simplified results [...] » (page 95)

The presence of this type of verbs is particularly evident in paragraphs 4.1 and 4.5, but is also dominant in paragraphs 4.3 and 4.4: *procedural verbs* seem to convey an image of studying physics as a following of procedure, whose results are mathematical formulas, and it seems to have its aim in developing equations.

However, there are also several verbs related to the semantic field of *conjecturing, thinking* and *reasoning*, like “to consider”, “to notice”, “to suppose”, etc., as follows:

> «Notice that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second.» (page 93)
>
> « [...] then observe its motion carefully.» (page 93)
>
> «From Equation 4-12 we see that R varies with angle as $sin2\theta$.» (page 104)

This fact reveals how the author tries to engage the reader into the reasoning, to show her/him the mathematical model.

To conclude, it is possible to notice that paragraph 4.2 is instead the one where the verb *to be* as copula[7] is dominant: in its first part, in fact, some properties of the projectile are presented and commented:

> «The acceleration due to gravity is constant [...] » (page 92)
>
> «Air resistance can be significant if a projectile moves with relatively high speed [...] » (page 93)
>
> « [...] had no effect on the ball’s vertical motion—the motions are independent.» (page 93)

The procedural image of physics is thus substituted with the description of a concept to be understood.

As a supplementary phase of the work, we also considered the need to look not only at individual words, but at a set of them. It was necessary to contextualize the

[7]A copula (or a linking verb) is a verb which links the subject of a clause and a complement. Verbs like “be”, “seem” and “become” are examples of copula (Collins Online Dictionary).

word in the discourse, looking at them in their syntactic context. We detected the entire syntagma[8].

Sometimes, indeed, it is the context that gives meaning to individual words: a semantically "neutral" term from the point of view of the field to which it belongs may lean towards one of the two disciplines, negotiating its meaning and aligning itself with the words surrounding it. For instance:

> «We've chosen the positive sign for the square root because the projectile was launched in the positive x direction, and hence it lands at a positive value of x.» (page 97)

The term "positive", taken alone, could be associated with both physics and mathematics, but also with a number of other disciplines, such as philosophy, biology, economics, etc. In the context of the sentence just quoted, however, the term belongs unequivocally to mathematics, since we are talking about the square root and the direction of the axis.

Negotiation of meaning within the context occurs not only when dealing with neutral terms, but also for terms that belong distinctly to different fields. For instance:

> «Similarly, replacing each x in Equation 4-1 with y converts it to Equation 4-2, the y equation of motion.» (page 90)

The noun "equation" is a technicality of mathematics, "motion" belongs to physics. However, if we take the whole sentence and its context into account, we end up saying that the dominant field is physics, because the specification of "motion" brings "equation" into the field of physics. We are not talking about how a mathematical equation works, but we are referring to those specific to motion.

2.3.4 Summary of the results

To conclude the analysis conducted on the textbook *Physics*, the main results obtained are briefly reported.

The application of the grid reveals an articulated structure of the chapter's contents, as shown in Fig. 2.3. The reader is guided in following the progression of information both through the anticipations and in structuring procedures to be carried out in order to obtain results.

The analysis of the syntax confirms some typical tendencies of modern school textbooks, such as the brevity of the periods, but it shows a difference of Walker compared to other textbooks, that is a more pronounced use of subordinates: this is important as it is an index of the author's attention to avoiding implicit content, constructing more parts dedicated to explanations.

The analysis of the nouns reveals a high semantic density, due to the presence of a large number of technical terms, typical of the fields of physics and mathematics, as well as terms that lie on their boundary and that, depending on the context, may belong to the former or to the latter discipline. Then, the verb analysis helps to bring out the role that mathematics plays in the construction of physical knowledge.

[8]A syntagma is a word or phrase forming a syntactic unit (Collins Online Dictionary).

Physics and mathematics are in constant dialogue in the text and, depending on the paragraph, their relationship takes on different nuances.

The first paragraph is dominated by mathematics: the aim is to recall the equations for one-dimensional motion, in order to derive those for two-dimensional motion. So, the use of mathematics is purely instrumental.

In the second paragraph mathematics plays a structural role, providing the criteria to shape the model of the phenomenon, starting from physics assumptions: actually, the projectile motion in two dimensions is studied and the mathematical equations are derived.

Paragraph 4.3 presents the actual demonstration of the equivalence between the physical equation of the projectile trajectory and the mathematical equation of the parabola: in this part of the text, mathematics plays a role in the argumentation process.

In the last paragraph, mathematics may boost epistemological considerations about the phenomenon, stimulating a deeper comprehension: in fact, this section mainly studies the symmetry of the parabola, which originates as an object of geometry, but crosses the border and enters the world of physics, providing properties to the trajectory of the projectile.

Some of these results, which are only hypothesised here on the basis of a linguistic analysis, will then be taken up and studied in Section 3.3, through the epistemological lens.

2.4 Application of the tool to a historical text

At this point, after having tested and used the grid on modern textbooks, we decided to apply it to a very different text from the school textbook, both for the textual genre and the distance in terms of the time: Galilei's *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, published in 1638, in Leiden.

The choice of analysing texts written in a different historical period and addressed to readers belonging to different socio-cultural contexts aims at observing the differences between the construction of a disciplinary discourse in a totally different context.

2.4.1 The work of Galilei and its content

Discorsi e dimostrazioni matematiche intorno a due nuove scienze is a dialogue between the scientist Filippo Salviati, the aristocrat Giovanni Francesco Sagredo and the invented character Simplicio, already the protagonists of the *Dialogo sopra i due massimi sistemi del mondo*, wrote in 1632, about the Galilei's Latin treatise. The treatise is technical and intended for the scientific community; the dialogue is written in the Italian Vulgar. The three characters read and comment on the Latin treatise and the various phenomena considered within it, including the motion of a projectile. The dialogue takes place in Venice, over the duration of a week, and it is divided into four days. A summary of the contents of the work was previously given in Table 1.1.

The linguistic analysis focuses on the first part of the fourth day, in particular the part in which the properties of projectile motion are described and its trajectory geometrically demonstrated.

In the fourth day, starting from the physical principles, definitions, axioms and theorems of the third day, Galilei moves on to the proof of the parabolic trajectory of the projectile motion. The physical assumptions are introduced in the following lines, which are an explanation of what will happen in the text:

> «In the preceding pages we have discussed the properties of uniform motion and of motion naturally accelerated along planes of all inclinations. I now propose to set forth those properties which belong to a body whose motion is compounded of two other motions, namely, one uniform and one naturally accelerated; these properties, well worth knowing, I propose to demonstrate in a rigid manner. This is the kind of motion seen in a moving projectile; its origin I conceive to be as follows: [...] »[9] (Crew & De Salvio, 1914, p. 244)

The independence of the two motions is postulated, as is the possibility of combining them:

> «Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits. But if the plane is limited and elevated, then the moving particle, which we imagine to be a heavy one, will on passing over the edge of the plane acquire, in addition to its previous uniform and perpetual motion, a downward propensity due to its own weight; so that the resulting motion which I call projection [*projectio*] is compounded of one which is uniform and horizontal and of another which is vertical and naturally accelerated. We now proceed to demonstrate some of its properties, the first of which is as follows: [...] »[10] (Crew & De Salvio, 1914, pp. 244−245)

Then, the author proceeds with a theorem, which establishes the properties of such motion:

[9] Tr. from Latin: «Nella trattazione, che ora comincio, cercherò di presentare, e di stabilire sulla base di salde dimostrazioni, alcuni fenomeni notevoli e degni di essere conosciuti, che sono propri di un mobile, mentre si muove con moto composto di un duplice movimento, cioè di un movimento equabile e di uno naturalmente accelerato: tale appunto sembra essere quello che chiamiamo moto dei proietti; la generazione del quale cosi stabilisco» (Galilei, 1638, p. 236)

[10] Tr. from Latin: «Immagino di avere un mobile lanciato su un piano orizzontale, rimosso ogni impedimento: gia sappiamo, per quello che abbiamo detto piu diffusamente altrove, che il suo moto si svolgera equabile e perpetuo sul medesimo piano, qualora questo si estenda all'infinito; se invece intendiamo [questo piano] limitato e posto in alto, il mobile, che immagino dotato di gravita, giunto all'estremo del piano e continuando la sua corsa, aggiungera al precedente movimento equabile e indelebile quella propensione all'ingiu dovuta alla propria gravita: ne nasce un moto composto di un moto orizzontale equabile e di un moto deorsum naturalmente accelerato, il quale [moto composto] chiamo proiezione. Ne dimostreremo parecchie proprietà: la prima delle quali sia [la seguente].» (Galilei, 1638, pp. 236−237)

«THEOREM 1, PROPOSITION I

> A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola.»[11] (Crew & De Salvio, 1914, p. 245)

At this point in the text, the Vulgar dialogue between Salviati, Simplicio and Sagredo begins. The latter takes its turn to speak, to express his perplexity about the comprehension of what is explained: he admits he does not have a complete knowledge of Apollonius' studies on conics and, therefore, asks Salviati to present the fundamental notions that will come into play in the demonstration of the theorem. Simplicio agrees, adding that he does not understand even the most elementary terms of the discourse. Salviati reassures them, telling them that the prerequisites are reduced to the knowledge of only two fundamental properties of the parabola, and demonstrating them geometrically in a simple way, without having to go through long and complicated reasoning. In this way, the characters alternate their turns in asking and answering questions about concepts, notions, knowledge needed to understand Galilei's treatise. Salviati takes the role of guiding the others in reading and demonstrating the theory:

> «Let us imagine an elevated horizontal line or plane *ab* along which a body moves with uniform speed from *a* to *b*. Suppose this plane to end abruptly at *b*; then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular *bn*. Draw the line *be* along the plane *ba* to represent the flow, or measure, of time; divide this line into a number of segments, *bc*, *cd*, *de*, representing equal intervals of time; from the points *b*, *c*, *d*, *e*, let fall lines which are parallel to the perpendicular *bn*. On the first of these lay off any distance *ci*, on the second a distance four times as long, *df*; on the third, one nine times as long, *eh*; and so on, in proportion to the squares of *cb*, *db*, *eb*, or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from *b* to *c* with uniform speed, it also falls perpendicularly through the distance *ci*, and at the end of the time-interval *bc* finds itself at the point *i*. In like manner at the end of the time-interval *bd*, which is the double of *bc*, the vertical fall will be four times the first distance *ci*; for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space *eh* traversed during the time *be* will be nine times *ci*; thus it is evident that the distances *eh*, *df*, *cl* will be to one another as the squares of the lines *be*, *bd*, *bc*. Now from the points *i*, *f*, *h* draw the straight lines *io*, *fg*, *hl* parallel to *be*; these lines *hl*, *fg*, *io* are equal to *eb*, *db* and *cb*, respectively; so also are the lines *bo*, *bg*, *bl* respectively equal to *ci*, *df*, and *eh*. The square of *hl* is to that of *fg* as the line *lb* is to *bg*; and the square

[11] Tr. from Latin: «TEOREMA 1, PROPOSIZIONE I. Un proietto, mentre si muove di moto composto di un moto orizzontale equabile e di un moto deorsum naturalmente accelerato, descrive nel suo movimento una linea semiparabolica.» (Galilei, 1638, p. 237)

of fg is to that of io as gb is to bo; therefore the points i, f, h, lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola. Q.E.D.»[12] (Crew & De Salvio, 1914, pp. 248–250)

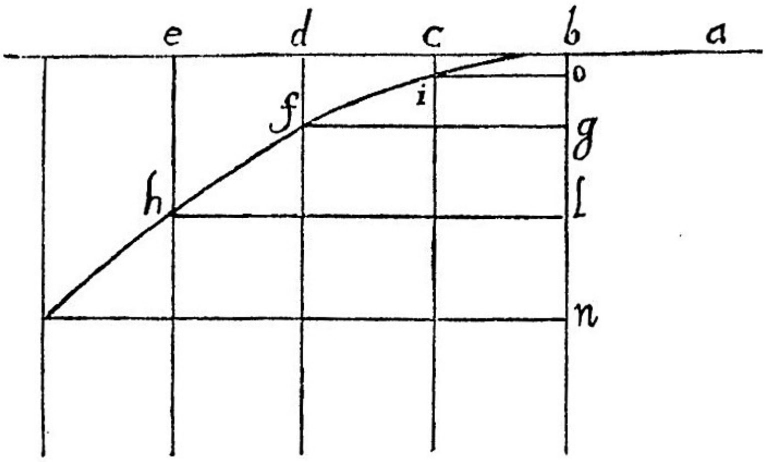


Fig. 2.12: Application of the laws of motion and the definitions of equable and equably accelerated motion; reading the curve in a geometric key (from Galilei, 1638, p. 242).

