IDENTITIES

Enlightening Interdisciplinarity in STEM for Teaching

Student activity - O3: Parabola and parabolic motion

Parabolic motion and the birth of Physics as discipline

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Parabola
|
The conics

⟺

Parabolic motion
|
Motion of the planets

Parabola

|

The conics

⟺

Parabolic motion

|

Motion of the planets

School subjects, for example in physics…

Parabolic Path

RWP Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

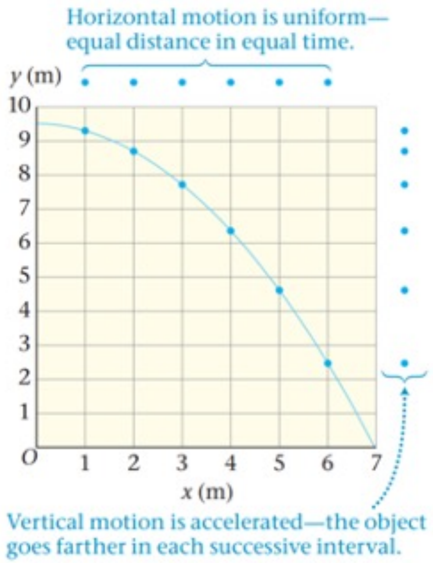
$$t = \frac{x}{v_0}$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2{v_0}^2}\right)x^2 \qquad \text{4-8}$$

It follows that y has the form

$$y = a + bx^2$$



◀ **FIGURE 4-7 Visualizing Concepts**
Parabolic Trajectories Lava bombs (left) and fountain jets (right) trace out parabolic paths, as is typical in projectile motion. The trajectories are only slightly altered by air resistance.

In this expression, $a = h$ = constant and $b = -g/2{v_0}^2$ = constant. This is the equation of a *parabola* that curves downward, a characteristic shape in projectile motion. Examples of parabolas produced by real-world projectiles are shown in **FIGURE 4-7**.

Parabola

|

The conics

⇔

Parabolic motion

|

Motion of the planets

School subjects

But not only!

Parabola
|
The conics

⟺

Parabolic motion
|
Motion of the planets

School subjects

But not only!

They are at the core of important changes **in the establishment of physics as a discipline**:
- the transition from the medieval to the modern conception of science (from a world of quality to a world of quantity, from uniform movement as a process to a state - the formulation of the "revolutionary" first law....)

Parabola
|
The conics

⟺

Parabolic motion
|
Motion of the planets

School subjects

But not only!

They are at the core of important changes **in the history of mathematics**:
- the genesis of projective geometry, thanks to the contribution of a physicist like Kepler (Laura)

