IDENTITIES

Enlightening
Interdisciplinarity
in STEM
for Teaching

Introductory lecture on cryptography elements

Cryptography module

Co-funded by the
Erasmus+ Programme
of the European Union

Grant Agreement n°2019-1-IT02-KA203-063184

Cryptography used to *ensure secure communication* (**confidentiality**, **integrity**, **authenticity**)

- Internet security
- Credit cards (smart cards)
- Electronic passports
- Electronic currency (bitcoins)
- Electronic voting
- Medical data

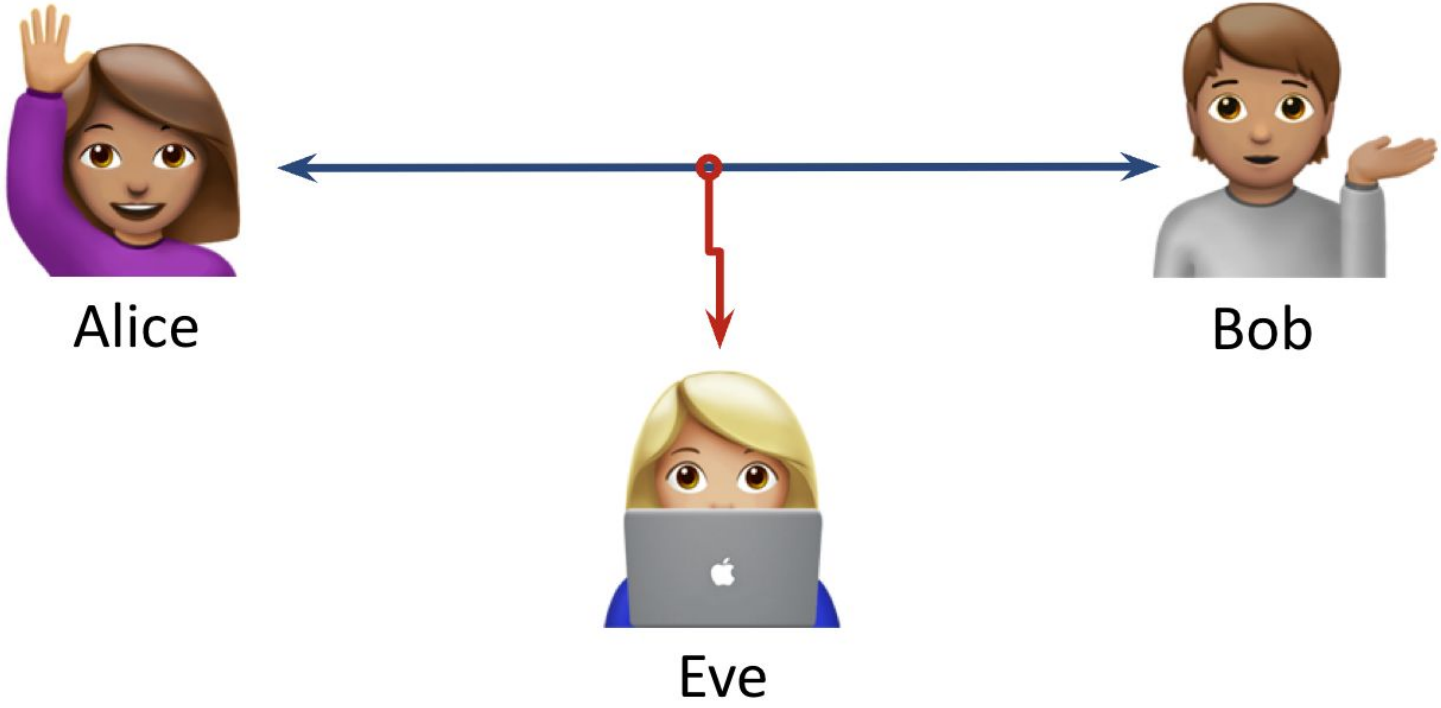
Alice
Bob
Eve

Figure out the answer to the riddle below.
Use the cipher table to show how the letters were encrypted.
Find a pattern to complete the table.

What is a clumsy bee?

