IDENTITIES

Enlightening
Interdisciplinarity
in **STEM**
for Teaching

Ambiguities, negation, interpretation, logical aspects

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Linguistic and epistemology activators

Ambiguities – negation, interpretation, logical aspects

Facing ambiguities with negation of various statements in different disciplines – addressing translation issues - relevance of logical analysis for discussing ambiguities.

Individual – small groups – collective discussions

First activity - Translation of mathematics sentences

For each of the following sentences

A - Translate the sentence in your preferred language else than English.

B - Provide the negation of the sentence you have just translated.

C – Provide the negation of the sentence in English.

S1 - All the balls in the urn are red.

S2 - Some of the triangles in this tessellation are isosceles.

S3 - If a whole number is divisible by 4, then it ends in 4.

S1 - All the balls in the urn are red (1)

- Toutes les boules dans l'urne sont rouges (2)

Toutes les boules dans l'urne ne sont pas rouge (3)

Not all the balls in the urn are red (4)

(3) Is correct following the French linguistic norm

It means exactly : not all the balls in the urn are red

Sometimes interpreted as : no ball is the urn is red.

Corresponding logical form by word to word translation of (3)

Pour tout x dans U, x n'est pas rouge

For all x in U, x is not red

$\forall x \in U, not\ R(x)$

Logical translation according to the linguistic norm of (3)

Not for all x in U, x is red

$Not\ \forall x \in U, R(x)$

$\exists x \in U, not\ R(x)$

S2 - Some of the triangles in this tessellation are isosceles (1)

- Certains triangles de ce pavage sont isocèles (2)

Certains triangles de ce pavage ne sont pas isocèles (3)

Aucun triangle de ce pavage n'est isocèles (4)

Tous les triangles de ce pavage ne sont pas isocèles (5)

Some triangles of this tessellation are not isosceles (6)

No triangle of this tesselation is isocele (7)

All the triangles of this tessellation are isosceles (8)

Corresponding logical form of (3 & 6) : $\exists x \in T, not\ I(x)$

It is not the negation of (1) – both sentences might be true

Corresponding logical form of (3 & 6): $\forall x \in T, not\ I(x)$

It is the negation of (1 & 2)

In French, according to the norm, 5 is not the negation of (2)

S3 - If a whole number is divisible by 4, then it ends in 4 (1)

- Logical formalisation of (1) considering that there is an implicit quantification

Two properties :

D : to be divisible by 4

E : to end by 4

N the set of whome numbers

For all x in N, if D(x), then E(x) - $\forall x \in N\ (D(x) \Longrightarrow E(x))$

Negation of (1) expresses the existence of a counterexample to (1)

There exist a whole number divisible by 4 that does not by 4

$\exists x \in N\ (D(x)\ and\ not\ E(x))$

Students often provide negation with conditional

If a whole number is divisible by 4, then it does not en by 4

If a whole numbers is not divisible by 4, then it does not end by 4.

Second activity in small groups of 3 or 4 participants

Negation of definition in Physics and Computer science and links with proof

A - We recall the definition of *Uniform motion* in Physics

An object is said to have *uniform motion* when this object covers equal distance in equal interval of time within exact fixed direction.

Question - How would say that an object has not a uniform motion?

B - A graph G is defined by a set S of vertices and a set A of edges connecting two vertices.

Definition - An undirected graph G = (S, A) is said to be connected *if and only if* whatever the vertices *u* and *v* of S are, there exists an edge connecting *u* to *v*.

C1- Complete - An undirected graph G = (S, A) is not connected *if and only if*...

C2 - How would you prove that an undirected graph is not connected ?

Negation of definition in Physics

A - We recall the definition of *Uniform motion* in Physics

An object is said to have *uniform motion* when this object covers equal distance in equal interval of time within exact fixed direction.

Question - How would say that an object has not a uniform motion?

An object has not a uniform motion when this object

- does not covers equal distance in equal interval of time within exact fixed direction (1)
- covers inequal distance in equal intervals of time within exact fixed direction (2)
- covers equal distance in inequal intervals of time within exact fixed direction (3)
- covers equal distance in equal interval of time within not exact fixed direction (4)

Else ?

Negation of definition in Computer science and links with proof

B - A graph G is defined by a set S of vertices and a set A of edges connecting two vertices.

Definition - An undirected graph G = (S, A) is said to be connected *if and only if* whatever the vertices *u* and *v* of S are, there exists an edge connecting *u* to *v*.

C1- Complete - An undirected graph G = (S, A) is not connected *if and only if*

There exists at least two vertex u and v such that there is no edge connecting them.

C2 - How would you prove that an undirected graph is not connected ?

- By providing explicitly a pair of verteces so that there is no edge connecting them
- By proving that it is impossible that all the pair of verteces are connected by at least one edge (reductio ad absurdum)

Else ?

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