Parabola

|

The conics

⟺

Parabolic motion

|

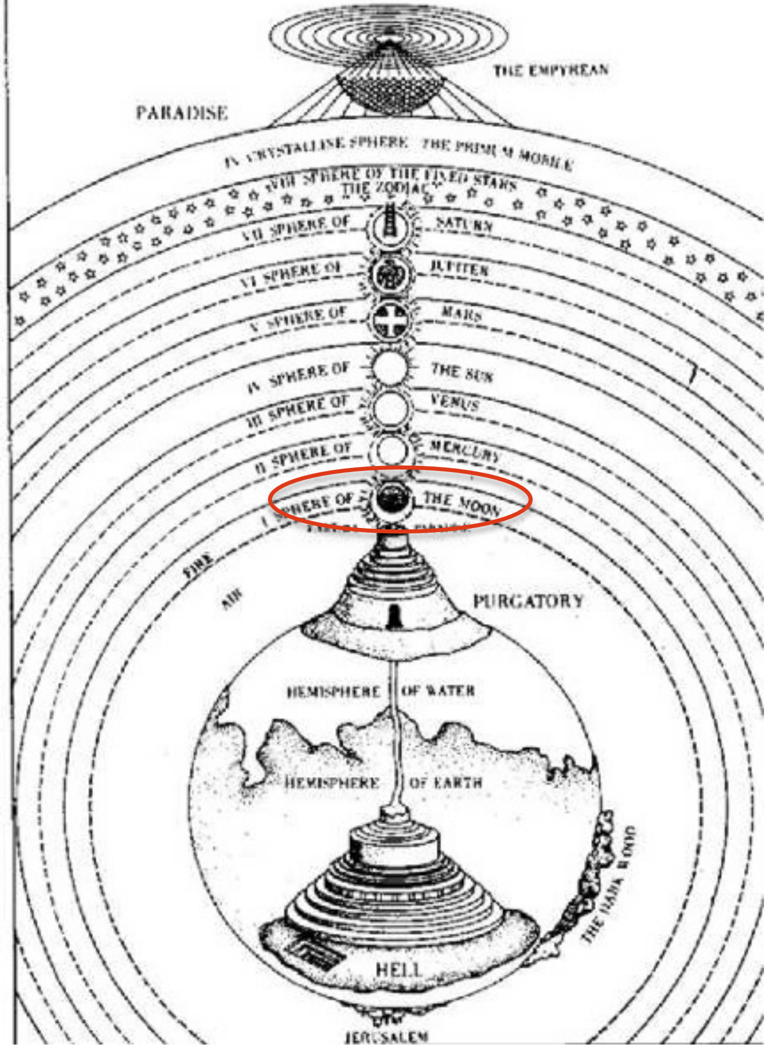
Motion of the planets

In physics!

Scientific thought - in particular, the physical sciences - does not develop "in vacuo", but within a framework of ideas, fundamental principles, axiomatic evidences which, usually, have been considered to belong to philosophy.

The transition to modern science does not mean fighting wrong or insufficient theories, but revolutionizing the **intellectual attitude** that is natural by replacing it with another that was not natural.

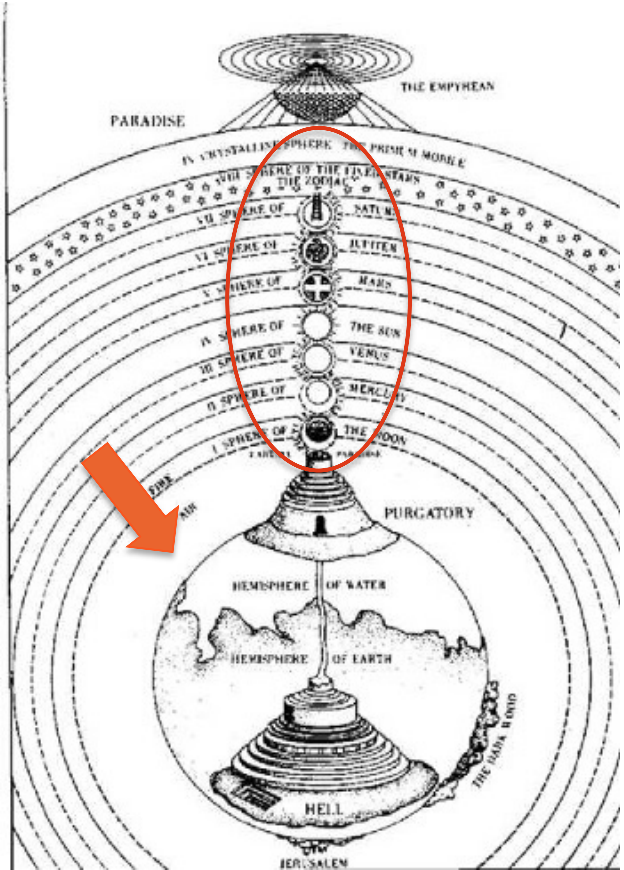
Freely taken from Koyré, “La verità degli eretici”



The medieval view of the universe in a (incredible semplified) nutshell

The medieval view of the universe and its legacy from ancient greek philosophy (Plato and Aristotle):