B CVNCMJOH CFF

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z

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B		C	V	N	C	M	J	O	H		C	F	F

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z

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What is a clumsy bee?

a		b	u	m	b	l	i	n	g		b	e	e
B		C	V	N	C	M	J	O	H		C	F	F

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z

*How do we **encrypt**?*

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A

*How do we **encrypt**?*

- Shift the alphabet by 1
- C = m +1 (mod 26)

Example: the 11th letter is encrypted by the 12th letter (k -> L)

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A

We can shift by another number k

Additive Ciphers

- Plaintext **m**
- Ciphertext **C**
- Key **k**
- **Encryption** C = m+k (mod 26)
- **Decryption** m = C-k (mod 26)

L MFXMWTYR MPP

Example: Shifting by 11

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K

PROBLEM with this system

To determine what the shift is, we need to guess *only one letter*

(...there are 25 guesses at most!)

L		M	F	X	M	W	T	Y	R		M	P	P

We need to add some more confusion ...

Multiplicative Ciphers

- Plaintext **m**
- CIPHERTEXT **C**
- Key **k**
- **Encryption** C = m*k (mod 26)
- **Decryption** m = C*k^-1 (mod 26)

Multiplicative Ciphers (examples of enc dec)

- Plaintext **m**
- CIPHERTEXT **C**
- Key **k** (here **k**=5)

Encryption **C** = m*k(mod 26)

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	5	10	15	20	25	4	9	14	19	24	3	8	13	18	23	2	7	12	17	22	1	6	11	16	21
A	F	K	P	U	Z	E	J	O	T	Y	D	I	N	S	X	C	H	M	R	W	B	G	L	Q	V

m = C*k^-1(mod 26) **Decryption**

5 * 21 = 105 (mod 26) = 1 (mod 26) => k^-1 = 21 i.e. 21 is the multiplicative inverse of k=5
For example to decipher D => 3 * 21(mod 26) = 63 (mod 26)= 11 (mod 26) => l

Multiplicative Ciphers

- The KEY has to be coprime with 26 (alphabet length)
  - The possible keys are only the numbers coprime with 26
- If not, **encryption is NOT injective**
  - Example (table below): the **KEY is 2** (*not coprime with 26*).
    Because of the modulo operation the encryption produces only half of the letters.

Encryption **C** = m*k(mod 26)

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0	2	4	6	8	10	12	14	16	18	20	22	24	0	2	4	6	8	10	12	14	16	18	20	22	24
A	C	E	G	I	K	M	O	Q	S	U	W	Z	A	C	E	G	I	K	M	O	Q	S	U	W	Z

k = 2 example a => A, but also n => A because 13*2=26=0mod26

Problem:
Each plaintext letter is always encrypted with the same ciphertext letter.

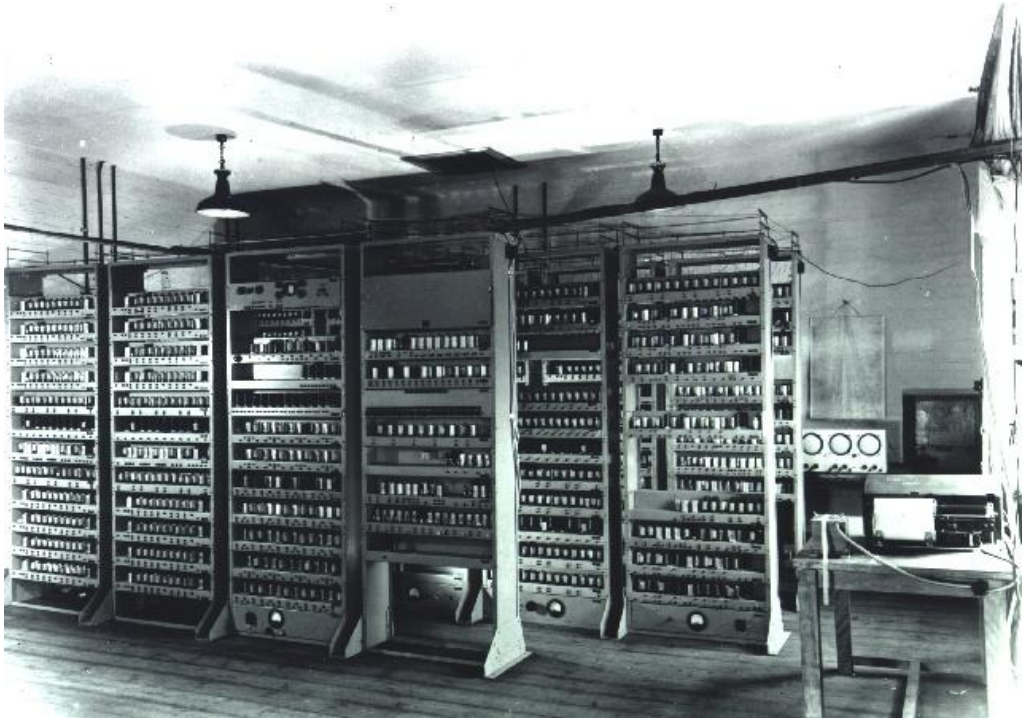
Solution:
Polyalphabetic Ciphers like the Vigenere Code

