



# Complex systems and simulations

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# The module

- Introduction to complex systems (between physics and other disciplines)
- Students' difficulties in learning complex systems
- Explanation models for complex systems
- Introduction to simulations: equation-based and agent-based approaches
- Epidemiological models as *boundary objects*: equations, agents and networks

# Complex Systems

A complex system is a system composed of several **individual elements** (agents) which, **interacting with** each other according to **non-linear** relationships, give the resulting complex **system** certain **properties** that classical systems do not have (Cilliers, 2007)

## Simple systems

- If a simple system evolves over time, the way it evolves is predictable given the initial conditions (**determinism**)
- If two simple systems start from different initial conditions, their development over time will be all the more different the more different the initial conditions are

## Complex Systems

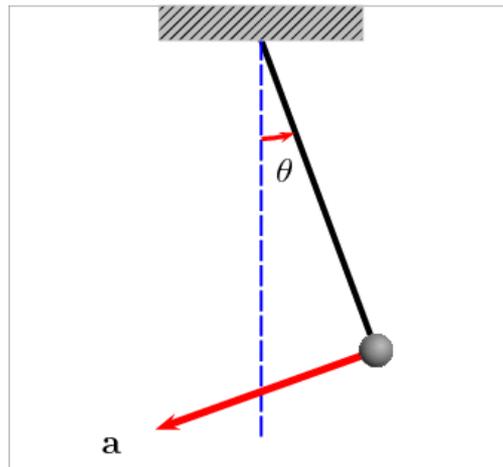
- Future states of a complex system are generally not predictable on the basis of initial conditions
- Complex systems are very sensitive to their initial conditions: two complex systems starting from close initial conditions can develop in very different ways over time (**deterministic chaos**)

# Examples of complex systems in physics

- Let us start with the simple system model par excellence: the harmonic oscillator

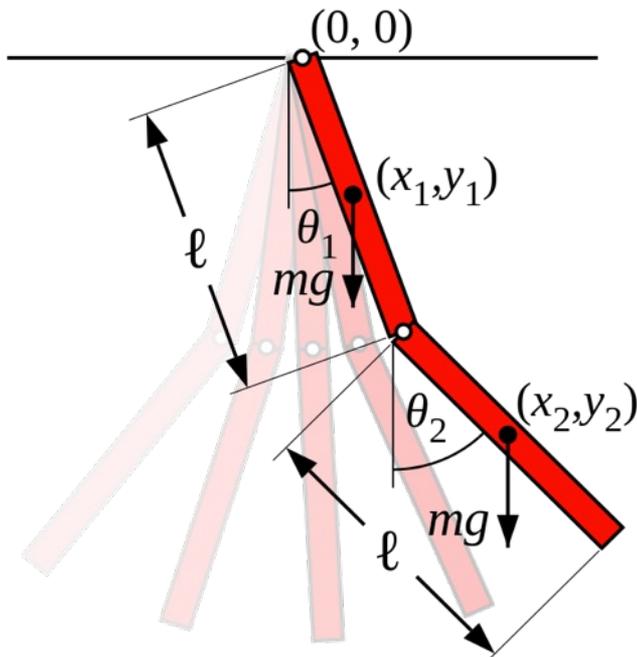
$$\ddot{x}(t) = -\omega_0^2 x(t)$$

- The same equation of linear motion applies to the simple pendulum in the Taylor approximation for small angles



# The double pendulum

- By connecting two simple pendulums, we obtain the double pendulum



$$\dot{\theta}_1 = \frac{6}{m\ell^2} \frac{2p_{\theta_1} - 3 \cos(\theta_1 - \theta_2)p_{\theta_2}}{16 - 9 \cos^2(\theta_1 - \theta_2)}$$