[12]Tr. from Latin: «Si intenda la linea orizzontale ossia il piano *ab* posto in alto, e un mobile si muova su di esso da *a* in *b* di moto equabile; mancando ora il sostegno del piano in *b*, sopravvenga al medesimo mobile, per la propria gravità, un moto naturale deorsum secondo la perpendicolare *bn*. Si intenda inoltre che la linea *be*, la quale prosegue il piano *ab* per diritto, rappresenti lo scorrere del tempo, ossia [ne costituisca] la misura, e su di essa si segnino ad arbitrio un numero qualsiasi di porzioni di tempo eguali, *bc*, *cd*, *de*; inoltre dai punti *b*, *c*, *d*, *e* si intendano condotte linee equidistanti dalla perpendicolare *bn*: sulla prima di esse si prenda una parte qualsiasi *ci*; sulla [linea] successiva se ne prenda una quattro volte maggiore, *df*; [sulla terza,] una nove volte maggiore, *eh*; e così di séguito sulle altre linee secondo la proporzione dei quadrati delle [porzioni di tempo] *cb*, *db*, *eb*, o vogliam dire in duplicata proporzione delle medesime. Se poi intendiamo che al mobile, il quale si muove oltre *b* verso *c* con moto equabile, si aggiunga un movimento di discesa perpendicolare secondo la quantità *ci*, nel tempo *bc* [esso mobile] si troverà situato nell'estremo *i*. Ma continuando a muoversi, nel tempo *db*, cioè [in un tempo] doppio di *bc*, sarà disceso per uno spazio quattro volte maggiore del primo spazio *ci*; abbiamo infatti dimostrato nel primo trattato, che gli spazi percorsi da un grave, con moto naturalmente accelerato, sono in duplicata proporzione dei tempi: e parimenti, il successivo spazio *eh*, percorso nel tempo *be*, sarà nove [volte maggiore del primo spazio]: sì che risulterà manifesto che gli spazi *eh*, *df*, *ci* stanno tra di loro come i quadrati delle linee *eb*, *db*, *cb*. Si conducano ora dai punti i, f, h le rette *io*, fg, *hl*, equidistanti dalla medesima *eb*: le linee *hl*, fg, *io* saranno eguali, ad una ad una, alle linee *eb*, *db*, *cb*; e così pure le linee *bo*, *bg*, *bl* saranno eguali alle linee *ci*, *df*, *eh*; inoltre il quadrato di *hl* starà al quadrato di fg come la linea *lb* sta alla *bg*, e il quadrato di fg starà al quadrato di *io* come *gb* sta a *bo*; dunque, i punti i, f, h si trovano su un unica e medesima linea parabolica. Similmente si dimostrerà che, preso un numero qualsiasi di particole di tempo eguali di qualunque grandezza, i punti, che il mobile mosso di un simile moto composto occuperà in quei tempi, si troveranno su una medesima linea parabolica. È dunque manifesto quello che ci eravamo proposti.» (Galilei, 1638, pp. 241–242)

2.4.2 The linguistic analysis

From a textual point of view, the excerpt shows the level of discourse codified into different languages: Latin and the Vulgar Italian. Latin is the language spoken by educated people, in particular by the scientific community contemporary with Galilei, and it is therefore used to state theorems, demonstrations, etc. The second is the language of ordinary people, used in the dialogues between the protagonists of the work, to give voice to doubts and perplexities concerning the notions explained in Latin. This expedient allows the author to explicit what is left as implicit content by the scientific community, as it is considered already known to it (Bagaglini et al., 2021). For this reason, none of the theories and concepts remain in implicit content: as Apollonius does in his work, Galilei states and proves all (and only) the propositions he uses. In this way, Galilei does not allow the readers to make mistakes: he does not only inform, but also argues and leads the readers into the topic.

Although not largely frequent in the whole text, in Galilei's work also other codes are used to convey information. As shown in Fig. 2.12, the motion of the projectile is described geometrically: the physical entities of time and space are mathematized through segments on the horizontal and vertical dimension, and in particular the time is explicitly "spatialized". This construction contains the intuition that time and space on the horizontal plane are proportional and that it is possible to measure the time by establishing a unit of distance.

Galilei mathematizes his space-time intuition, in order to geometrically demonstrate the theorems: this operation succeeds in creating a correspondence between the mathematical objects and the phenomenological aspects that lie at their foundation. In this way, mathematics provides a logical-deductive mode of reasoning for physics, since the information proceeds from initial assumptions to conclusions: this is what we may call a structural role and not an instrumental one, in the sense of being a computational tool only, observed already in textbooks.

Looking at the sentences, it is noticeable that they are rather long by today's standards (there is an average of 40 words per sentence) and rich in both coordinates and subordinates, especially explicit relative ones. This syntactic abundance also contributes to the author's intention not to leave anything implicit, to explain in detail and clarify as much as possible any passage, notion or concept. The demonstration provided by Salviati is a proof of this fact: all the steps leading to the creation of Fig. 2.12 are carefully described, assigning a name to each drawn segment, to make it immediately recognisable, and recall it when needed in the text.

As far as lexical analysis has been conducted in textbooks, I first looked at the nouns: despite the large number of words in each sentence, there are very few nouns (around 12% of the text), most of which can be categorised as technical vocabulary. Table 2.2 illustrates how this lexicon is distributed in the fields of physics and mathematics. What is immediately noticeable is the fact that it was not always possible to find a single word expressing a technical term, but it was necessary to consider a syntagma and then reconstruct its scientific meaning.

Table 2.2: Lexicon fields.

Lexicon of physics	• uniform motion and of motion naturally accelerated; perpetual motion; motion is compounded; • body; particle; • projectile; • friction; • a downward propensity due to its own weight; natural motion downwards; • freely falling body; • uniform speed; • time; intervals of time.
Lexicon of mathematics or geometry	• horizontal / vertical plane or line; • semi-parabola; parabola; • the perpendicular / parallel line; • segments; points; • distance four time as long; nine time as long; the double; equal; ... • in proportion to the squares of ...; square ratio.

Most of the physical vocabulary is found in the introductory part of the text, in which the assumption of the motion of a body in free fall is given by the composition of a uniform motion and a naturally accelerated motion. In stating the theorem, Galilei begins to "geometrize" the discussion, associating uniform motion with horizontal displacement and naturally accelerated motion with vertical displacement. Most of the mathematical vocabulary is consequently in the proof, where the trajectory presented in Fig. 2.12 is created by connecting points through segments.

The linguistic analysis concludes with the study of verbs. As might be expected, the verbs that appear predominantly are "to move", which is constantly linked to actions relating to particles in motion, and "to be", especially in the last part of the demonstration, where the results are defined. Among verbs of perception, only the verb “to imagine” stands out, which is repeated four times, while verbs introducing practices include “to draw”.

As can be seen by comparing this vocabulary with the modern scientific terminology, the lexicon of the time can appear “not mature” under a contemporary scientific point of view: there are still no definite references to consolidated concepts, but rather there are periphrases or more detailed descriptions of an already existing but not established term (Altieri Biagi, 2013, pp. 3−16). Indeed, we are in a particular historical moment, when the topic was being approached for the first time with different eyes from those of tradition.

2.5 Comparison of results between the two kinds of text

From the results shown in preceding paragraphs, it is possible to draw some conclusions and make considerations regarding the texts examined through the grid, and the images they give of physics and mathematics. Summing up:

- In Section 2.2, the text taken into consideration was the chapter entitled *Richiami di cinematica* (from Cutnell et al., 2015), belonging to a textbook for high school students. This chapter represents a synthesis of the arguments usually found in a book dealing with kinematics, and for this reason it only provides the main definitions and formulas.

- In Section 2.3, the text examined was the chapter *Two-dimensional kinematics* (from Walker, 2017), found in a more complex high school textbook. Unlike the previous case, in fact, this text is intended for students who are approaching the topic of parabolic motion for the first time.

- In Section 2.4, the text examined was an extract from the fourth day of *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, a work written by Galilei (1638) in the form of both treatise and dialogue. In particular, the passage under consideration shows the demonstration of the parabolic trajectory of the projectile motion.

By looking at these distinct types of text, which have different structures and objectives, and applying the linguistic tool to them, different results were obtained, which made it possible to identify factors and parameters leading to different images of the intertwining between physics and mathematics.

The analysis started with the Cutnell, a textbook characterized by very few sentences in each paragraph and very few words in each sentence. It was seen that most of the words were technical terms of the two disciplines: thus, the conciseness of this textbook is accompanied by a very high semantic density. This is a characteristic which is rather shared by modern textbooks (even if in Cutnell it is accentuated by the purpose to be a synthesis).

Also Walker’s shows similar characteristics, even if not so strongly: in fact, it has a rather variegated syntax and is composed of a good balance between coordinated and subordinated sentences. This is relevant, since the presence of subordinates can more typically be linked to the presence of explicit explanations, contextualization, arguments, etc.

Anyway, in both textbooks, references to mathematical processes and underlying theories are often taken for granted and therefore left implicit; in addition, links to the typical forms of mathematical thinking do not appear. Moreover, if physical principles are mentioned, they are very rarely brought into line with the proposed mathematization and used in a meaningful way in the physical explanation.

On the contrary, in the short text extracted from the dialogue of Galilei, an opposite tendency is evident: the sentences are long, composed of several periods, and this is not just a characteristic of the writing of that period but denotes the scientist's desire to be as exhaustive as possible, and not to leave information implicit or take it for granted. The text flows more slowly and concepts are often repeated, with different languages (both in Latin and in the Vulgar Italian), thus giving the feeling of wanting to discuss the contents in detail.

These observations, together with those elaborated in detail in the respective sections, made it possible to interpret the role played by mathematics within each text.

Most modern textbooks, including Cutnell, conveys an image of mathematics as mainly instrumental, useful as a calculating tool for solving exercises. In general, these texts fail to make sufficiently explicit the interdisciplinary reasoning underlying the study of the phenomena they deal with, and mathematics ends up being presented almost only as a formula; moreover, the argumentative structure is very difficult to highlight, as sentences seem to be constructed more to convey information rather than to argue. Walker partly distances himself from this limited use of mathematics by showing, paragraph by paragraph, how the discipline can be made to play a different role according to need.

What is missing in the modern textbooks is the epistemological role of proof: at the end of paragraph 4.3 of *Physics*, the author demonstrates that the trajectory for projectile motion is parabolic, deducting it algebraically. Comparing this demonstration with Galilei's, one can see how Walker skipped the crucial steps that led to the transformation of physical properties into quantities. In particular, Galilei's proof included the step where time was transformed into space and speed became a "quality" that characterised different types of motion.

Moreover, this proof in Walker's textbook is difficult to detect, for two reasons:

1. The author decided to title a certain subsection of the text *Demonstrating Independence of Motion*, thus making one think that the proof of the parabolic shape of the trajectory is to be found there; but in the way it is presented, this part is more like a thought experiment than a proper formal proof. On the contrary, the part identified as a proof can be found under the title *Real World Physics*, at the end of the third paragraph.

2. The proof structure of this passage is difficult to identify: "an argument, in order to be considered the proof of some statement, must refer to a list of axioms (given as true) and be included a theory of reference" (Mariotti, 2000, in Gombi, 2020, p. 53). In the specific case, not all of these elements are explicit in the text and, if we are looking for a demonstrative structure, they have to be reconstructed and retraced.

In the case of the historical excerpt, the initial axioms, the statement, and the proof itself are all objects made explicit in the work; furthermore, Galilei introduces all (and only) those elements needed, in order to demonstrate the various theorems. All these elements, unlike in Walker's textbook, contribute to bringing out the demonstration explicitly as a mathematical meta-object (Mariotti, 2000, in Gombi, 2020, p. 56).

Galilei's proof of the parabolic trajectory seems to bring out disciplinary and interdisciplinary elements than the one proposed by Walker. The way of reasoning is synthesized in *Physics* and in the modern textbooks. What remains from the historical study is the vision of the motion of a projectile as a parabolic motion and, from the methodological point of view, the importance of the experimental method that was emerging in those years and that allowed a real scientific revolution.

As far as the lexicon is concerned, Galilei's work is strongly conditioned by the historical period in which it was written. In the analysis, it was not always possible to find a single word that expressed a technical term, as we mean it today, but rather it was necessary to analyse each syntagma to reconstruct its scientific meaning. The verbs used by Galilei are also not those we would expect to find in a scientific text today: "to imagine", "to draw"; they do not convey the idea of physics as procedure as it happens in the other two texts. The two textbooks take an entirely different approach, limiting themselves to presenting the results in a definitive and consolidated manner.