Each plaintext letter is encrypted by different ciphertext letters.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

But even that was broken and ....

COMPUTER SCIENCE came in the 50s



Binary ciphers

- **Every letter is translated to a binary vector**
- The message is splitted into blocks
- Every block is encrypted using a binary cipher

ASCII Alphabet			
A	1000001	N	1001110
B	1000010	O	1001111
C	1000011	P	1010000
D	1000100	Q	1010001
E	1000101	R	1010010
F	1000110	S	1010011
G	1000111	T	1010100
H	1001000	U	1010101
I	1001001	V	1010110
J	1001010	W	1010111
K	1001011	X	1011000
L	1001100	Y	1011001
M	1001101	Z	1011010

Binary ciphers

- Every letter is translated to a binary vector
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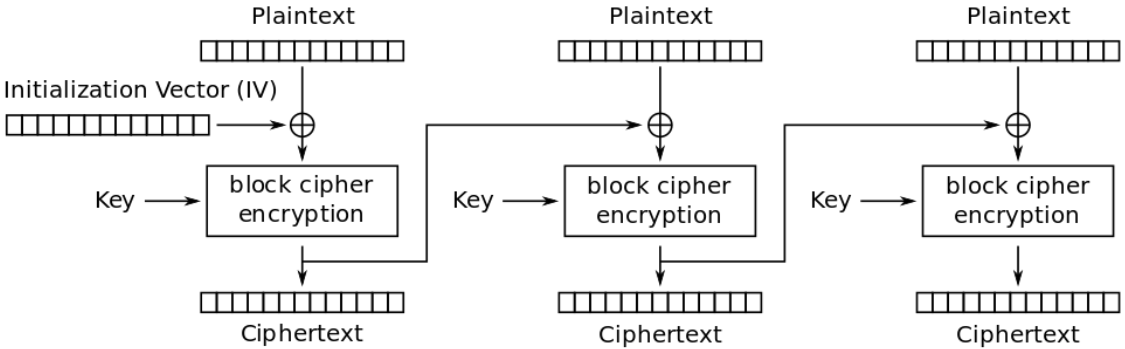
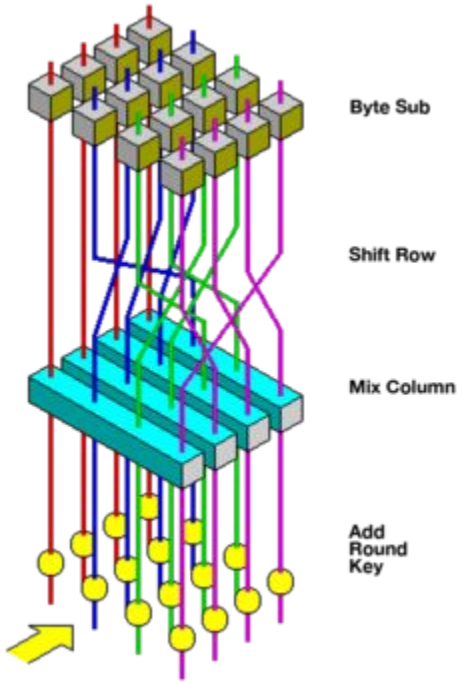
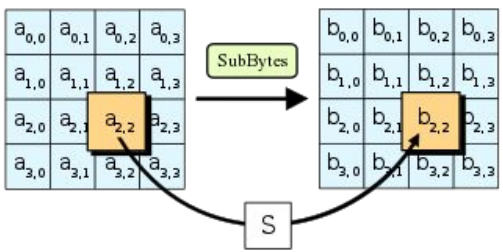
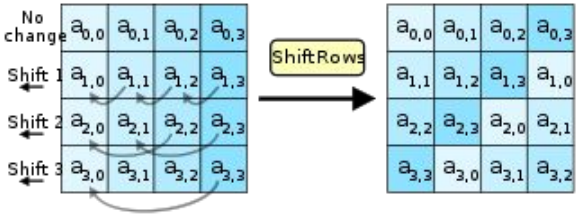