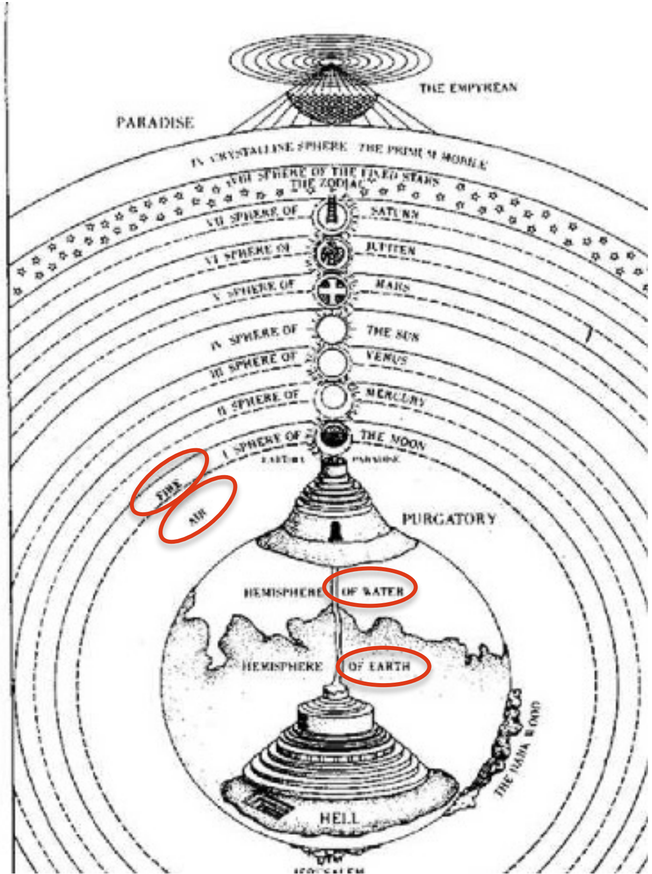
- The distinction between the celestian and the sub-lunar world;
- The different role of mathematics in the two worlds;
- Spheres and straight lines as the only «shapes» allowed to describe the world;
- The classification of the «basic motions» of sublunar world was based on the distinction between natural and violent.



The Aristotelian two-sphere universe

„This conception considered a finite and completely full universe limited by a sphere of stars. The majority of its interior was supposed to be filled with a simple element, the ether, aggregated in a set of nesting shells containing the planets. The sphere of stars formed the outer surface of that aggregate of shells, and the sphere containing the moon (the lowest planet) formed its inner surface. The earth rested in the centre of this universe.“

Gilbert, J. K., & Zylbersztajn∗, A. (1985). A conceptual framework for science education: The case study of force and movement. The European Journal of Science Education, 7(2), 107-120., pp.110-111)

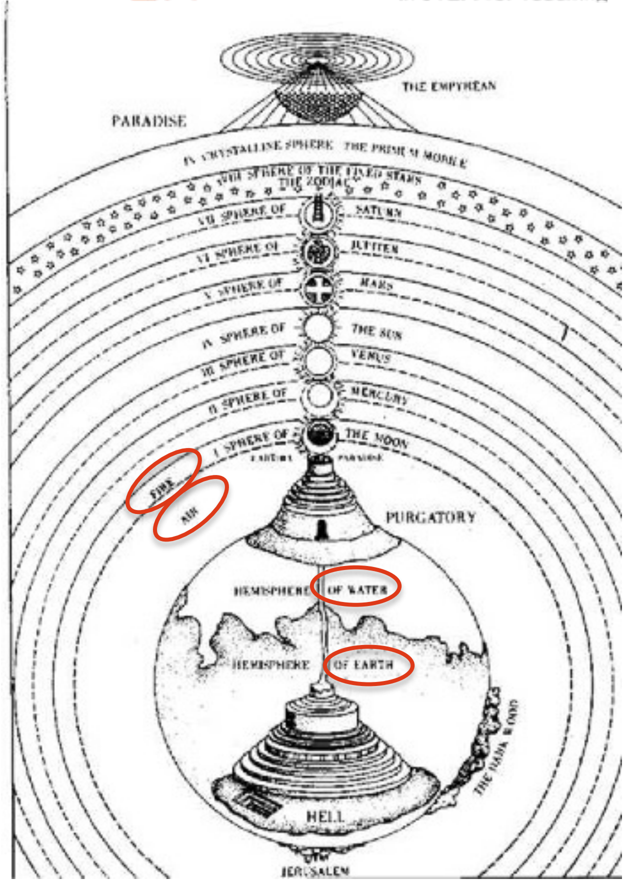


The Aristotelian two-sphere universe: the sublunary region“

„The sublunary region was filled with the four Aristotelian elements: earth, water, air and fire. At every point of this universe some sort of substance was present. Matter and space were inseparably linked, with the result that the very notion of a vacuum was absurd. Motion was considered differently with regard to the celestial and sublunary regions. In the former, which was eternal and changeless, motion was supposed to be perfect, that is, uniform, circular and perpetual. Terrestrial or sublunary motion, in its turn, was divided into natural and violent.“