Chapter 3

The epistemological tool

The purpose of the linguistic tool designed in Chapter 2 was to have an instrument able to highlight the textual and syntactic structures and to explore the lexicon in chapter 4, *Two-Dimensional Kinematics*, of the textbook *Physics* (Walker, 2017). The aim was to identify what image of physics and mathematics emerges from this text. The application of the tool seemed to make sense, as the results were consistent with those already obtained by Gombi (2020) in his Master thesis.

To produce a more comprehensive analysis, another tool has been considered to explore the textbook from the disciplinary perspective, trying not to focus on the linguistic structures but to grasp also the physical and mathematical *meaning* of the words.

The epistemological tool for the analysis has been designed starting from the idea of Family Resemblance approach (FRA), as a framework to reflect on the Nature of Science (NOS). The framework has been inspired by the philosopher Ludwig Wittgenstein (1958) and developed by Irzik & Nola (2011, 2014); then, it was expanded and reconceptualized to NOS by Erduran & Dagher (2014) within science education (Reconceptualized Family resemblance approach to Nature of science, RFN). The questions that the framework addresses are typical of NOS, such as "What is science?" and "What ideas about nature of science should be taught and learned?" (Osborne et al., 2003). These are complex questions and their answer requires not only understanding of scientific knowledge and its processes, but also reflecting on how we get to understand what science is. Over the years, scientists, philosophers of science, sociologists, etc., have worked to find ways to delineate what science is and what should be handed down, gaining a wide, even opposing, variety of results, that confirms the difficulty of the task (Erduran & Dagher, 2014).

The RFN has been widely employed in science education for many different purposes. It was used to analyse the Turkish (Kaya & Erduran, 2016), American, Korean and Taiwanese (Yeh et al., 2019; Park et al., 2020) biology and physics curricula, and the Italian physics curriculum (Caramaschi et al., 2021). Furthermore, the RFN was used as an analytical tool for textbooks: in Mcdonald & Abd-El-Khalick (2017), one chapter explores Lebanese middle school chemistry, life and earth science, and physics textbooks for their representations of NOS, while another chapter explores Australian junior secondary textbooks representations of NOS in the specific topic of genetics.

In Section 3.1, I will describe the FRA and all its components that lead to the picture of science as a cognitive-epistemic and social-istitutional system; in particular, I will illustrate four aspects of this system that form the basis of my epistemological tool. In Section 3.2, I will discuss the phases that we have followed to build the epistemological tool, describing the choices and methodologies undertaken. In Section 3.3, the elaboration of the analytic grid and the epistemological contents highlighted by the its application will be reported and discussed. In Section 3.4, I will conclude the chapter with some consideration about what we gained: the results of the linguistic analysis shown in Section 2.3 will be taken up and integrated with those obtained through the epistemological tool.

3.1 The theoretical tool for the epistemic analysis: RFN for science education

From the 1960s, different studies have been carried out in science education regarding the definition of NOS, inspired by philosophers and their works, like Kuhn (1970), Feyerabend (1975), Lakatos (1976), etc. In the last 30 years, this research area has become increasingly relevant in science education, in coherence with the goals to promote and foster students "to a) understand the process of science, b) make informed decisions on socio-scientific issues, c) appreciate science as a pivotal element of contemporary culture, d) be more aware of the norms of the scientific community, and e) learn science content with more depth" (Driver et al, 1996, in Erduran & Dagher, 2014, p. 24).

Although both the science education literature and the science standard documents confirm the list above, there is little agreement about what NOS is. Several researchers and science educators adopt the so-called *consensus view*, which aims to teach students only those widely shared characteristics of science, such as "scientific knowledge is empirical (relies on observations and experiments), reliable but tentative (i.e. subject to change and thus never absolute or certain), partly the product of human imagination and creativity, theory-laden and subjective (that is, influenced by scientists background beliefs, experiences and biases) and socially and culturally embedded (i.e. influenced by social and cultural context). Finally, there is no single scientific method that invariably produces secure knowledge" (Irzik & Nola, 2011, p. 592).

Although its success in science education, this view is often criticized on the basis of arguments like the following. First of all, it provides students with an incomplete standpoint, because it offers a limited image of science, that does not take into account the differences among scientific disciplines (i.e. astronomy and chemistry are very different sciences, given that the first one is a non-experimental discipline). It follows that NOS seems to be fixed and timeless, giving the impression that sciences cannot change or evolve over time, together with social requirements. Finally, the issues the view presents are neither interconnected nor sufficiently addressed: actually, questions arise (i.e. "Is objectivity impossible?", "How can there be general knowledge if science is influenced by the society in which it develops?", etc.) and they need to be carefully analysed, in order to have a sophisticated understanding of NOS (Irzik & Nola, 2011).

As other possible approach to NOS, the philosophers of science Irzik & Nola (2011) suggest to address the questions by pursuing a FRA: basing their idea of family resemblance on Wittgenstein's work, they argue that their approach is comprehensive enough to accommodate a variety of scientific features, including the epistemic, cognitive and social aspects of science. They described the FRA as follows:

> «Consider a set of four characteristics A, B, C, D. Then one could imagine four individual items which share any three of these characteristics taken together such as $(A\&B\&C)$ or $(B\&C\&D)$ or $(A\&B\&D)$ or $(A\&C\&D)$; that is, the various family resemblances are represented as four disjuncts of conjunctions of any three properties chosen from the original set of characteristics. This example of a polythetic model of family resemblances can be generalized as follows. Take any set S of n characteristics; then any individual is a member of the family if and only if it has all of the n characteristics of S, or any $(n-1)$ conjunction of characteristics of S, or any $(n-2)$ conjunction of characteristics of S, or any $(n-3)$ conjunction of characteristics of S and so on. How large n may be and how small $(n-x)$ may be is something that can be left open as befits the idea of a family resemblance which does not wish to impose arbitrary limits and leaves this to a "case by case" investigation. In what follows we will employ this polythetic version of family resemblance (in a slightly modified form) in developing our conception of science.» (Irzik & Nola, 2011, pp. 594–595)

The central idea of a FRA turns on the fact that the members of a family can resemble each another in some features, but not in others. The same happens for sciences, where there are characteristics common to all: for example, Table 3.1 shows that "Data collection" is common to all the sciences under consideration, whereas "Experimentation" is a feature of only two of them; "Prediction" is shared by all these sciences too, even if it is performed in different ways (Irzik & Nola, 2011).

Table 3.1: Example of FRA ability of representing differences and similarities among disciplines (from Irzik & Nola, 2014, p. 1015). The use of * and ** indicates differences in predictive powers.

	Astronomy	Particle Physics	Earthquake Science	Medicine
Data collection	x	x	x	x
Inference making	x	x	x	x
Experimentation		x		x
Prediction	x	x	x*	x**
Hypothetic-deductive testing	x	x	x	
Blinded randomized trials				x

Nevertheless, these characteristics cannot define a science nor distinguish it from another one: for example, although "Observation" is a practice shared by all sciences, the act of observing is not exclusive to science and therefore cannot be considered a defining characteristic of scientific disciplines.

Thus, Irzik & Nola (2011) offered polythetic sets of characteristics within which each individual scientific discipline could be compared with the others, to find similarities or differences. The original FRA framework (2011a) included four categories, focused on the cognitive aspects of science, called "Activities", "Aims and values", "Methods and methodological rules" and "Products", providing a structural description of NOS. Later (2011b) the two philosophers of science introduced a fifth aspect to consider, based on institutional and social norms, which was next redesigned by Erduran & Dagher (2014) and divided into four categories too: "Social values", "Scientific ethos", "Professional activities" and "Social certification and dissemination".

In adapting this approach for science education purposes and making the social-institutional dimension more complete, Erduran & Dagher (2014) included three further categories: "Social organizations and interactions", "Political power structures" and "Financial systems". This greater variety of categories is designed to include aspects of science that would otherwise be lost, and that are taken for granted as part of our culture (i.e. colonial science).

This plurality of categories offers teachers and educators an articulated picture of science dimensions, that can be explored, according to the content being taught, and can be a resource for providing students with a more comprehensive view of the discipline they are learning. Therefore, the RFN by Erduran & Dagher (2014) represents a conceptual tool for organizing the different aspects of the nature of science, bearing in mind, however, that these principles are not definitive and may vary according to the context.

3.1.1 FRA wheel and the systems of science

In the research work carried out by Erduran & Dagher (2014), the categories just mentioned are schematically represented and organized in the FRA wheel (Fig. 3.1).

The cognitive-epistemic core is comprised of four categories: *"Aims and values", "Practices", "Methods and methodological rules"* and *"Knowledge"*. These aspects will be described in detail in Section 3.1.2.

The two external rings represent the social-institutional system. The innermost one represents the four categories originally proposed by Irzik & Nola (2011):

- *"Social values"*. It refers specifically to social values like social utility, improvement of people's health and quality of life, respecting the environment, freedom, decentralizing power, honesty, addressing human needs and equality of intellectual authority.

- *"Scientific ethos"*. It refers to norms that scientists observe during their own work like skepticism, universalism, communalism and disinterestedness, freedom and openness, intellectual honesty, respect for research subjects and respect for the environment.

- *"Professional activities"*. It refers to activities performed by scientists in order to communicate their research, including conference attendance and pre-

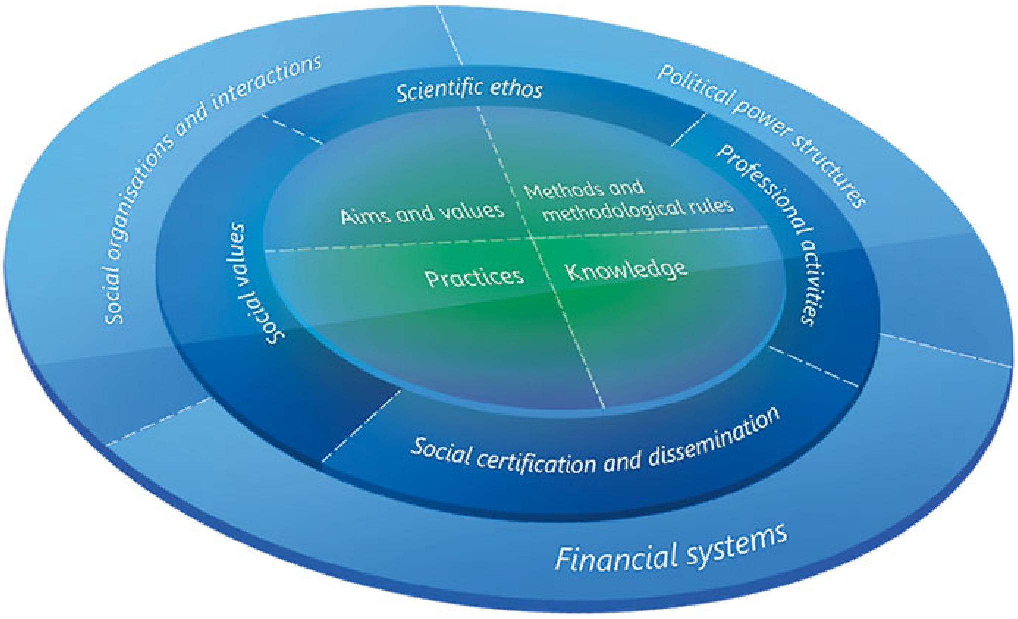


Fig. 3.1: FRA wheel: science as a cognitive-epistemic and social-institutional system (from Erduran & Dagher, 2014, p. 28).

sentation, writing manuscripts for peer-reviewed journals, reviewing papers, developing grant proposals and securing funding.

- *"Social certification and dissemination".* It refers to peer review process, which is a form of control and validation of new scientific knowledge by the broader scientific community.

The outermost ring represents three new categories added by Erduran & Dagher (2014):

- *"Social organizations and interactions".* It refers to social organizations in which scientists meet and work, like universities and research centres. The nature of social interactions among members of a research team working on different projects is governed by an organizational hierarchy.
- *"Political power structures".* It refers to all political environments that influence the scientific enterprise: it is considered important to unveil the political heritage of science and promote a legal and fair science.
- *"Financial systems".* It refers to the fact that science lays on economic factors: scientists require funding in order to carry out their work, and governments provide funding to universities and research centres. As such, these organizations have an influence on the types of scientific research funded.

The overall view of the FRA wheel for NOS coheres with the assumption that the boundaries among both categories and systems are porous, and all compartments can flow naturally in every direction: in this way, every feature of NOS can interact with one another, enhancing or influencing scientific activity (Erduran & Dagher, 2014).