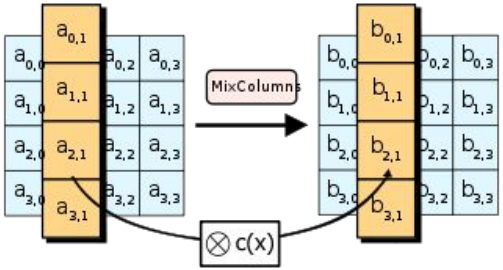
Cipher Block Chaining (CBC) mode encryption

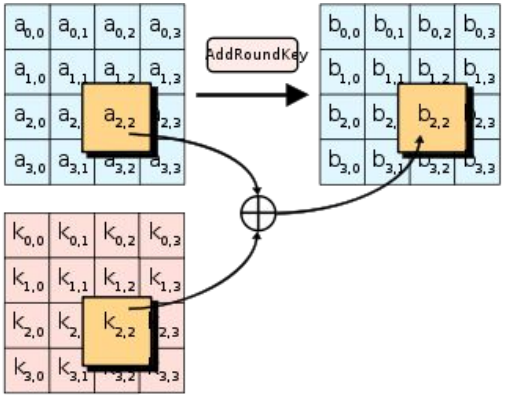


Nowadays we are using **AES**
(***A**dvanced **E**ncryption **S**tandard*)