Gilbert, J. K., & Zylbersztajn∗, A. (1985). A conceptual framework for science education: The case study of force and movement. The European Journal of Science Education, 7(2), 107-120., pp.110-111)

The Aristotelian two-sphere universe: the sublunary region“



„Natural motion was directed to the 'natural places' of objects. In the case of rocks and earthly materials, that destiny was considered to be the centre of the universe. According to the Aristotelian view, a stone falls naturally towards the earth, not because it is attracted by it, but because the earth occupies the centre of the universe. The earth occupied this position because it was, itself, composed of rocky and earthly materials. […] All motions which were not natural were considered violent in the Aristotelian framework. In this case, a **force** was needed to keep a body moving against its 'natural' inclination, and the greater the force the greater the velocity.“

Gilbert, J. K., & Zylbersztajn∗, A. (1985). A conceptual framework for science education: The case study of force and movement. The European Journal of Science Education, 7(2), 107-120., pp.110-111)

Radical duality

The celestial world was the world of precision, made of incorruptible matter (ether), where the motion was *ontologically governed by rigid laws of geometry (the realm of upper-class culture)*

The terrestrial world was the world of imprecision, made of corruptible matter, where mathematics was *a "practical tool" forced into it by art (the realm of craftsmanship, middle-law class, …)*

Liberal arts in medieval schools

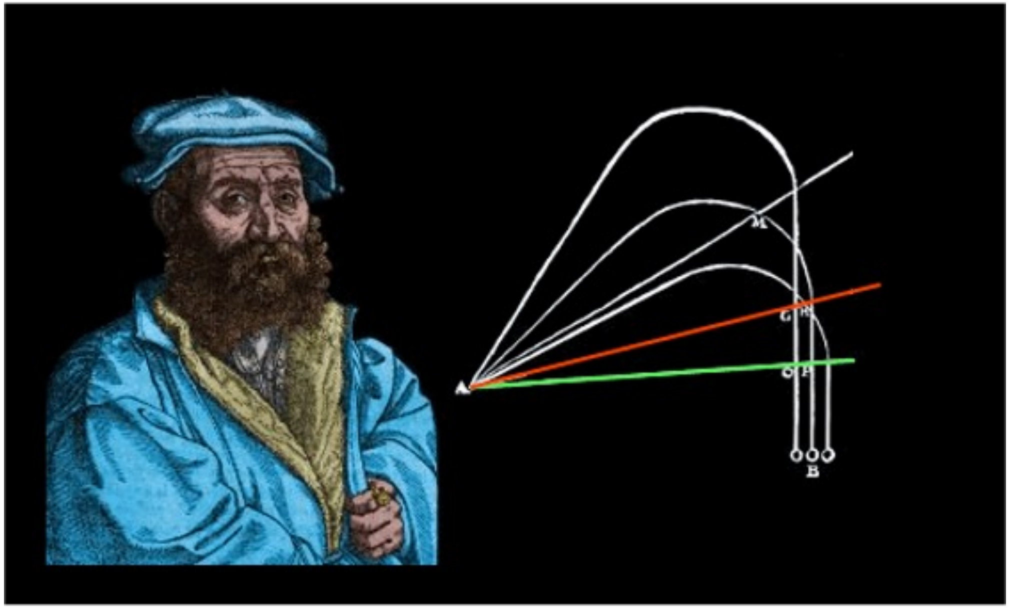
Quadrivium

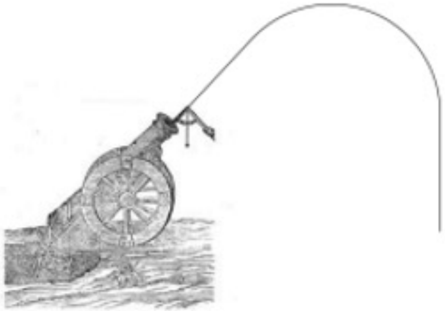
Trivium

In the Aristotelian perspective as well as in the Platonic one (albeit with due distinctions) *mathematics is used for the understanding of the celestial world and is not suitable for explaining the "essence" of natural sublunary phenomena*. The sublunary world in its imperfections does not allow a foundational mathematical treatment. It is therefore impossible to mathematically describe the terrestrial world.