3.1.2 The cognitive-epistemic system

Since the aim of this thesis is to elaborate tools to analyse texts (textbooks in particular), with the purpose to point out what epistemic identity of physics and mathematics emerges, we have focused our attention only on the cognitive-epistemic nucleus. In the following, the categories of the cognitive-epistemic system are described and discussed, in order to present and clarify the lenses we used for the analysis, grounding on the work of Erduran & Dagher (2014), which is topic of Section 3.2.

3.1.2.1 Aims & Values

In their first paper, Irzik & Nola (2011) review the aims and values of science in the following way:

> «Being able to make predictions and providing explanations are among the well-known aims of science [...]. The aims in question are not moral but cognitive. Of course, there are many other aims in science such as consistency, simplicity, fruitfulness and broad scope (Kuhn, 1977); high confirmation, as emphasized by logical empiricists (Hempel, 1965); falsifiability and truth or at least verisimilitude (i.e. closeness to truth) (Popper, 1963 and 1975); empirical adequacy (Van Fraassen, 1980), viability (Von Glasersfeld, 1989), ontological heterogeneity and complexity, as emphasized by empiricist feminists like Longino (1997).» (Irzik & Nola, 2011, p. 597)

Values can intersect with scientific knowledge and knowing in several ways. First, there are epistemic and cognitive perspectives which characterize both scientific knowledge and the forms of reasoning at its basis, such as consistency, simplicity, objectivity, empirical adequacy; for example, values like accuracy, testability and novelty can guide scientists in making judgments about knowledge claims. Moreover, science is situated in a particular cultural, social, political, moral and ethical context, so it is inevitably made of values which people who does science share. In this sense, values can influence theory choices, impact on how scientists interact with their environments and affect methodological decisions and interpretations. On the other hand, science itself will generate values that can contribute to society, like "being free from inductive bias, honesty, applicability to human needs and decentralization of power with respect to race and gender" (Erduran & Dagher, 2014, p. 48).

It is neither straightforward nor significant to search for a set of aims and values which can be considered complete and representative of all sciences. In fact, the FRA does not pursue completeness: it merely requires setting out as many aims of individual sciences as possible, and understanding their role in the characterization of that science. In this way, it will be possible to have a family resemblance with respect to the aims and values of science, highlighted by different philosophical interpretations or stances. It mainly concerns the appropriation of a set of understandings about how to conduct or to understand scientific inquiry from a comprehensive and articulated perspective.

According to Erduran & Dagher (2014), science educators should care about the aims and values of science. Firstly, it can promote students' awareness about the values that guide the scientific research, and consequently to check and control bias and stereotypes in communication. Secondly, the acquisition of epistemic values through appropriate teaching methods could facilitate a deeper engagement with the scientific activities. The authors point out the importance of creating and developing classroom cultures, where both teacher and students are enabled to build a common language in approaching, conducting and interpreting explicitly the shared epistemic values that lead scientific activities.

This framework is based on the idea that the aims and values can have three different natures: an epistemic, a cognitive and a social nature. Fig. 3.2 shows how the trichotomy (epistemic-cognitive-social) builds a space of possibility where they can intertwine and mix.

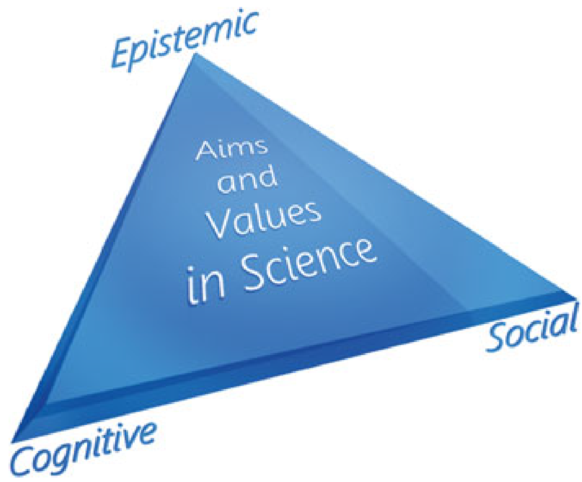


Fig. 3.2: Aims & Values for science education (from Erduran & Dagher, 2014, p. 49).

3.1.2.2 Scientific practices

The term *practices* has replaced the terms *activities* and *processes*, adopted by Irzik & Nola in their papers of 2011 and 2014 respectively, to align this category within the science education curricular policy. Even if they are often used interchangeably, they should be differentiated because are situated in different theoretical assumptions (Matthews, 2013). In particular, the term *processes* was used to denote how scientific research is done, but in teaching it to the students, it was always simplified and made to coincide with the idea of science process skills. On the other hand, the term *practices* place the aspects of science "into broader epistemic and discursive practices, such as making sense in patterns of data and coordination of theory and evidence" (Erduran & Dagher, 2014, p. 69).

School science is usually proposed to students in two main ways, that are mutually exclusive: the first one focuses on knowledge, in terms of the products of science (theory, laws, models), while the second one focuses on the processes involved in achieving those products. Both the approaches, taken individually, can generate educational problems. In the first case, the products of science can be presented in a decontextualized manner, disconnected from the processes that led to those results, and thus failing to convey a sense of relationships between different forms of scientific knowledge and their evolution. In the second case, skills related to the scientific process are mainly taught as individual practices, without explaining how such individual practices are part of a larger, interconnected whole that aims to produce meaningful scientific knowledge (Erduran & Dagher, 2014). They, hence, argue to what extent it is relevant to teach science in a more holistic view, which is able to enhance and justify both processes and knowledge, through appropriate teaching techniques.

There are numerous activities that underlie scientific practices, but Erduran & Dagher (2014) put a particular emphasis on observation, classification, and experimentation.

- *Observation.* Some scientists make direct observations of phenomena (i.e., botanists who study plant species or astronomers who study galaxies), but the scientists of twentieth century, like cognitive scientists, have contemplated the nature of observation asking questions as "What is the role of observation in getting to know the physical world?" and "What is the relationship between human perception and the real world?", generating a great variety of perspectives to critically examine. When embedded in scientific theories and flanked by other practices (i.e. modeling), observation has nothing to do with the generic "human activity" of understanding the world through sensory experience, but becomes a scientific practice able to generate knowledge.

- *Classification.* Classification can facilitate the stage of inquiry whether treated as a tool of discovery, analysis and theorizing. An example of a classification system is the *periodic table* of elements: it was first presented according to the insights of the time, but little by little changed with the refinement of knowledge, becoming even a predictive tool for new elements. The example illustrates how classification is also a scientific practice constituted by an epistemic purpose, because all the criteria chosen for deciding where to place an element, if a concept belongs to hierarchy or not, etc. are much more than just a sorting and describing process. However, it loses power if the epistemic dimensions is not pointed out in school science, losing a contribution for the knowledge generation.

- *Experimentation.* There are many research which highlight the role of the experiment in science and how it has changed starting from the studies of light, heat, etc. Moreover, the interaction between science and technology increased hugely, especially after the second half of the nineteenth century, allowing scientists to invent new approaches. Under the point of view of science teaching and learning, experimentation can be an important scientific practice rather than a common activity whereby students must follow a predetermined set of procedures (the "cookbook" approach).

These scientific practices also involve cognitive practices like explaining, modeling and predicting, which are closely linked to the discursive practices of argumentation and reasoning, working altogether in a complex set of interactions, including collection and analysis of data, certification of subsequent knowledge claims, etc.

As the importance of these practices does not always emerge from an educational point of view, Erduran & Dagher (2014) propose a heuristic that "a) brings together the often disparate components of science (e.g. modeling, social certification), and b) redefines the "discarded" process skills aspects (e.g. experimentation, classification) with the newer practices aspects (e.g. epistemic operations like argumentation and modeling) into one representation that capitalizes on the interrelatedness of scientific practices" (Erduran & Dagher, 2014, pp. 80−81). What is proposed should be broad enough and include potential interdisciplinary links (i.e. with economics, politics and the history of science) that can contribute to a better understanding of scientific practices. Hence, "in approximating a heuristic that conveys a range of scientific practices, a systemic approach bringing together the epistemic, cognitive and social-institutional aspects of science is essential for communicating to students a representative account of science" (Erduran & Dagher, 2014, p. 81). The heuristic can be visualized in Fig. 3.3, as the structure showing the relationship between the components of benzene.

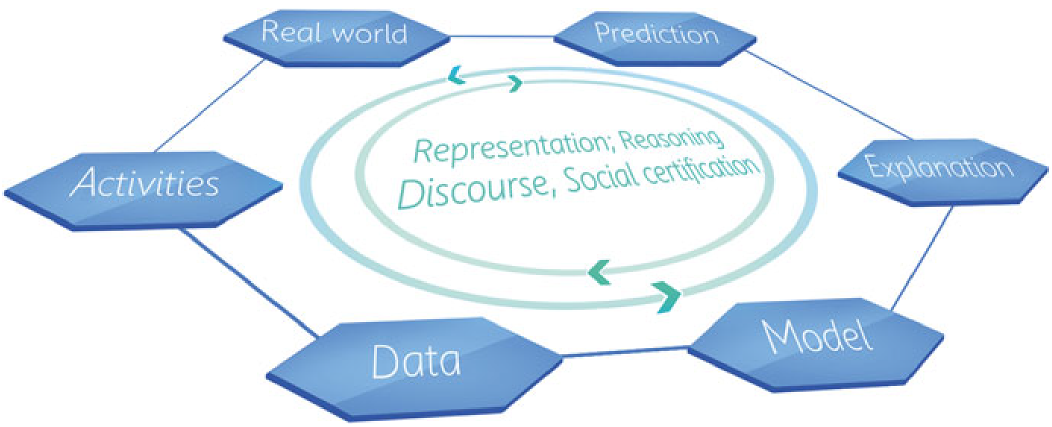


Fig. 3.3: "Benzene Ring" of scientific practices (from Erduran & Dagher, 2014, p. 82).

The analogy, made clear in Table 3.2, expresses how the practices of science are interconnected within a range of epistemic, cognitive and social-institutional practices. Overall, the heuristic has two primary purposes: "a) it illustrates a holistic approach to representing scientific practices, and b) it provides a pedagogical tool for communicating about scientific practices" (Erduran & Dagher, 2014, p. 83). A significant aspect is that "a) it communicates a dynamic set of interactions between the data, models, explanations and predictions that underlie the characterizations of phenomena occurring in the real world, and b) it integrates the social-institutional and cognitive processes that mediate such interactions through discursive practices like argumentation as well as norms such as social certification" (Erduran & Dagher, 2014, p. 85).

Table 3.2: Benzene Ring analogy (from Erduran & Dagher, 2014, p. 82).

Analog: Benzene ring	Heuristic: Scientific practices
Six-carbon hexagonal ring with three double bonds	Each of the carbon atoms in the hexagonal structure represents a scientific practice
Double bonds flip around a circle	Scientific practices are not confined to a definitive location in the representation
Benzene ring is represented as a hexagon with *pi* electrons moving around the ring	Representation, reasoning, discourse, social certification and similar processes correspond to the *pi* electrons. They float around the practices "ring" but in essence they are integral to and interact with scientific practices

3.1.2.3 Methods and methodological rules

Science employs many methods and methodological rules to achieve its various aims and check, internally, the solidity, coherence, and reliability of the practices. Grounding on the historical debate in philosophy of science (see Nola & Sankey, 2007; Nola & Irzik, 2005, for further details), the FRA focuses on unpacking methodological rules that stay behind, articulates and/or unites the macro-categories of inductive, deductive, and abductive methods.

The following are some significant examples of methodological rules:

- "construct hypotheses/theories/models that are highly testable;
- avoid making ad-hoc revisions to theories;
- other things being equal, choose the theory that is more explanatory;
- choose the theory that makes novel true predictions over the theory that merely predicts what is already known;
- reject inconsistent theories;
- other things being equal, accept simple theories and reject more complex ones;
- accept a theory only if it can explain all the successes of its predecessors;
- use controlled experiments in testing casual hypotheses;
- in conducting experiments on human subjects always use blinded procedures." (Irzik & Nola, 2011, p. 599)

There are two main lines of thought on the scientific method in the context of science education: on one hand, there are those who believe that the scientific method is not a linear process, and its representation is problematic; on the other, there are those who want to convey a simple and cognitively less demanding representation of the methods used in science, and for this reason it is often chosen in school science. According to this last idea, a typical representation of "the scientific method" is made by the steps illustrated in Fig. 3.4.

