“What is it possible to say about the divisors of two consecutive natural numbers?”

First student’s answer

The natural numbers are alternated in odd and even; for this reason, two consecutive natural numbers will be one odd and one even. They will have, thus, as common divisor the number one. Moreover, the even number will have two as a divisor.

Es. 1781 and 1782 are divisible by one and themselves, 1782 is divisible by two, being even.

There exist established criteria, that, once known, facilitate the task to find the divisors, or it is possible to go on decomposing the numbers in prime factors. An odd number ending with the digit “5” is for sure divisible for 5, its consecutive and precedent numbers are for sure divisible for 2. The even numbers ending with 0 are divisible for 2 and 5. If I had 1100 and 1101 I could divide 1100 for 2, for 4, for 5, for 10, for 11 while I could divide 1101 only for 3.

Summing up, it seems to me that two consecutive natural numbers can not have common divisors apart from one.

If I consider 17-18, 17 is divisible only for 1 and for itself, 18 is divisible for 1, for 2, for 3 and for 18. ….

IDENTITIES Enlightening Interdisciplinarity in STEM for Teaching

Co-funded by the Erasmus+ Programme of the European Union
Grant Agreement n°2019-1-IT02-KA203-063184

Second student's answer

Examples:

divisors of 14 → 1;2;7;14

divisors of 15 → 1;3;5;15

divisors of 24 → 1;2;3;4;6;8;12;24

divisors of 25 → 1;5;25

Odd numbers have only odd divisors.

Being the two consecutive numbers one even and one odd, only the even one will be divisible by 2.

Divisors of 6 →1;2;3;6

Divisors of 7 → 1;7

The even numbers can have both even and odd divisors, apart from the divisor 2.

Divisors of 19 → 1;19

Divisors of 20 → 1,2,4,5,10,20

Properties: two consecutive natural numbers have only one common divisor, the number 1.

To prove this, it is possible to affirm that, to start, that for sure two consecutive numbers don't have an even common divisor, since the odd numbers don't have even numbers as divisors. Then they do not have common divisors apart from 1 because between two numbers there is only one unit of difference. If a number is divisible by 3, the following divisible by 3 will be bigger of 3 units, not only 1. Since 3 is the odd number consecutive of 1, there are no other values that can be considered as divisors of consecutive numbers

IDENTITIES Enlightening Interdisciplinarity in STEM for Teaching

Co-funded by the Erasmus+ Programme of the European Union
Grant Agreement n°2019-1-IT02-KA203-063184

Third student's answer

(Examples: 1-2, 2-3, 3-4, …)

Conjecture: the only divisor in common between two natural numbers is 1.

Proof:

Two consecutive natural numbers are "constituted" by an even number, i.e. multiple of 2 ($= 2n,\ n \in N$), and an odd number ($= 2n + 1,\ n \in N$).

Hypothesizing *ab absurdum* that 1 is not the only divisor in common, i.e. $\exists k$: $k|2n$ e $k\ |2n + 1 \ \Rightarrow$

$$2n = k * a,\ a \in N$$

$$2n + 1 = k * b,\ b \in N$$

$2n = k * a$ also in the number to the right it must appear the factor 2 ⇒ $k = 2 * c,\ c \in N$

oppure $a = 2 * d,\ d \in N$

$2n + 1 = k * b$ being k in common ⇒ $k = 2 * c,\ c \in N$

oppure $b = 2 * e,\ e \in N$

but only the product of two odd numbers is an odd number ⇒ $k = 1$ is the only possibility

I wasn't able to complete the proof because of the time.

Fourth student's answer

Given $n \in N$, if it is divisible by $d \in N$, then the remainder of the division of n by d is 0, that is to say $n \ mod \ d$ is 0, that is to say in Z_d, $n = 0$.

When I consider $n + 1$, reasoning in the same way I realize that dividing by d I get remainder 1, that is to say $n + 1 = 1$ in Z_d, $\forall d \neq 1$. Then, the only common divisor for n and $n + 1$ is 1

Fifth student's answer

The only common divisor of n and $n + 1$ with $n \in N$ is 1.

Proof:

I hypothesize that y is a divisor of n and $n + 1$, thus:

$y|n$ and $y|n + 1$

But $y|n \Rightarrow n = ya$

and $y|n + 1 \Rightarrow n + 1 = yb$, with $a, b \in N$

I create a system with the two conditions:

$$n = ya \Rightarrow yb - 1 = ya$$

$$n + 1 = yb \Rightarrow n = yb - 1$$

$yb - ya = 1 \Rightarrow y(b - a) = 1 \Rightarrow$ 1) $y|1 \Rightarrow y = 1$ OK

2)

$b - a|1 \Rightarrow b - a = 1 \Rightarrow b = a + 1 \Rightarrow n + 1 = y(a + 1) \Rightarrow$

$n + 1 = ya + y \Rightarrow y = 1$

In both cases I obtained $y = 1 \Rightarrow$1is the only common divisor of n and $n + 1$.

Enlightening Interdisciplinarity in STEM for Teaching

Co-funded by the Erasmus+ Programme of the European Union

Grant Agreement n°2019-1-IT02-KA203-063184