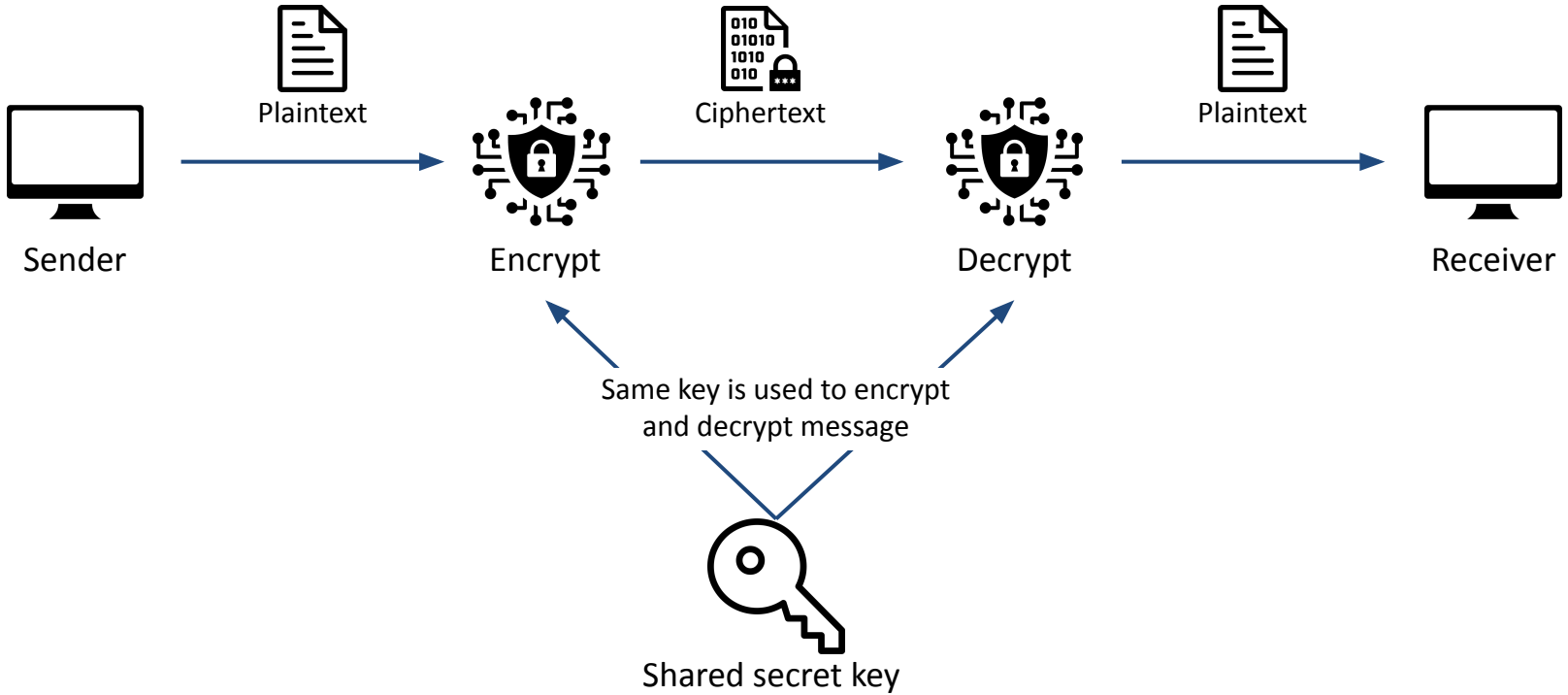




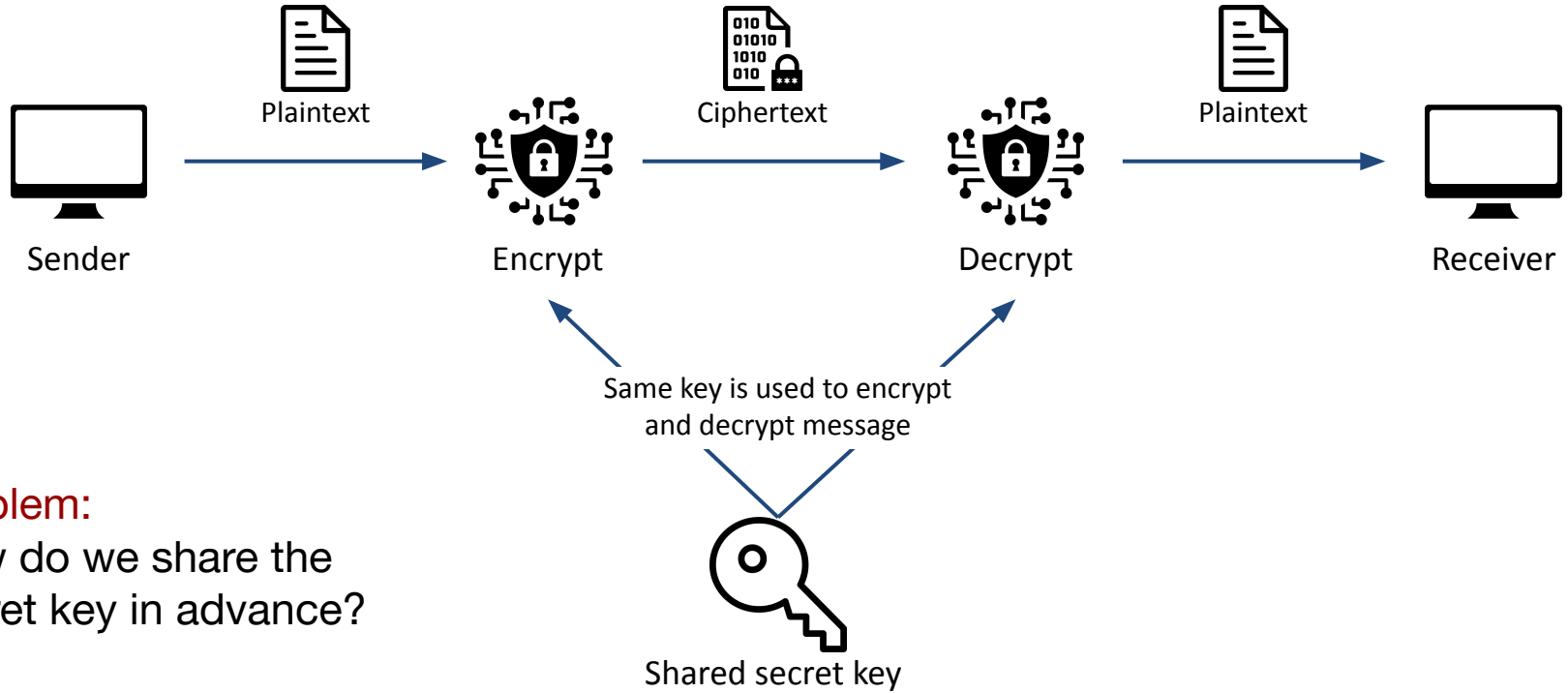




SYMMETRIC CRYPTOSYSTEM

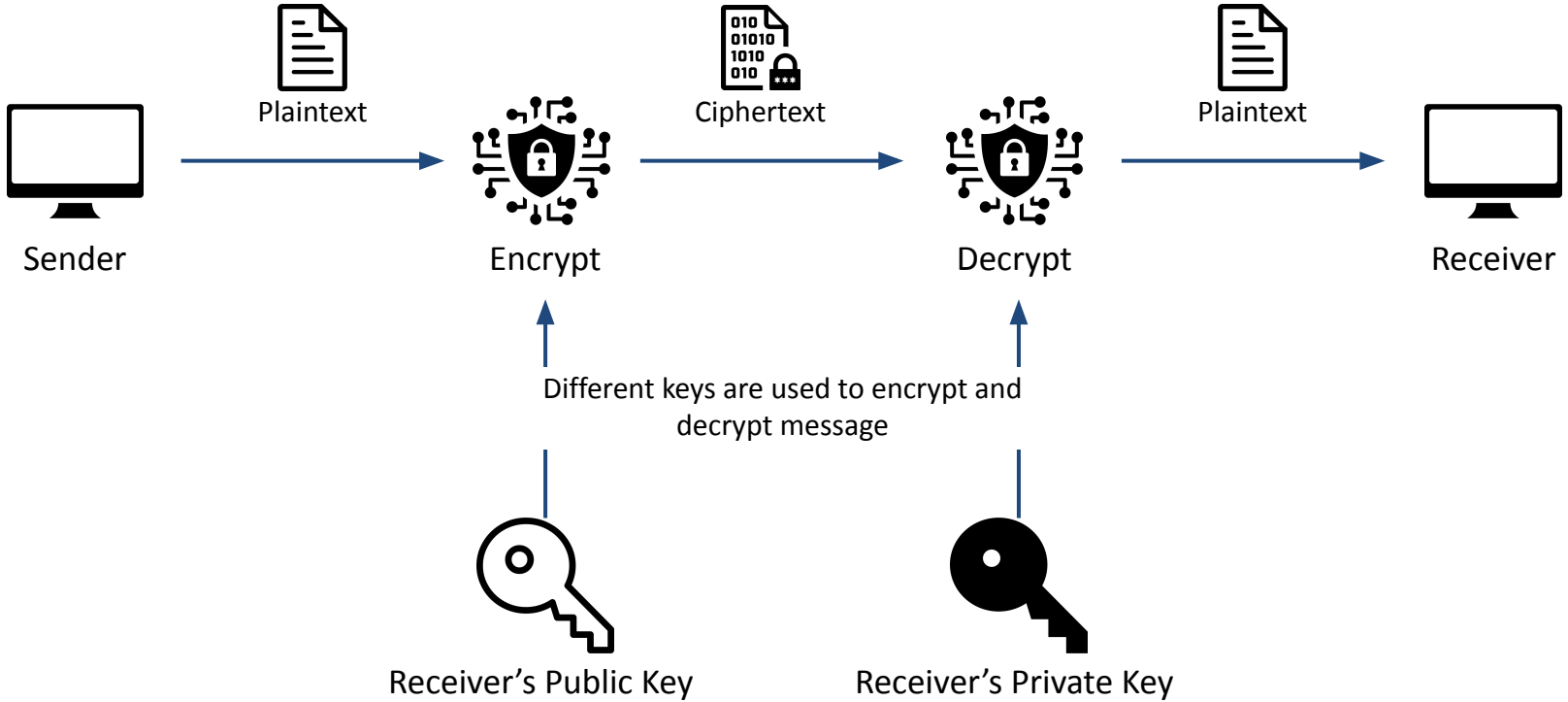


SYMMETRIC CRYPTOSYSTEM



Problem:
How do we share the secret key in advance?

ASYMMETRIC CRYPTOSYSTEM



Kerckhoff's Principle

A cryptosystem should be secure even if everything about the system EXCEPT THE KEY is public knowledge

Engineers should design their system assuming that their opponent knows the system in detail

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