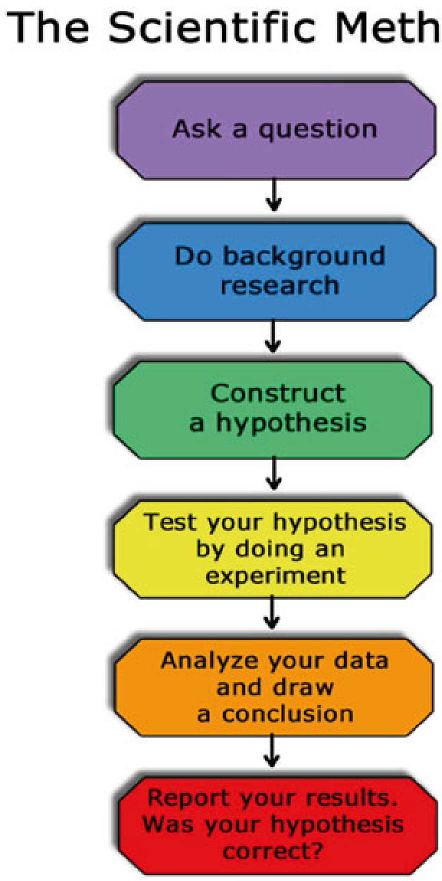


Fig. 3.4: A popular depiction of what is considered to be "the scientific method" (from Erduran & Dagher, 2014, p. 94).

Fig. 3.4 is not only problematic because it is a distorted, unrealistic and false image of how science works. It also hinders the diversity of methods through which knowledge obtained from experimental methods may be seen as more well-founded than that obtained from non-experimental ones (Erduran & Dagher, 2014).

Different scientific methods are argued to be discussed in teaching, so as to support a better understanding of scientific practices and to convey this message: no single method is sufficient to support complex theoretical statements, but there must be several proofs in order to achieve a theoretical rigor, typical of scientific knowledge. So "components of evidence from these different sources become gears, so to speak, that drive the "engine" (Fig. 3.5) of explanatory consilience" (Erduran & Dagher, 2014, p. 101).

The pluralistic nature of scientific methods is argued to be also reflected in

Fig. 3.5: The "gears" are a metaphor for how evidence from a variety of methods works synergistically (from Erduran & Dagher, 2014, p. 101).

diverse teaching practices, in order to widen also the opportunities for participation in science.

3.1.2.4 Scientific knowledge

Theories, laws and models are the products of scientific activity, and by working together they are able to build new scientific knowledge in explaining particular phenomena; the abbreviation *TLM* is used to indicate the complex network of relationships between these three different forms of knowledge.

- *Theory.* Inspired by Lakatos (1976), the framework of Erduran & Dagher (2014) considers theories as classified in three levels: the centre, the frontier and the fringe regions of science (Dushl, 1990, in Erduran & Dagher, 2014, p. 117). The ones at the first level (the centre) are part of the mainstream science and include the accepted foundational theories and their implications, like the theory of relativity, the Newton's laws of motion and Kepler's laws of planetary motion. The frontier ones are also part of mainstream science, but they can be challenged by other explanations, because of unresolved aspects which may or may not elevate them to the first level theory. They both constitute the hard core of science, because they are the key set of assumptions and standards on which scientists base their knowledge at a particular timeframe. Finally, the last level is made up of those theories that are making their way into science and are subject to investigation, before moving on to the higher levels.

- *Model.* While theories are classified according to level of development, models are classified according to perspectives (i.e. epistemic, educational, etc.). They are typically defined as a "representation between a source and a target", where the source is a familiar object or phenomenon that helps to understand the target, which is the unknown object or phenomenon that needs to be explained (Duit & Glynn, 1996; Grosslight et al., 1991; Justi, 2000, in Erduran & Dagher, 2014, p. 118). Models are fundamental in summarizing data, visualizing invisible structures and processes, making predictions, justifying outcomes and facilitating communication in science. They are considered by the philosophers of science as a middle way between theory, abstract, and experiment, practical and concrete (Redhead, 1980, in Erduran & Dagher, 2014, p. 118).

- *Law.* Laws can also be classified according to different criteria; aspects related to the meaning and nature of laws have changed over time and some are still under discussion. Laws, in teaching, are often defined as empirical regularities, coming from inductive observations or as theories confirmations, forgetting that laws of nature are also able to explain and predict (Dagher et al., 2004, in Erduran & Dagher, 2014, p. 120). The authors point out how "some laws can be expressed in algebraic form (e.g. Newton's laws of gravitation) while others are qualitative approximations (e.g. Mendeleev's periodicity); some are probabilistic (e.g. gas laws) while others are definitive (e.g. Avogadro's law)" (Erduran & Dagher, 2014, p. 121). There are various "senses of laws", and their nature can be rather different in each context.

Examples of *TML* in different scientific subjects are presented in Table 3.3.

Table 3.3: *TML* in different science domains (from Erduran & Dagher, 2014, p. 114).

Domain form of knowledge	Biology	Chemistry	Physics
Theory Model Law	Genetic Theory Genes Inheritance law	Atomic Theory Atomic model Periodic law	Thermodynamics Heat transfer Law of thermodynamics
TML explain	biological traits	structure of matter	heat

Erduran & Dagher (2014) claim that school science is characterized by theories, models and laws, often declined as "content knowledge", while no room is left for understanding how the various forms of scientific knowledge relate to each other. On the contrary, understanding how knowledge, seen as in Fig. 3.6, evolves, might enable students to grasp scientific knowledge as a coherent network of theories, models and laws, rather than as separate pieces of information. It could also give rise to questions such as "Are laws in chemistry and physics the same? If not, how are they different?", "Do theories in physics and biology have the same characteristics?", etc. Therefore, the epistemic dimensions of *TLM* can potentially promote not only the development of disciplinary knowledge but also of interdisciplinary knowledge, that is how it can be applied within and across science disciplines (Erduran & Dagher, 2014).

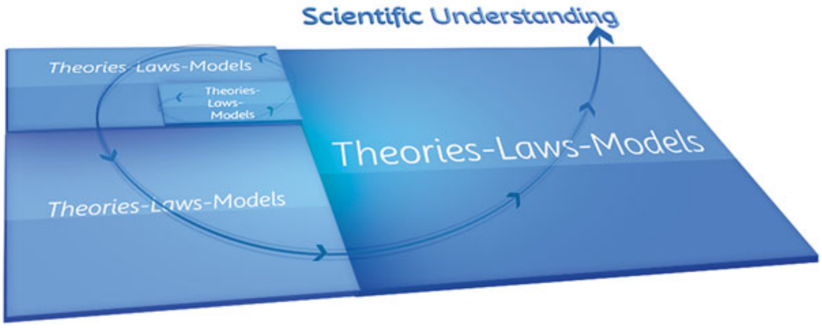


Fig. 3.6: Growth of scientific knowledge and scientific understanding (from Erduran & Dagher, 2014, p. 115).

3.2 Turning the theoretical framework into an analytic tool: methodological choices

In this section we describe the process that we have followed to turn a theoretical framework into an analytic tool. The main issue was to fill in the gap between rather broad categories (like aims & values, practices, etc.) and a textbook, where these elements are implicitly treated. The gap has been filled through a back-and-forth process, from the FRA wheel to the textbook, that led us to point out more and more specific and operational categories.

In order to build our epistemological tool through the analysis of Walker's chapter, it was decided to consider only the categories of the cognitive-epistemic system ("Aims & Values", "Scientific practices", "Methods and Methodological rules" and "Scientific knowledge") of the FRA wheel. Moreover, it was decided to neglect the aspects that appeared just linked to pragmatical choices made by the author: the sentences that invite the readers to look at the graphs, images or tables; the request to pay attention to some formulas or algebraic passages; some anticipations; etc. The choice was guided by the idea that these parts of the text do not embody significant typical cognitive-epistemic aspects of physics itself, but are rhetorical expedients useful to make the students follow the reasoning.

From a methodological point of view, the elaboration of the analytical grid involved the following steps.

The first step consisted in the transformation of the text into data. For this purpose, the text was divided in partitions. Figures, tables and the paratextual elements were not considered. From the beginning, it was quite clear that a text partition made by a single sentence was not always effective and productive, because the concepts, the ideas and the knowledge are often developed in more than one single sentence. For this reason, the partition of the text was "content-oriented", and the chapter, originally divided into 187 lines, was grouped together into (40) information blocks[13] (indicated in column A in Fig. 3.7).

The second step consisted in finding a structure that allowed us to have a "systemic view" of the chapter, in terms of cognitive-epistemic aspects. So, four columns

[13]Portion of text dealing with the same topic (Collins Online Dictionary).

were added, representing the four categories ("Aims & Values", "Scientific practices", "Methods and Methodological rules" and "Scientific knowledge"), each a of which a different color was assigned (Fig. 3.7). The color code (pink for "Aims & Values", blue for "Scientific practices", green for "Methods and methodological rules" and orange for "Scientific knowledge") was created to keep track, throughout the chapter, of words, expressions and portions of text which suggest one or more categories.

	A	B	C	D	E	F
1	IB	TEXT	Aims & Values	Scientific Practices	Methods and Methodological rules	Scientific Knowledge
2		4. TWO DIMENSIONAL KINEMATIC				
3	1	The main idea in this chapter is quite simple: horizontal and vertical motions are independent. That's it.				
4		This chapter develops and applies the idea of independence of motion to many common physical systems.				
5		4.1 – MOTION IN TWO DIMENSION				
6	2	In this section we develop equations of motion to describe objects moving in two dimensions.				
7		First, we consider motion with constant velocity, determining x and y as functions of time.				
8		Later, we investigate motion with constant acceleration.				
9		In both cases, we show that the one-dimensional kinematic equations of chapter 2 can be extended in a straightforward way to two dimensions.				
10	3	Constant velocity				
11		To begin, consider the simple situation shown in FIG 4-1.				
12		A turtle starts at the origin at t = 0 and moves with a constant speed v0 = 0.26 m/s in a direction 25° above the x axis.				
13		How far has the turtle moved in the x and y directions after 5.0 seconds?				

Fig. 3.7: How the work file originally looked like.

The third step of the analysis consisted in finding the approach to the text and, thus, the data. Through a bottom-up analysis of the chapter, an initial list of cognitive-epistemic aspects was carried out. These categories were then grouped into "thematical sets" (Section 3.3.1), that could unpack the larger categories of "Aims & Values", "Scientific practices", "Methods and Methodological rules" and "Scientific knowledge". The analysis was conducted separately by me and a Ph.D. student of the research group of *Physics Education and History of Physics* of the University of Bologna.

The fourth step consisted in applying systematically the sub-categories against the data, so as to validate and triangulate their effectiveness and efficiency, to point out interesting aspects from the data. This phase was carried out individually and then discussed collectively. Through an iterative process of comparisons and discussions, an almost complete agreement was reached both on the set of the emerged categories and on their individuation throughout the text. To reach the complete agreement, another Ph.D. student of the research group, who already worked with the RFN framework, was involved (Caramaschi et al., 2021).

The fifth step consisted in finalizing the process of naming the "Aims & Values", "Scientific practices", "Methods and Methodological rules" and "Scientific knowledge" in order to reach, on one hand, a fruitful and consistent image of each category and, on the other, a comprehensive image of physics through the categories.

These five phases led to the production of our first result: the analytic grid (see Section 3.3.1). The following step consisted in building the matrix of the epistemological analysis, by re-testing and refining the application of the grid on the text. A picture that gives back how the data matrix appeared is shown in Fig. 3.8. Finally, graphs of the matrix allowed us to build the epistemological profile of our chapter (see Section 3.3.2).

	A	B	C	D	E	F
1	IB	TEXT	Aims & Values	Scientific Practices	Methods and Methodological rules	Scientific Knowledge
2		4. TWO DIMENSIONAL KINEMATIC				
3	1	The main idea in this chapter is quite simple: horizontal and vertical motions are independent. That's it.	- simplicity - economy	readout strategy		statement about the theory
4		This chapter develops and applies the idea of independence of motion to many common physical systems.	generality			
5		4.1 – MOTION IN TWO DIMENSION				
6	2	In this section we develop equations of motion to describe objects moving in two dimensions.	mathematization	mathematize		theory confirmation
7		First, we consider motion with constant velocity, determining x and y as functions of time.		translation	reductionist rule	background components
8		Later, we investigate motion with constant acceleration.				
9		In both cases, we show that the one-dimensional kinematic equations of chapter 2 can be extended in a straightforward way to two dimensions.	generalizability			extendibility of previous results
10		Constant velocity				
11	3	To begin, consider the simple situation shown in FIG 4-1.	simplicity	visualize	reductionist rule	
12		A turtle starts at the origin at t = 0 and moves with a constant speed v0 = 0.26 m/s in a direction 25° above the x axis.		- readout strategy - action to model testing		- from reality to model - model presentation and assumptions
13		How far has the turtle moved in the x and y directions after 5.0 seconds?	predictive power			

Fig. 3.8: How the work file looks like after the analysis.

