IDENTITIES

Enlightening Interdisciplinarity in STEM for Teaching

Text Analysis

O3 – Parabola and parabolic motion

Co-funded by the Erasmus+ Programme of the European Union

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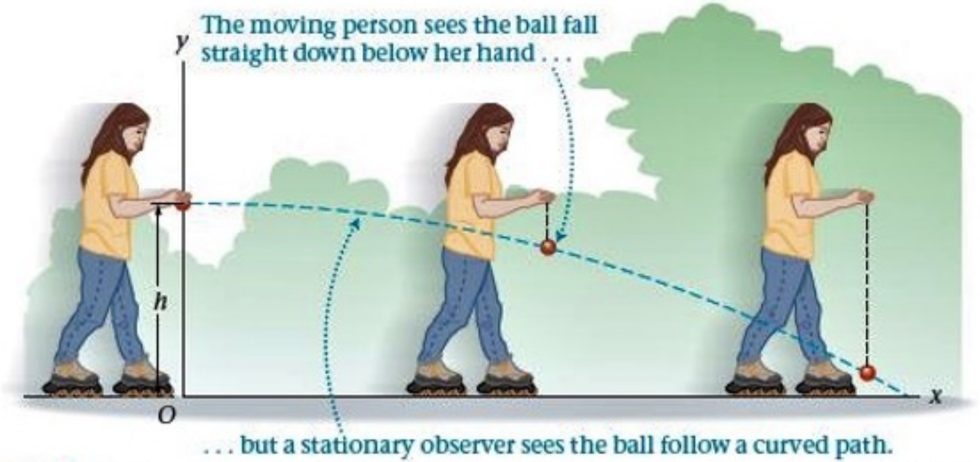
Part 2:

Read carefully the following texts.

Demonstrating Independence of Motion A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion. First, while standing still, drop a rubber ball to the floor and catch it on the rebound. Notice that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second.

Next, walk—or roller skate—with constant speed before dropping the ball, then observe its motion carefully. To you, its motion looks the same as before: It goes straight down, lands near your feet, bounces straight back up, and returns in about one second. This is illustrated in FIGURE 4-3. The fact that you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion—the motions are independent.



▲ **FIGURE 4-3 Independence of vertical and horizontal motions** When you drop a ball while walking, running, or skating with constant velocity, it appears to you to drop straight down from the point where you released it. To a person at rest, the ball follows a curved path that combines horizontal and vertical motions.

(a) (b) (c)

▲ **FIGURE 4-4 Visualizing Concepts Independence of Motion (a)** An athlete jumps upward from a moving skateboard. The athlete retains his initial horizontal velocity, and hence remains directly above the skateboard at all times. **(b)** The pilot ejection seat of a jet fighter is being ground-tested. The horizontal and vertical motions are independent, and hence the test dummy is still almost directly above the cockpit from which it was ejected. (Notice that air resistance is beginning to reduce the dummy's horizontal velocity.) **(c)** This rollerblader may not be thinking about independence of motion, but the ball she released illustrates the concept perfectly as it falls directly below her hand.

To an observer who sees you walking by, the ball follows a curved path, as shown. The precise shape of this curved path—a parabola—is verified in the next section. Additional examples of this principle are shown in **FIGURE 4-4**.

Parabolic Path

RWP Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = \frac{x}{v_0}$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2v_0^2}\right)x^2 \qquad 4\text{-}8$$

It follows that y has the form

$$y = a + bx^2$$

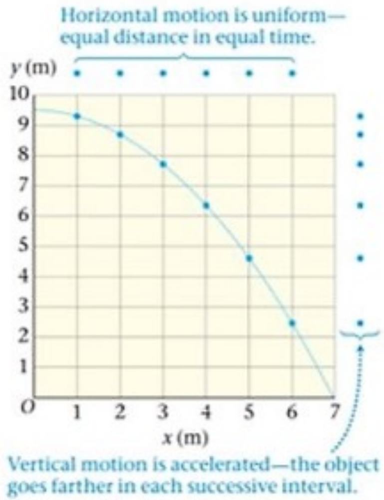


FIGURE 4-7 Visualizing Concepts
Parabolic Trajectories Lava bombs (left) and fountain jets (right) trace out parabolic paths, as is typical in projectile motion. The trajectories are only slightly altered by air resistance.

In this expression, $a = h =$ constant and $b = -g/2v_0^2 =$ constant. This is the equation of a *parabola* that curves downward, a characteristic shape in projectile motion. Examples of parabolas produced by real-world projectiles are shown in FIGURE 4-7.

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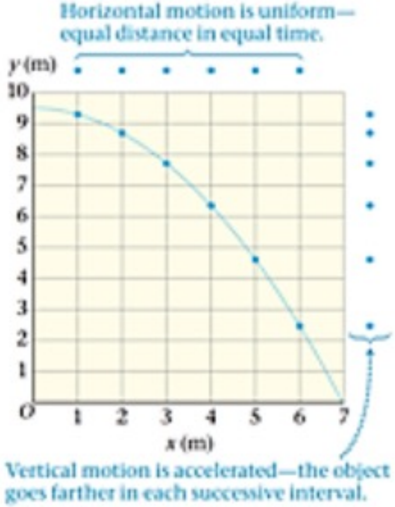

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PART 2:

QUESTIONS

1. What disciplinary elements of mathematics can be found in the texts?

2. How do you recognise them? How are they expressed?

3. Please, comment, in the light of the knowledge provided in the summer-school lectures, on the use of the word “demonstration” in Walker and on the proof elaborated to conclude that trajectory is parabolic

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4. Based on the analysis of the text, what arguments can be detected to recognise the ***proof*** as a ***boundary object***?