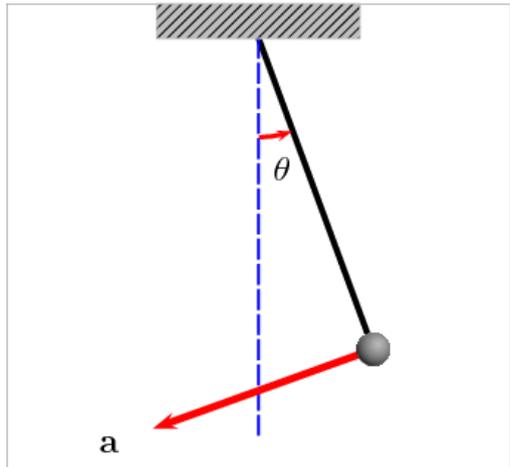
$$\dot{\theta}_2 = \frac{6}{m\ell^2} \frac{8p_{\theta_2} - 3 \cos(\theta_1 - \theta_2)p_{\theta_1}}{16 - 9 \cos^2(\theta_1 - \theta_2)}$$

$$\dot{p}_{\theta_1} = \frac{\partial L}{\partial \theta_1} = -\frac{1}{2}m\ell^2 \left[ \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + 3\frac{g}{\ell} \sin \theta_1 \right]$$

$$\dot{p}_{\theta_2} = \frac{\partial L}{\partial \theta_2} = -\frac{1}{2}m\ell^2 \left[ -\dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + \frac{g}{\ell} \sin \theta_2 \right]$$

- The positions of the pendulum elements can be obtained by numerically integrating the equations of motion

## A simple pendulum



Equation of linear motion for small angles (harmonic oscillator)

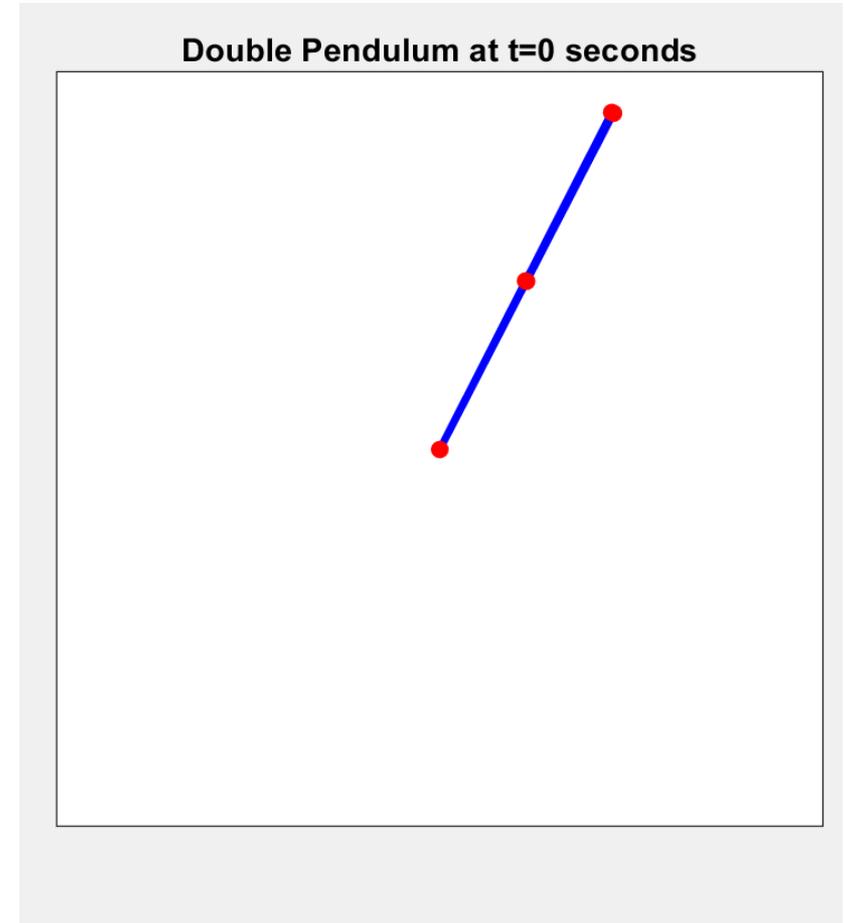
## A double pendulum



Non-linear equations of motion

# The double pendulum

- We see the evolution of the system from close but not coincident initial positions ( $+0.5^\circ$  and  $-0.5^\circ$  from the reference position) an example of **deterministic chaos**
- However, the evolution over time is not random
- The system remains deterministic but it is its non-linearity - together with the impossibility of determining the initial conditions exactly - that leads to a loss of predictability on the system