3.3 Presentation of the results

3.3.1 The analytic grid

The first result of the analysis is the construction of our epistemological tool, as elaboration of the FRA wheel. The resulted epistemological tool consists of an analytic grid comprised on bunches of specific "Aims & Values", "Scientific practices, "Methods and Methodological rules" and "Scientific knowledge".

In the following, the resulted lists of sub-categories are presented with a few explanatory sentences.

3.3.1.1 Aims & Values

1. *Simplicity.* It represents the assumption that scientific knowledge has to make things simple, since nature *per se* behaves in a simple way.

   «The main idea in this chapter is quite simple: horizontal and vertical motions are independent. That's it.» (page 88)

   «To begin, consider the simple situation shown in FIG 4-1» (page 89)

«The most direct way to answer this question is to set $y = 0$ in Equation 4-8, since $y = 0$ corresponds to ground level.» (page 97)

2. *Economy.* It represents the assumption that science uses as few as possible thinking and formal tools.

   «Though it may appear sometimes that we are writing new sets of equations for different problems, the equations aren't new at all [...] » (page 91)

3. *Generality.* It refers to the power of scientific knowledge to describe a variety of physical systems with the same theory, model or law.

   «As you might expect, this covers a wide variety of physical systems.» (page 92)

4. *Generalizability.* It describes how laws and models can be expanded from simplicity (for example from one dimension) to complexity (for example to two or three dimensions).

   «Replacing v_0 with the x component of the velocity, v_{0x}, yields Equation 4-1. Similarly, replacing each x in Equation 4-1 with y converts it to Equation 4-2, the y equation of motion.» (page 90)

   «In three dimensions we introduce a third coordinate direction and label it z. We would then simply replace x with z in Equation 4-3(a) to obtain z as a function of time.» (page 91)

5. *Mathematization.* Natural phenomena can be described through mathematics (i.e. equations), which, thanks to its own rules, can be used to describe and manipulate phenomena.

   «In this section we develop equations of motion to describe objects moving in two dimensions.» (page 89)

   «Using the trigonometric identity $sin2\theta = 2sin\theta cos\theta$, as given in Appendix A, we can write this more compactly as follows [...] » (page 104)

6. *Predictive power.* It refers to the effectiveness of scientific knowledge to make deterministic predictions.

   «How far has the turtle moved in the x and y directions after 5.0 seconds?» (page 89)

7. *Explanatory power.* It refers to the effectiveness of scientific knowledge to provide explanations for the observed phenomena.

«A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion.» (page 93)

8. *Multi-perspective.* It conveys the value that, in science, the same problem can be analysed and solved in different ways, in order to achieve a more robust result.

   «An alternative way to approach this problem is to treat the x and y motions separately.» (page 89)

   «One way to obtain the range, then, is as follows: [...] » (page 103)

9. *Applicability.* It represents a link to reality, which allows to "see" the effect and the impact of a model for a broader scope.

   «Real World Physics. One interesting application of the mathematical description of motion in three dimensions is the Traffic Collision Avoidance System [...] » (page 90)

10. *Beauty of understanding.*

    «Discovering such patterns and symmetries in nature is really what physics is all about. [...] detailed analysis reveals deeper, more subtle, and sometimes unexpected levels of beauty.» (page 106)

3.3.1.2 Scientific practices

1. *Readout strategies.* It is the most multifaceted practice. It refers to processes of setting up the initial conditions, choose the parameters, selecting variables and looking for their relationships, etc.

   «A turtle starts at the origin at $t = 0$ and moves with a constant speed $v_0 = 0.26m/s$ in a direction $25°$ above the x axis.» (page 89)

   «Notice that R depends inversely on the acceleration due to gravity, g—thus the smaller g, the larger the range.» (page 92)

   «Suppose you are walking with a speed v_0 when you release a ball from a height h. If we choose ground level to be $y = 0$ [...] » (page 94)

2. *Translation practices.* They refer to the search for the key variables and translate them into functions of some other quantity.

   « [...] These equations give x and y as functions of time.» (page 89)

3. *Mathematize.* It refers to practices like: turning a phenomenon into quantitative and formal equations; applying mathematical formalism to extrapolate data; taking mathematical definitions and using them to elaborate results and gain new information.

> «From the definitions of sine and cosine given in the previous chapter, we see that the horizontal (x) and vertical (y) distances are given by: $x = dcos25° = 1.2m$ and $y = dsin25° = 0.55m$» (page 89)

> «Substituting these specific values into our fundamental equations for projectile motion (Equations 4-6) gives the following simplified results for zero launch angle [...] » (page 95)

4. *Visualize.* It refers to using graphs and images to represent phenomena, processes, etc.

> «This is illustrated in Figure 4-3.» (page 95)

5. *Hypothesize, test or explore assumptions.* It refers to practices like: making assumptions; applying formalized knowledge to solve problems; modelling a real phenomenon to investigate it; explore the sense of a model by "situating it".

> «A situation illustrating the use of $x = x_0 + v_{0x}t$ and $y = y_0 + v_{0y}t$ is given in the following Example.» (page 90)

> «In studying projectile motion we make the following assumptions [...] » (page 92)

> «As expected, the range (Equation 4-12) and maximum range (Equation 4-13) depend strongly on the initial speed of the projectile—they are both proportional to v_0^2.» (page 104)

6. *Interpretation.* It refers to practices of making considerations related to the observed phenomena. It consists in interpreting and looking for consistencies and similarities; comparing mathematical models and experimental results; finding correspondence between the mathematical description and the physical model.

> «Noting that downward is the negative direction, it follows that $a_y = -9.81 m/s^2 = -g$.» (page 93)

> «To you, its motion looks the same as before: it goes straight down, lands near your feet, bounces straight back up, and returns in about one second.» (page 93)

> «The fact that you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion—the motions are independent.» (page 93)

3.3.1.3 Methods and methodological rules

1. *Reductionist rule.* It refers to the methodological rule of decomposing a phenomenon into simpler basic elements. The rule implies to recognize (first) the simplest cases and elements and (then) to move toward more complex situations by iterating, generalizing and/or recombining the various elements. Although not many subcategories were found within methods and methodological rules, it is evident that this is the most common one.

   > «First, we consider motion with constant velocity, determining x and y as functions of time. Later, we investigate motion with constant acceleration.» (page 89)

2. *Approaches' triangulation.* It refers to the rule of checking the results validity by applying a plurality of ways, to test the consistency of what is obtained.

   > «This is in agreement with our previous results.» (page 89)

3. *Demonstration.* It refers to the rule of proving assumptions/hypotheses through formal reasoning.

   > «It follows that y has the form: $y = a + bx^2$.» (page 96)

4. *Phenomenological assumptions checking.* It refers to the rule of checking the validity of the results by analysing the precision of the experimental procedure and the eventual impact of the phenomenological simplifications on the results.

   > «In studying projectile motion we make the following assumptions [...] » (page 92)

5. *Mathematical model checking.* It refers to the rule of checking the physical significance of mathematical results.

   > «Clearly, $t = 0$ is a solution to this equation—it corresponds to the initial condition— but the solution we seek is a time that is greater than zero.» (page 103)

3.3.1.4 Scientific knowledge

1. *Statement about the theory.* Theories have a certain structure (i.e. hypotheses, propositions, etc.), but the chapter only proposes statements, concerning the independence of the motions and the parabolic shape of the trajectory.

   > «The main idea in this chapter is quite simple: horizontal and vertical motions are independent. This chapter develops and applies the idea of independence of motion to many common physical systems.» (page 88)

   > «The precise shape of this curved path—a parabola—is verified in the next section.» (page 94)

2. *Theory confirmation.* It refers to all the regularities that validate a theory: demonstrations (experimental or algebraic), applications, new sets of equations, etc.

   > «This is in agreement with our previous results.» (page 89)
   >
   > «The fundamental equations in Table 4-1 are used to obtain all of the results presented throughout the rest of this chapter.» (page 91)
   >
   > «A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion.» (page 93)

3. *Theory type.* It emphasises whenever the nature of the theory appears. The chapter presents an example of observation-based theory, which in general emerges from the context and cannot be identified in specific words or expressions.

   > «First, notice that the turtle moves in a straight line a distance given by speed multiplied by time: $d = v_0t = (0.26m/s)(5.0s) = 1.3m$.» (page 89)
   >
   > «A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion. First, while standing still, drop a rubber ball to the floor and catch it on the rebound. Notice that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second. Next, walk—or roller skate—with constant speed before dropping the ball, then observe its motion carefully. To you, its motion looks the same as before: it goes straight down, lands near your feet, bounces straight back up, and returns in about one second.» (page 93)

4. *Extendibility of previous (or present) results.* It refers to equations, methods, etc., previously demonstrated and used, now adapted to the current situation of the motion in two dimension.

   > «To study motion with constant acceleration in two dimensions we repeat what was done in one dimension in Chapter 2, but with separate equations for both x and y.» (page 91)
   >
   > «Start with Equation 2-7, $v = v_0 + at$, and write it in terms of x and y components.» (page 91)

5. *Background components.* It represents what needs to be taken from previous studies (parameters, variables, etc.) and which I must continue to analyse in order to obtain results in this current case too.

> «First, we consider motion with constant velocity, determining x and y as functions of time. Later, we investigate motion with constant acceleration.» (page 89)

> «For example, to obtain x as a function of time we start with $x = x_0 + v_0 t + \frac{1}{2}at^2$ (Equation 2-11) and replace both v_0 and a with the corresponding x components, v_{0x} and a_x. [...] To obtain y as a function of time, we write y in place of x in Equation 4-3(a): $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ 4-3(b). These are the position-versus-time equations of motion for two dimensions.» (page 91)

6. *From reality to model.* It refers to the transition from observing the real phenomenon to creating a model that approximates it.

> «To begin, consider the simple situation shown in FIG 4-1. A turtle starts at the origin at $t = 0$ and moves with a constant speed $v_0 = 0.26m/s$ in a direction 25° above the x axis.» (page 89)

> «Let's incorporate the preceding assumptions into the equations of motion given in the previous section.» (page 93)

7. *Model presentation.* It highlights the type of model implemented, which is usually deterministic, capable of making certain predictions.

> «A turtle starts at the origin at $t = 0$ and moves with a constant speed $v_0 = 0.26m/s$ in a direction 25° above the x axis. How far has the turtle moved in the x and y directions after 5.0 seconds?» (page 89)

> «Well, a projectile is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity.» (page 92)

8. *Model assumptions.* It represents the approximations, both physical and mathematical, that are made to make the build model.

> «In studying projectile motion we make the following assumptions:
> - Air resistance is ignored.
> - The acceleration due to gravity is constant, downward, and has a magnitude equal to $g = 9.81m/s^2$.
> - The Earth's rotation is ignored.» (page 92)

> «We've chosen the positive sign for the square root because the projectile was launched in the positive x direction, and hence it lands at a positive value of x.» (page 97)

9. *Model confirmation.* It serves to test the validity of the proposed model and its applicability.

«As you might expect, this covers a wide variety of physical systems.» (page 92)

«In all cases, our results follow as a direct consequence of the fundamental kinematic equations (Equations 4-10) describing projectile motion.» (page 103)

3.3.2 The epistemological profile of the chapter

The application of the tool allowed us to build the epistemological profile of the chapter and check the effectiveness of the grid, to highlight subtle aspects that are not immediately visible. Fig. 3.9 shows the distribution of "Aims & Values", "Scientific practices", "Methods and Methodological rules" and "Scientific knowledge" throughout the chapter.

