Text 1

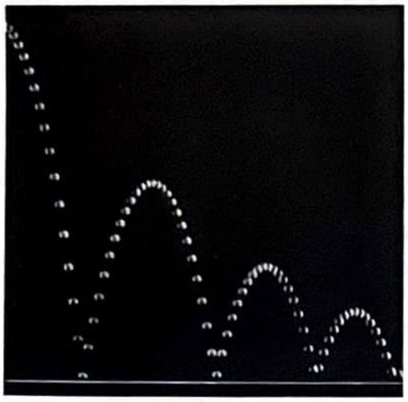


FIGURE 3–17 This strobe photograph of a ball making a series of bounces shows the characteristic "parabolic" path of projectile motion.

3–5 Projectile Motion

In Chapter 2, we studied the motion of objects in one dimension in terms of displacement, velocity, and acceleration, including purely vertical motion of falling bodies undergoing acceleration due to gravity. Now we examine the more general motion of objects moving through the air in two dimensions near the Earth's surface, such as a golf ball, a thrown or batted baseball, kicked footballs, and speeding bullets. These are all examples of **projectile motion** (see Fig. 3–17), which we can describe as taking place in two dimensions. Although air resistance is often important, in many cases its effect can be ignored, and we will ignore it in the following analysis. We will not be concerned now with the process by which the object is thrown or projected. We consider only its motion *after* it has been projected, and *before* it lands or is caught—that is, we analyze our projected object only when it is moving freely through the air under the action of gravity alone. Then the acceleration of the object is that due to gravity, which acts downward with magnitude $g = 9.80\ \mathrm{m/s^2}$, and we assume it is constant.[†]

Horizontal and vertical motion analyzed separately

Galileo was the first to describe projectile motion accurately. He showed that it could be understood by analyzing the horizontal and vertical components of the motion separately. For convenience, we assume that the motion begins at time $t = 0$ at the origin of an xy coordinate system (so $x_0 = y_0 = 0$).

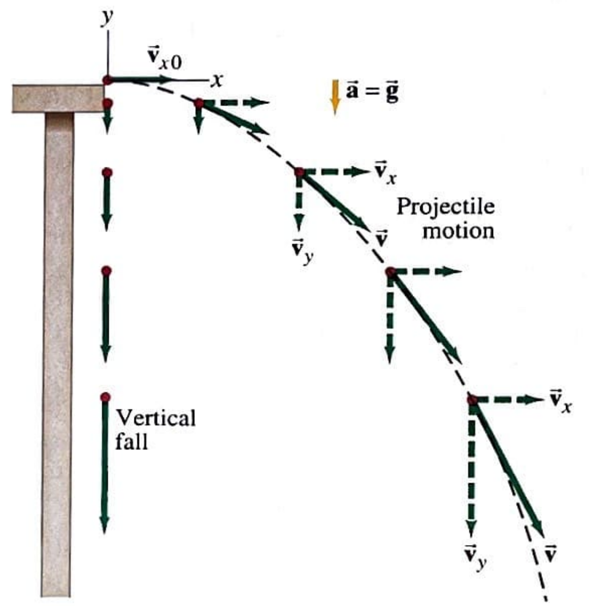


FIGURE 3–18 Projectile motion of a small ball projected horizontally. The dashed black line represents the path of the object. The velocity vector $\vec{v}$ at each point is in the direction of motion and thus is tangent to the path. The velocity vectors are green arrows, and velocity components are dashed. (A vertically falling object starting at the same point is shown at the left for comparison; v_y is the same for the falling object and the projectile.)

$\vec{v}$ is tangent to the path

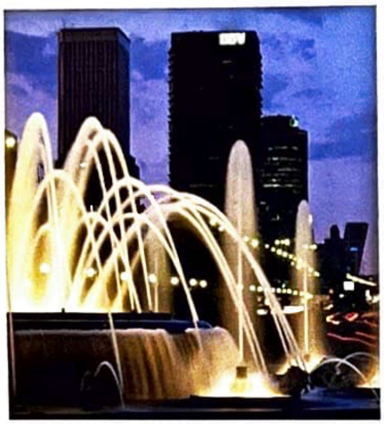
Let us look at a (tiny) ball rolling off the end of a horizontal table with an initial velocity in the horizontal (x) direction, v_{x0}. See Fig. 3–18, where an object falling vertically is also shown for comparison. The velocity vector $\vec{v}$ at each instant points in the direction of the ball's motion at that instant and is always tangent to the path. Following Galileo's ideas, we treat the horizontal and vertical components of the velocity, v_x and v_y, separately, and we can apply the kinematic equations (Eqs. 2–11a through 2–11c) to the x and y components of the motion.