# Complex Systems

A complex system is a system composed of several **individual elements** (agents) which, **interacting with** each other according to **non-linear** relationships, give the resulting complex **system** certain **properties** that classical systems do not have (Cilliers, 2007)

## Simple systems

- A simple system tends not to be involved in mutual causal relationships (**linear causality**)
- One element of the system influences another, but not vice versa

## Complex Systems

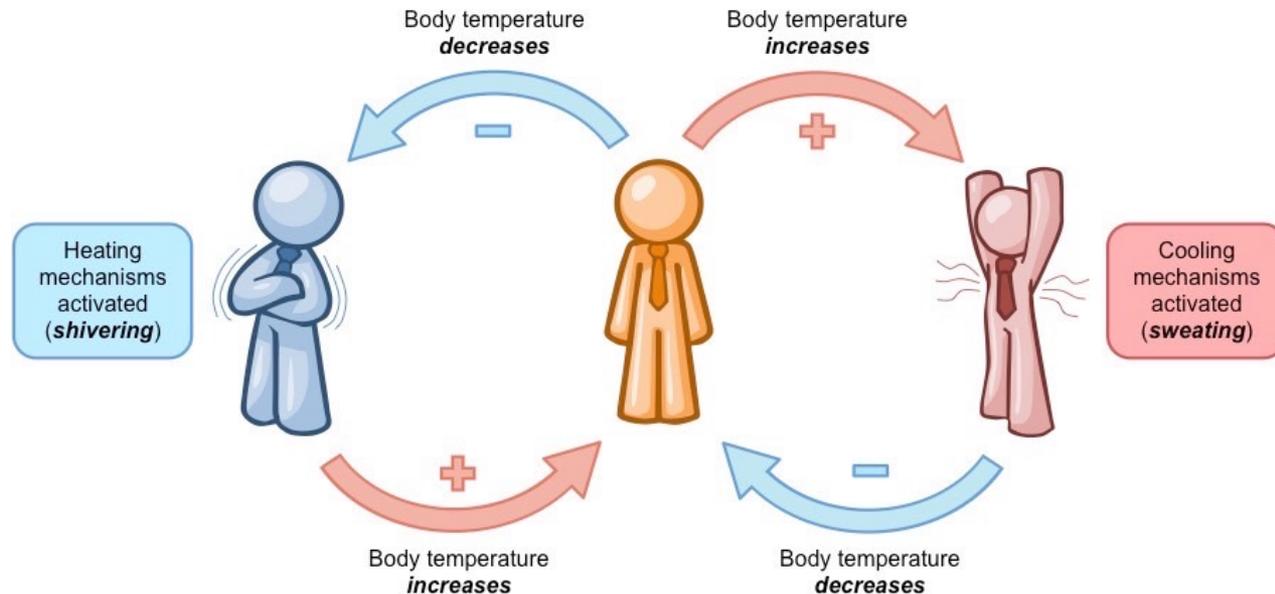
- Complex systems are involved in mutual causal relationships that include positive or negative feedback effects (**circular causality** and **feedback**)
- One element of the system influences another element and is influenced by it

# Two types of circular causality

Positive feedback



Negative feedback



# Complex Systems

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## Simple systems

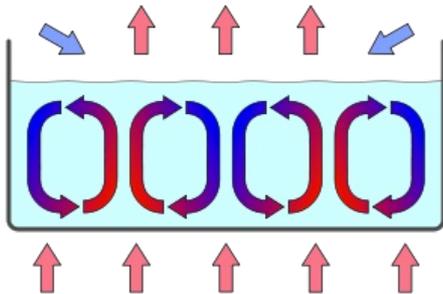
- A simple system consists of elements whose role in determining the overall behaviour of the system can be clearly identified
- The systemic properties of a simple system can be reconstructed and explained from the properties of the individual elements