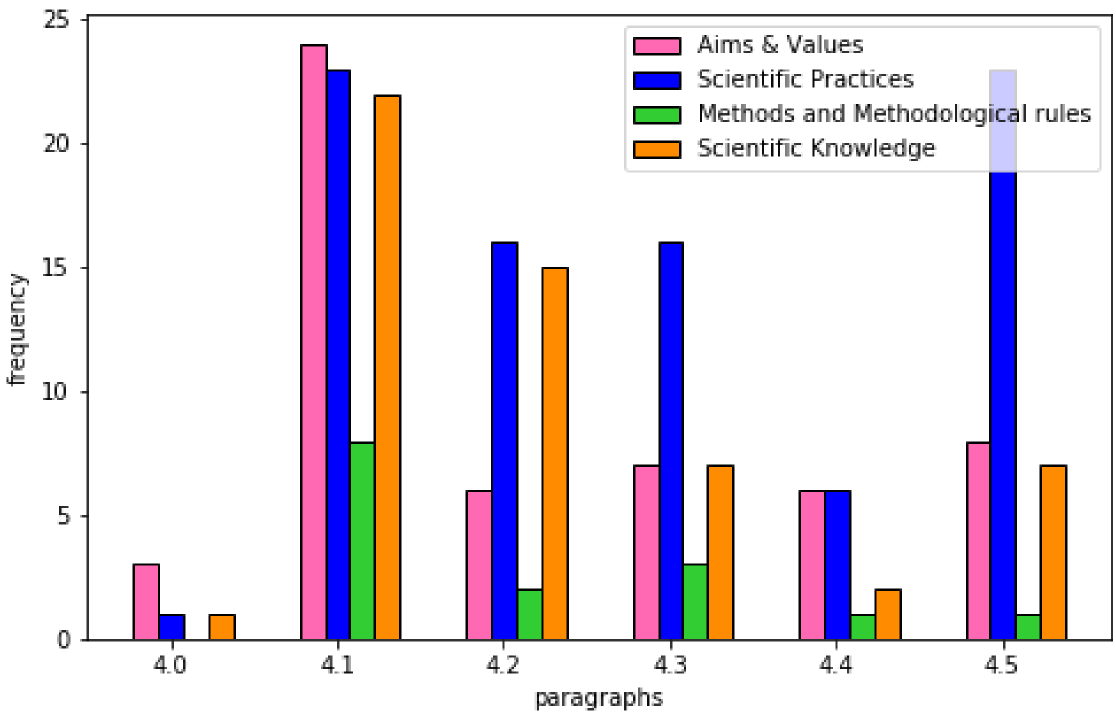


Fig. 3.9: Frequency distribution of the four cognitive-epistemic categories in each paragraph.

Moreover, the graph in Fig. 3.10 shows how these four aspects sometimes intertwine, generating very rich portions of text from an epistemological point of view.

The following is a description, category by category, of all the declinations of each cognitive-epistemic aspect which have been highlighted in the analysed text.

The category "Aims & Values" is quite present within the whole chapter, but it is especially in the introductory page and in the first paragraph that it dominates over the other cognitive-epistemic aspects; as one proceeds through the paragraphs, however, they become less and less. In addition, Fig. 3.10 shows how aims and values frequently overlap with practices and knowledge, describing and characterising them.

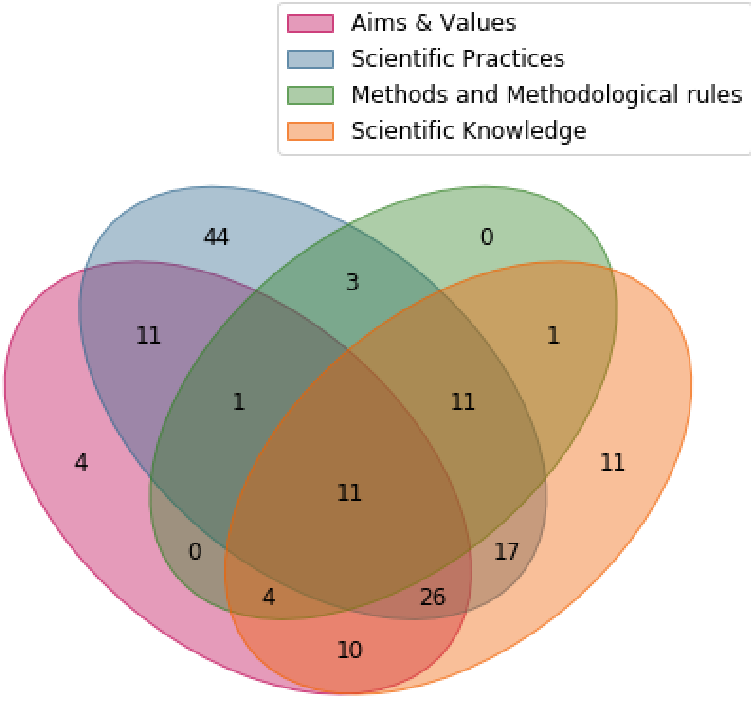


Fig. 3.10: Intersections among the four cognitive-epistemic categories.

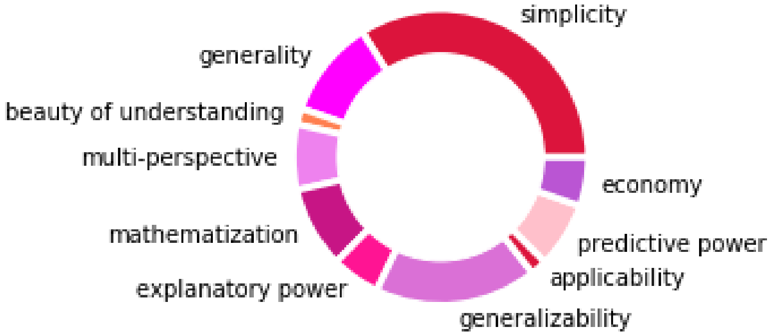


Fig. 3.11: Identified types of aims and values.

In this category, *simplicity* permeates the chapter, overlapping both with other values and with the other cognitive-epistemic aspects of FRA: the idea seems to be to convey how any aspect of physics (practices, methods or knowledge) can be easily implemented and understood, often from previous techniques. The criterion of *generalizability* is highly present too, underlining the fact that what is explained in this specific case has general validity.

However, it is also interesting to note that there is a great variety of values within the category, as shown in Fig. 3.11. This fact can be seen as representative of how, even in a textbook, it is possible to bring out certain aspects of physics that may seem superfluous in the school context; teachers can find in the textbook the opportunity to discuss and to unveil, with the students, this layer of epistemic discourse.

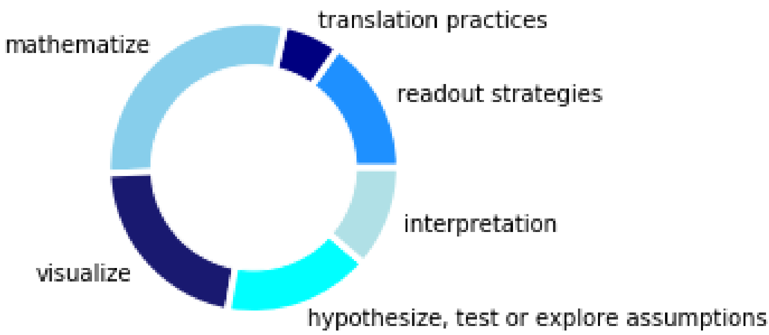


Fig. 3.12: Identified types of scientific practices.

The category "Scientific Practices" is always present in high frequency, if not dominant. This fact is mainly responsible for the major image that physics takes on in a high school textbook: the idea that understanding physics means learning and reproducing a list of procedures, aimed at achieving a result. This was partly foreseeable and in line with what was explained in Section 3.1.2.2, concerning the structure of modern textbooks. These procedures are accompanied by other cognitive-epistemic aspects, but mostly characterise portions of the text on their own, worsening the perception also from the point of view of epistemological richness.

Also in this case, we can find a rich variety of elements that a teacher can stress to problematize the scientific processes. Fig. 3.12 shows the frequencies of the different scientific practices carried out from the bottom-up analysis. In each paragraph, these always follow a certain pattern: *readout strategies* and *translation practices* introduce the initial conditions and significant parameters and variables; considerable space is given to *mathematize* and *visualize*, the most present practices, which allow equations to be formalised and situations to be visualised, graphically or through images; finally, the results are observed and commented.

The category "Methods and Methodological rules" is the least present in the chapter: after the first paragraph, in fact, it was found in very few other information blocks. As Fig. 3.10 shows, it is never found alone to characterise a part of the text, but is always overlapped with all the other categories.

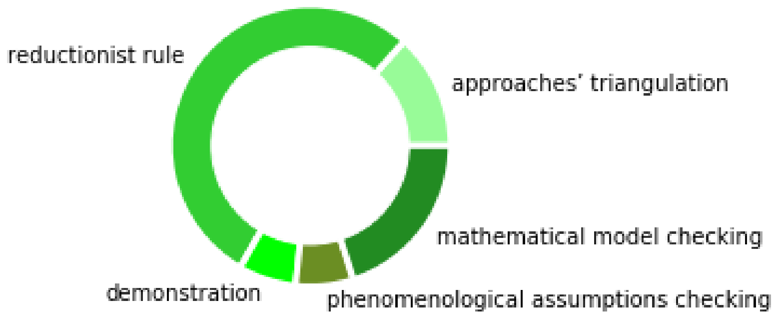


Fig. 3.13: Identified types of methods and methodological rules.

It is mostly declined as *reductionist rule*, as Fig. 3.13 shows: this fact is consistent with the large presence of practices presented in the form of steps to be performed, and therefore it is necessary to give an order to the actions to be carried out.

The fact that there are not many references to this aspect of physics seems to confirm the idea that today there is a tendency not to pass down the scientific method, neither explicitly nor through the study of a school textbook, but rather to propose the achievement of results and knowledge as a set of linear processes. As argued in Section 3.1.2.3, this fact provides a unique picture of how the research develops, shown in Fig. 3.4, which seems rigid and unchanging.

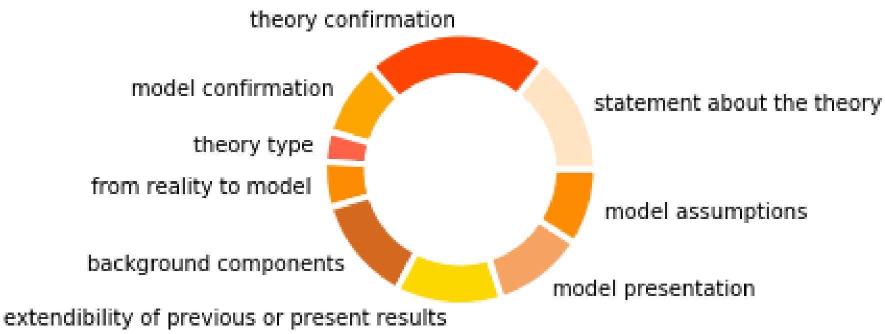


Fig. 3.14: Identified types of scientific knowledge.

The category "Scientific Knowledge" is very frequent in the first two paragraphs, where theory and model are presented and described, while it decreases drastically from the third paragraph onwards. In this case, there is considerable overlap between this category and that of practices, which highlights how some form of knowledge immediately follows from a physical process.

In the introductory page, the idea of the independence of motions is presented: unlike what happens in a real construction of a theory, which is usually composed of hypotheses, propositions, etc., here the statement is proposed directly, already given as true and never questioned: actually Fig. 3.14 shows that *statement about the theory* is a frequently repeating sub-category. Clearly, this choice strongly conditions the whole of the following discussion. The chapter focuses on the presentation of confirmations both that horizontal and vertical motions can be treated separately and that the shape of the trajectory followed by a projectile is parabolic: consequently *theory confirmation* recurs more than any other and constantly throughout the chapter.

Another important form of knowledge in the text, however, is related to the description of the model and the presentation of its features. It is mainly located in the second paragraph, where the projectile is introduced and all assumptions concerning it are made, but is also found in other paragraphs, where mathematical choices are made (i.e. direction of the axes of the reference system).

It should be emphasised that what has been found within "Scientific Knowledge" cannot be often traced back to a word or sentence, but instead remain rather implicit, to be read between the lines, and its meaning has to be grasped or reconstructed.

3.4 Discussion of the results achieved with the joint application of the tools

The results show that the FRA wheel was an extremely rich framework, that allowed us to recognize subtle epistemic elements, which did not appear immediately. The tool, through a bottom-up approach, has proved to be fruitful in identifying a list of categories, that appears particularly rich and nuanced. In particular, the analysis highlights that the textbook is rich of "Aims & Values" and "Scientific Knowledge", whereas it reflects some tendencies of modern textbooks (i.e. abandonment of the scientific method, not strictly structured demonstrations, little inclination towards interdisciplinarity) when it comes to "Methods and Methodological rules", and a widespread use of "Scientific Practices", albeit varied, that tend to convey a procedural idea of physics.

Walker's textbook was previously analysed by Gombi (2020) in his Master's thesis. From his work, the textbook was found to be very rich by applying disciplinary and interdisciplinary lenses. Furthermore, he found out that the book shares with the historical text of Galilei (1638) some essential elements of the interdisciplinary treatment. The results of his analysis are briefly summarized in Table 3.4:

Table 3.4: Brief description of Gombi (2020)'s results.