Vertical motion (a_y = constant = $-g$)

First we examine the vertical (y) component of the motion. At the instant the ball leaves the table's top ($t = 0$), it has only an x component of velocity. Once the ball leaves the table (at $t = 0$), it experiences a vertically downward acceleration g, the acceleration due to gravity. Thus v_y is initially zero ($v_{y0} = 0$) but increases continually in the downward direction (until the ball hits the ground). Let us take y to be positive upward. Then $a_y = -g$, and from Eq. 2–11a we can write $v_y = -gt$ since we set $v_{y0} = 0$. The vertical displacement is given by $y = -\frac{1}{2}gt^2$.

[†]This restricts us to objects whose distance traveled and maximum height above the Earth are small compared to the Earth's radius (6400 km).

(a)



(b)

(c)

FIGURE 3–27 Examples of projectile motion—sparks (small hot glowing pieces of metal), water, and fireworks. All exhibit the parabolic path characteristic of projectile motion, although the effects of air resistance can be seen to alter the path of some trajectories.

* 3–7 Projectile Motion Is Parabolic

We now show that the path followed by any projectile is a parabola, if we ignore air resistance and assume that $\vec{\mathbf{g}}$ is constant. To show this, we need to find y as a function of x by eliminating t between the two equations for horizontal and vertical motion (Eq. 2–11b), and we set $x_0 = y_0 = 0$:

$$x = v_{x0}t$$
$$y = v_{y0}t - \tfrac{1}{2}gt^2.$$

From the first equation, we have $t = x/v_{x0}$, and we substitute this into the second one to obtain

$$y = \left(\frac{v_{y0}}{v_{x0}}\right)x - \left(\frac{g}{2v_{x0}^2}\right)x^2.$$

If we write $v_{x0} = v_0 \cos\theta_0$ and $v_{y0} = v_0 \sin\theta_0$, we can also write

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2.$$

In either case, we see that y as a function of x has the form

$$y = Ax - Bx^2,$$

where A and B are constants for any specific projectile motion. This is the well-known equation for a parabola. See Figs. 3–17 and 3–27.

The idea that projectile motion is parabolic was, in Galileo's day, at the forefront of physics research. Today we discuss it in Chapter 3 of introductory physics!

4-2 Projectile Motion: Basic Equations

We now apply the independence of horizontal and vertical motions to projectiles. Just what do we mean by a projectile? Well, a **projectile** is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity. As you might expect, this covers a wide variety of physical systems.

In studying projectile motion we make the following assumptions:

- Air resistance is ignored.
- The acceleration due to gravity is constant, downward, and has a magnitude equal to $g = 9.81\ \mathrm{m/s^2}$.
- The Earth's rotation is ignored.

Air resistance can be significant if a projectile moves with relatively high speed or if it encounters a strong wind. In many everyday situations, however, like tossing a ball to a friend or dropping a book, air resistance is relatively insignificant. As for the acceleration due to gravity, $g = 9.81\ \mathrm{m/s^2}$, this value varies slightly from place to place on the Earth's surface and decreases with increasing altitude. In addition, the rotation of the Earth can be significant when we consider projectiles that cover great distances. Little error is made in ignoring the variation of g or the rotation of the Earth, however, in the examples of projectile motion considered in this chapter.

Equations of Motion for Projectiles Let's incorporate the preceding assumptions into the equations of motion given in the previous section. Suppose, as in **FIGURE 4-2**, that the x axis is horizontal and the y axis is vertical, with the positive direction upward. Noting that downward is the negative direction, it follows that

$$a_y = -9.81\ \mathrm{m/s^2} = -g$$

Gravity causes no acceleration in the x direction. Thus, the x component of acceleration is zero:

$$a_x = 0$$

With these acceleration components substituted into the fundamental constant-acceleration equations of motion (Table 4-1) we find:

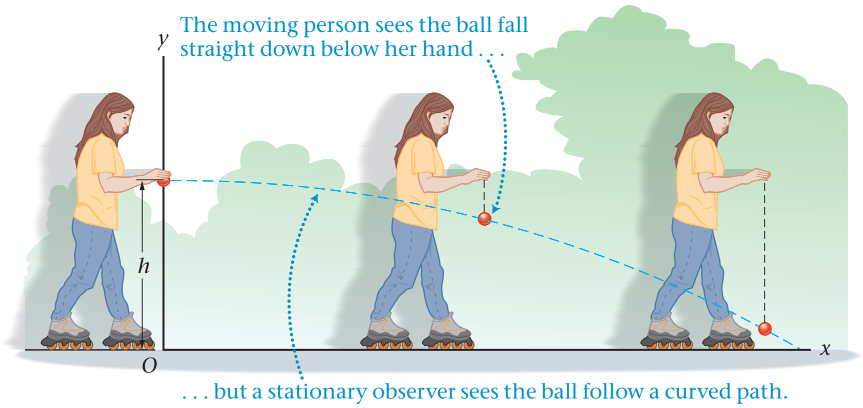
Projectile Motion ($a_x = 0$, $a_y = -g$)

$$x = x_0 + v_{0x}t \qquad v_x = v_{0x} \qquad v_x{}^2 = v_{0x}{}^2$$
$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2 \qquad v_y = v_{0y} - gt \qquad v_y{}^2 = v_{0y}{}^2 - 2g\Delta y \qquad \text{4-6}$$

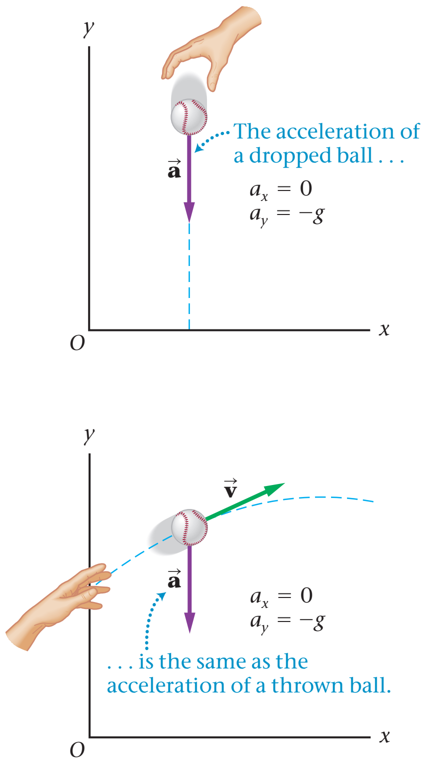
In these expressions, the positive y direction is upward and the quantity g is positive. *All* of our studies of projectile motion use Equations 4-6 as our fundamental equations—again, special cases simply correspond to substituting different specific values for the constants.

Demonstrating Independence of Motion A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion. First, while standing still, drop a rubber ball to the floor and catch it on the rebound. Notice that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second.

Next, walk—or roller skate—with constant speed before dropping the ball, then observe its motion carefully. To you, its motion looks the same as before: It goes straight down, lands near your feet, bounces straight back up, and returns in about one second. This is illustrated in **FIGURE 4-3**. The fact that you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion—the motions are independent.



▲ **FIGURE 4-3 Independence of vertical and horizontal motions** When you drop a ball while walking, running, or skating with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.



▲ **FIGURE 4-2 Acceleration in free fall** All objects in free fall have acceleration components $a_x = 0$ and $a_y = -g$ when the coordinate system is chosen as shown here. This is true regardless of whether the object is dropped, thrown, kicked, or otherwise set into motion.

Big Idea 2 Projectiles are objects that move under the influence of gravity alone. Projectiles can be dropped from rest or thrown at some angle to the horizontal. Once they are launched, they have all the characteristics of projectile motion, irrespective of how their motion started.

PHYSICS IN CONTEXT
Looking Ahead

The basic idea behind projectile motion is used again in Chapter 12, when we consider orbital motion.

PROBLEM-SOLVING NOTE
Acceleration of a Projectile

When the x axis is chosen to be horizontal and the y axis points vertically upward, it follows that the acceleration of an ideal projectile is $a_x = 0$ and $a_y = -g$.

(a) (b) (c)

▲ **FIGURE 4-4 Visualizing Concepts Independence of Motion (a)** An athlete jumps upward from a moving skateboard. The athlete retains his initial horizontal velocity, and hence remains directly above the skateboard at all times. **(b)** The pilot ejection seat of a jet fighter is being ground-tested. The horizontal and vertical motions are independent, and hence the test dummy is still almost directly above the cockpit from which it was ejected. (Notice that air resistance is beginning to reduce the dummy's horizontal velocity.) **(c)** This rollerblader may not be thinking about independence of motion, but the ball she released illustrates the concept perfectly as it falls directly below her hand.

To an observer who sees you walking by, the ball follows a curved path, as shown. The precise shape of this curved path—a parabola—is verified in the next section. Additional examples of this principle are shown in FIGURE 4-4.