## Complex Systems

- The role that each element of a complex system plays in determining the overall behaviour of the system is not clearly identifiable
- The systemic properties of a complex system cannot be reconstructed or explained from the properties of individual elements (**emergent** and **self-organising phenomena**)

# Rayleigh-Bénard convection as an example of self-organisation

- It is a convective motion that occurs when a thin horizontal layer of liquid is heated from below
- The effect of this motion on the fluid is the creation of a regular pattern on the surface (Bénard cells)
- The pattern is typical of the macroscopic level of the system and can only be observed only "from above" property emergent property of the system that is self-organising without order being "imposed"



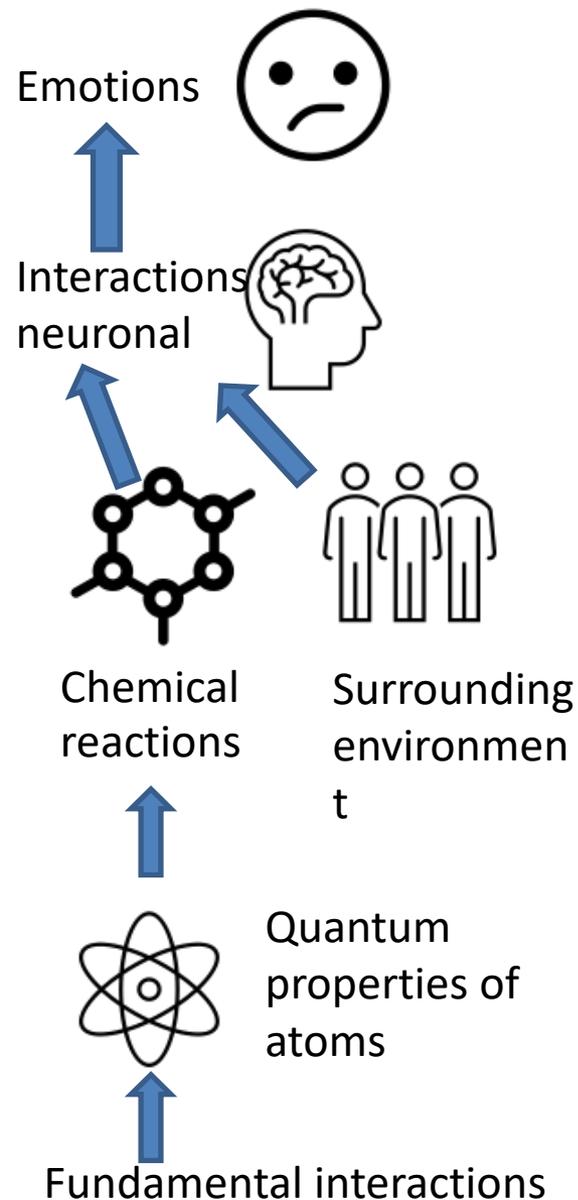
# Modelling complexity: reductionism or holism?

Consider an example of a financial market composed of individual and institutional investors whose actions are driven by the desire for profit and risk aversion. Although it is more or less known what 'forces' govern investors (in the simplest view greed and fear) and what interactions take place between them (exchange of information and transactions), in practice this knowledge is far insufficient to be able to construct a realistic model of the market and, on the basis of it, accurately predict future movements in share prices.