Level of analyses	Results
Ontological	The entity which the disciplinary discourse is based on, the *projectile*, is introduced through a rigorous definition in paragraph 4.3; the definition of all the other concepts and quantities had to be inferred from the context.
Epistemological	The *independence of motions* and the *symmetries* related to the parabola are the epistemological activators of the chapter. These are the elements of Walker that allow comparison with the historical texts of Guidobaldo and Galilei.
Explanatory and argumentative	In modern textbooks, the mathematical structure of the discourse does not explicitly emerge; the same happens in Walker, where the demonstration structure of the parabolic trajectory of projectile motion remains for the most part implicit. On the contrary, the historical text of Galilei explicits it rigorously, because he believed the proof to be a mathematical meta-object in physics. Moreover mathematical concepts are constructed without paying any attention to their underlying physical meaning: in Walker too, the abstract formalism of the analytic functions is always preferred to one focused on the expression of proportions. On the contrary, Galilei used geometry to build the parabolic trajectory in a way that could be as intuitive as possible.

The application of both the linguistic and the epistemological tools in the present thesis enhances the analysis from a methodological point of view, making individual tools even more powerful and producing more comprehensive results. I introduce in the following the main results carried out from the joint application of the tools.

From the very beginning of the text, the first few lines lead to a first important observation. Both the analyses focus on the presence of the word "simple": it is not typical for the sciences to use the connotation, but the adjective represents the most frequent value within the examined text and is often associated with the key concept of the chapter («horizontal and vertical motions are independent»). In this case, its use is mixed with the author's wish to put the reader at ease, but in general, *simplicity* is a goal of physics, which aims to present nature's behavior as it is (namely simple).

In the first paragraph, the linguistic analysis shows the highest concentration of terms related to mathematics (i.e. "equation", "x and y components", etc.) and also a very high number of procedural verbs (i.e. "to apply", "to substitute", etc.): the aim is in fact to extrapolate equations for two-dimensional motion, starting from the one-dimensional case. So, a procedural image of physics seems to emerge, accompanied by an instrumental use of mathematics. This idea seems to be confirmed by the epistemological analysis, given the high presence of practices (i.e. *mathematize, hypothesize, test or explore assumptions,* etc.) and the presence of methods mainly declined as *reductionist rule* (namely all those portions of sentences introduced by "to begin", "first", "next", etc.). This actually contributes to the feeling that the paragraph is just presenting a to-do list, in order to carry out the equations of motion. This part is also the one where the highest level of knowledge can be observed: it can be explained by the educational choice to present the main concept of the chapter immediately, and to confirm it in the form of algebraic steps to be carried out.

The second paragraph presents a high number of verbs "to be" in function of copula, and the dominant lexicon is that of physics (i.e. "projectile", "motion", etc.). After introducing the concept of projectile, the textbook led to move from the main concept to the construction of a model that represents the central idea of the chapter. The knowledge that emerges from this section is mainly related to the description of how to move from the reality to the model, the presentation of the model and the necessary physical and mathematical assumptions. From a procedural image of physics, which had previously emerged, here we switch to one that aims to construct a model (i.e. that of the projectile) able to approximate reality. Consequently, mathematics is no longer merely an algebraic tool, but serves to give structure to the model, allowing it to be formed and conceptualized.

Third and fourth paragraphs are very similar from a linguistic point of view: mathematical language and procedural verbs are prevalent. This is in line with the purpose of this part of the chapter, in which the equations for projectiles with zero and non-zero launch angle, respectively, are derived. What is interesting to note, however, is the role that mathematics takes in a section of the third paragraph, where the proof of the parabolic shape of the trajectory is presented. Although algebraic passages tend to provide the familiar idea that mathematics is only a tool for calculation, in this case its role is not reduced to this. The presence of this actual demonstration, counted among the methods of physics, although it remains implicit for many of its components, reveals a very important argumentative role of mathematics, which provides physics with a solid structure on which to base itself.

The fifth paragraph again shows the prevalence of the physical lexicon, since it presents certain characteristics of projectile motion. Despite the very high number of both procedural verbs and types of procedures, the knowledge outlined suggests that this paragraph deals with a topic that can highlight further roles that the two disciplines can assume. In fact, this section mainly studies the symmetry of the parabola, which originates as an object of geometry but crosses the border and enters the world of physics, providing properties to the trajectory of the projectile. In this case, mathematics can boost epistemological considerations about the phenomenon, stimulating a deeper comprehension.

Conclusions

My personal motivation

The reasons behind this thesis stem from my own experience. My first contact with physics at school was at the beginning of the 4th year of "Liceo Classico" high school: it was taught by the same professor of mathematics for only 2 hours per week, so in the collective of the class it was not perceived as an important subject. Yet, it was the trickiest one for most of my classmates, who had to work hard on it to achieve a passing grade. Moreover, we also perceived physics merely as formulas to apply and problems to be solved, leading most of us to feel the subject non-attractive.

Nevertheless, I enjoyed "that physics", because I was proficient in it. This was one of the main reasons why I chose physics at university. But in several years I could experience the most exciting and powerful aspects of the discipline that I had never met at school: observing the lengthy and arduous process which led to the birth of scientific method and to the formulation of the greatest theories; discovering how science and philosophy are deeply intertwined and affect each other; realizing that skills resulting from physics contribute to making aware decisions in everyday life and to improve people's attitude as informed and responsible citizens. As I discovered the implications and perspectives of physics, I wondered what I missed during high school: the rigid image of physics that emerged in those years does not do justice to the rich one I discovered later.

All this got me thinking about the importance of providing an authentic idea of what physics is. Lessons and school activities should be designed to involve not only those students who, like me, are successful in the more computational part, but also all those who might be fascinated by the many other aspects that physics can offer. This would certainly benefit the acceptance of the discipline by a wider group of students and the understanding of its significance, also from a social point of view.

That is why, little by little, I turned my attention to scientific communication and teaching related issues. My beliefs have been guiding most of my choices during the Master's degree, pushing me to design a certain study program, until I have found out how interdisciplinarity could be the key to providing a truthful and comprehensive image of physics. It is from this idea that my thesis—which for me is itself an example of interdisciplinarity—was born, and which I was glad to present.

Final remarks

The thesis is part of the IDENTITIES project, an Erasmus+ project which aims to design novel teaching approaches on interdisciplinarity, through innovative teaching modules. The central theme is the interdisciplinarity both in STEM and in curricular topics, with a focus on the links and interweaving between physics and mathematics.

What this thesis set out to do, in order to contribute to the purposes of the IDENTITIES project, was to design two different tools to analyse scientific written texts. In particular, the tools were applied to the chapter *Two-dimensional motion* of the textbook *Physics* (Walker, 2017), with the aim of exploring the structure of a high school physics textbook and bringing out its disciplinary and interdisciplinary characteristics.

The first tool developed was linguistic. The study of scientific texts and specialised language has enabled us to create a grid (Table 2.1) for analysing a text on three levels: textual, syntactic and lexical. Each level highlighted relevant information, contributing to shape an image of physics and mathematics. The textual level aimed to show to what extent the two disciplines dialogue with each other, and the characteristics of this interaction. The syntactic level, which describes the structure of sentences and their articulation, allowed us to establish the degree of accuracy and completeness of the content. The lexical level was fundamental both to observe the frequency with which each of the two disciplinary areas appears in the text and to analyse the verbs, giving an image of how the message is conveyed also through the actions required of the reader.

The second tool was epistemological. In this case, the starting point was a well-known framework within science education, the Family Resemblance approach (FRA), described by Irzik & Nola (2011, 2014); then, to adapt it for teaching purposes, Erduran & Dagher (2014) reconceptualized it (Reconceptualized Family resemblance approach for the Nature of science, RFN) and classified the various aspects of science in a wheel (Fig. 3.1). For the purposes of this thesis, only the cognitive-epistemic system of the FRA wheel was considered. The four categories of this system, "Aims & Values", "Scientific Practices", "Methods and Methodological rules" and "Scientific Knowledge", became the epistemological lenses with which analyse the textbook, allowing us to recognize the epistemological profile of physics that emerges and the role played by mathematics in this.

These tools were used to analyse the textbook *Physics* (Walker, 2017), that, as the analysis carried out, stands out from other modern textbooks. In recent studies, as in the analysis conducted by Bagaglini et al. (2021) on how the topic of parabolic motion is covered in textbooks, it is shown that modern textbooks all resemble each other, mainly with regard to the amount and ways of presenting information and the engagement with the recipient. Moreover, the researchers carried out that modern textbooks do not usually address historical-epistemological aspects, that are instead present in historical texts, such as *Discorsi e dimostrazioni matematiche intorno a due nuove scienze* by Galilei (1638). These choices could impact negatively both on the understanding and on the interest in the disciplines (Bagaglini et al., 2021).

By applying the two tools to Walker's textbook, it proved to offer a rich chapter and to distinguish itself in several aspects.

From the linguistic perspective, the tool showed that the dialogue between physics and mathematics is constant, through different linguistic codes (iconographic, written, formular). The conciseness and tendency to use parataxis, typical of modern textbooks, is not found to a significant extent. For this reason, Walker's textbook proved to be fruitful and effective in the explanation and the argumentation, and in avoiding to leave implicit content or take it for granted. From the lexical point of view, it was found that the different paragraphs of the chapter are populated by words and expressions that belong both to mathematics (i.e. "equation", "x and y components", "positive/negative direction", etc.) and physics (i.e. "projectile", "motion", "time", etc.). In particular, we recognized that many nouns are more related to the field of mathematics and most of the verbs belong to the procedural rather than the reasoning domain. However the discrepancy is never very large, thus also from this point of view there is a difference between Walker's textbook and the others: physics does not only emerge as a to-do list and mathematics as a merely algebraic tool, but there is a deeper image of discipline, that we analysed through the epistemological tool.

In order to make further considerations, the linguistic tool was also applied to another high school textbook (Cutnell et al., 2015) and to a historical text (Galilei, 1638), to make a comparison not only between high school textbooks, but also to highlight similarities and differences between modern textbooks and historical resources. This has allowed us to observe that Walker positions himself in an intermediate way between the two, showing tendencies typical of modern textbooks, but also an attention to the argumentation and to the clarification of all the content necessary for understanding the new topic, which is typical of historical texts.

To analyse the disciplinary aspects we applied the epistemological tool, through which it was possible to observe first of all a great semantic abundance, not visible through the use of linguistics. Actually, the application of the epistemological tool allowed us to show that the textbook is very rich in a RFN perspective, which enhances and helps to build a more comprehensive and less rigid image of the disciplines.

By the application of both the tools, some deeper consideration about the image of physics and, consequently, the role mathematics has in the physical context, can be presented.

We noted that there is a high density of aims and values typical of physics, such as *simplicity, generalizability,* and *multi-perspective*: it is not common for these aspects to be given prominence, as the focus is usually on practices and knowledge, but they may be used as a starting point to provide students with diversified pictures of physics.

As already mentioned, in some parts of the text—as well as in most school textbooks (Bagaglini et al., 2021)—the idea emerges that physics is a set of steps to be performed, in order to obtain a result, and that mathematics only acts as a computational tool to reach the goal. This conception is prompted both by linguistic elements (i.e. procedural verbs, short sentences, etc.) and by epistemological aspects (i.e. large number of practices, methods mostly declined as "to begin", "then", etc.). However, careful analysis has brought to light other portrayals of the disciplines.

A first example is given in the section where the projectile motion is presented. In this part, physical knowledge is represented by the transition from reality to the creation of a model (i.e. the projectile), capable of approximating the observed phenomenon of a two-dimensional motion. Assumptions and equations are thus the foundations of this model, so mathematics contributes in a structural way to its construction, providing the criteria to shape the model.

Then, there is a section where the actual proof of the equivalence between the physical equation of the projectile trajectory and the mathematical equation of the parabola is presented. Although many of the components that make up this type of demonstration remain implicit, this part is included in the methods of physics. Moreover, despite the large amount of procedural verbs and practices, it reveals a very important argumentative role of mathematics, which provides physics with a solid structure on which to base itself.

Finally, the chapter includes a section about some characteristics of the two-dimensional motion (i.e. range, time of flight, etc.). In particular, the symmetry of the parabola is studied, which originates as an object of geometry but crosses the border and enters the world of physics, providing properties to the trajectory of the projectile. In this case, mathematics can boost epistemological considerations about the phenomenon, stimulating a deeper comprehension. As far as physics is concerned, the intention here is to give relevance to one of its aims, that of being able to reveal "deeper, more subtle, and sometimes unexpected levels of beauty" (Walker, 2017, p. 106).

All these analyses reveal important information about both disciplinary and interdisciplinary aspects. *Physics* textbook has proved to be a very rich source of insights, and if treated with due care can open the way to a more complete and truthful presentation of physics and mathematics and their interaction.

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