[https://www.researchgate.net/profile/Stanislaw-Drozdz/publication/233757348\\_Physical\\_approach\\_to\\_complex\\_systems/links/5c01b049299bf1a3c159330c/Physical-approach-to-complex-systems.pdf](https://www.researchgate.net/profile/Stanislaw-Drozdz/publication/233757348_Physical_approach_to_complex_systems/links/5c01b049299bf1a3c159330c/Physical-approach-to-complex-systems.pdf)

Diving deeper into market structure, investors' emotions are undoubtedly a product of neuronal interactions in their brains. These emotions, however, can neither be explained nor predicted in such a 'microscopic' way. The activity of neurons is in turn a consequence of certain chemical reactions that take place in the neurons themselves and in the surrounding environment; the chemical reactions in a living cell are a higher-level effect of the quantum properties of atoms, which themselves consist of electrons and nucleons, and the latter are perhaps no more than the triples of strongly interacting quarks. Reasoning along these lines, it is easy to see that, in theory, all laws of financial economics must be a strict mathematical consequence of the four fundamental interactions between elementary particles. Of course, from a practical point of view, this way of thinking is highly misleading, since it is impossible to derive those laws using such a bottom-up approach. Therefore, **in order to fully explain financial market behaviour, it is necessary to neglect the deeper levels of organisation without any significant loss of information.**



# Examples of systems with emergent properties

The formulation of independent but complementary descriptions on different levels of organisation of matter is indispensable. This stems from the fact that 'the whole is something beyond the parts' or 'the whole is different': phenomena occurring at higher levels may not be a direct product of the lower-level structure and dynamics of the system's components. Examples of such phenomena are abundant: convection, turbulence, phase transitions, friction, fractal coastlines, DNA self-replication, metabolic cycles in living cells, multicellular organisms, population dynamics in ecosystems, brain potentials and cognition, natural language syntax, money, economic cycles in the economy, social structure, etc. Their common property is emergence, i.e. a spontaneous occurrence of macroscopic order from a vast array of randomly interacting elements at the microscopic level. This is possible if the elements interact in a strongly non-linear manner and the interaction can be propagated over long distances. Local fluctuations can thus be transformed into collective behaviour, depending on the state of the environment.

[https://www.researchgate.net/profile/Stanislaw-Drozdz/publication/233757348\\_Physical\\_approach\\_to\\_complex\\_systems/links/5c01b049299bf1a3c159330c/Physical-approach-to-complex-systems.pdf](https://www.researchgate.net/profile/Stanislaw-Drozdz/publication/233757348_Physical_approach_to_complex_systems/links/5c01b049299bf1a3c159330c/Physical-approach-to-complex-systems.pdf)

# Complex systems and physics: what relationships?

Complex systems are commonly found in nature; they can also be products of human technology, such as the Internet, communication networks, financial markets and so on. Their ubiquity means that various complex systems are traditionally the **subject of interest in many different fields of science** with their different analytical tools and languages: physics, chemistry, physiology, genetics, linguistics, economics, sociology, information theory and many others. Within each of these fields, there are **well-established laws concerning the macro- and microscopic properties of the relevant systems**, but, from the classical perspective, their domain does not exceed a horizon of interest of the corresponding scientific field. Adopting such a perspective **inevitably loses the possibility of a parallel study of distinct complex systems** and the derivation of more general laws.

# Complex systems and physics: what relationships?

Only recently has this paradigm changed, mainly due to a growing interest in interdisciplinary research. Some have even begun to consider complex systems as a topic of study in their own right and a new field of science: complex systems research. It has been discovered that many complex systems, sometimes of very distinct structure, have surprisingly common properties. This may suggest that such properties are universal for a broad class of systems and can be considered a starting point for studies aimed at formulating specific laws for such systems. Importantly, among the many scientific disciplines that contribute to the study of complex systems, physics seems to be the best equipped.

# Complex systems and physics: what relationships?

Physics has developed theories and tools that can be useful in this field of research: non-linear dynamics, theory of critical phenomena, renormalisation theory, theory of self-organised criticality, synergy and so on.

This is why physics, which until fairly recently was regarded as a deep-level (reductionist) science by many other disciplines, e.g. economics and neuroscience, and therefore wholly inadequate to describe the higher organisational levels of matter, such as financial markets or the human brain, is now appreciated as a discipline of equal importance.