Mathematical astronomy is possible,
mathematical physics is not possible

Tartaglia's representation of the projectile motion in his *Nova Scientia* (1537).



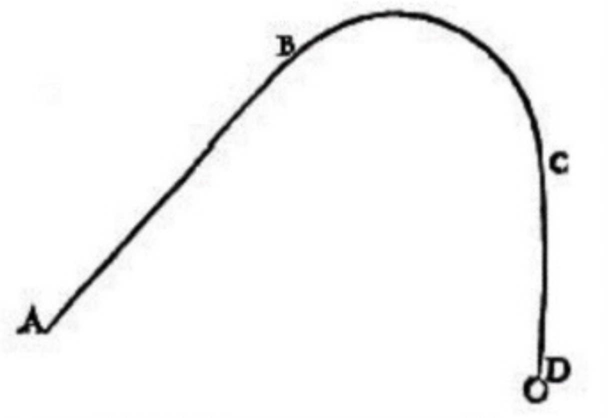


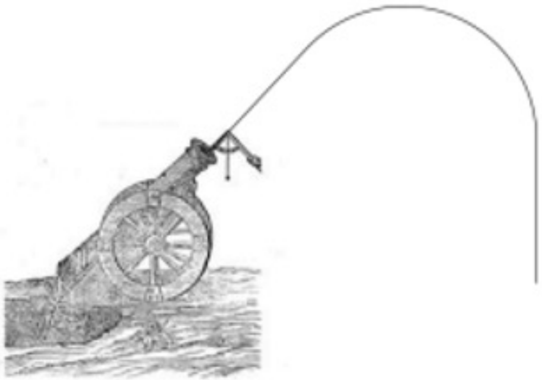
Tartaglia's representation of the projectile motion in his Nova Scientia (1537).

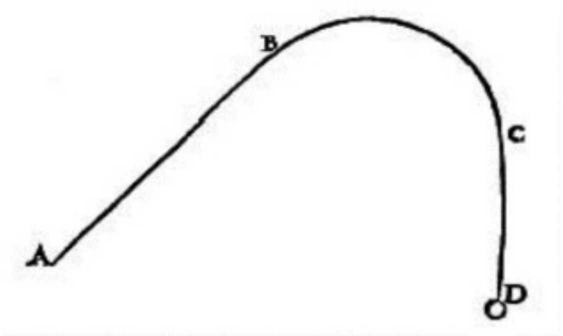
Tartaglia's work provided an important bridge between Aristotle and Galileo.
In the 1537 book "*New Science*", he founded modern ballistics – the science of projectiles.

Tartaglia had started thinking about ballistics five years earlier, when a soldier asked him a question: what angle should a cannon make with the ground to achieve the maximum shooting range? Tartaglia used geometry to answer the question: 45 degrees. He then began a more careful study of ballistics, and in doing so powerfully affected the future development of physics.

Framing „Tartaglia's representation“ in an historical perspective.
What can we see, now, in this representation? What can we take from here to answer our question „what curve is represented here“?







„According to Tartaglia's theory, the trajectory of a projectile consists of three parts. It begins with a straight part that is followed by a section of a circle and then ending in a straight vertical line (see figure [on the right]). This form of the trajectory also corresponds to <u>Tartaglia's adaptation of the Aristotelian dynamics to projectile motion in the case of artillery</u> […]. […] The first part of the trajectory was conceived by Tartaglia as reflecting the initially dominant role of the violent motion, whereas the last straight part is in accord with the eventual dominance of the projectile's weight over the violent motion and the tendency to reach the center of the earth. […] He claimed instead the curved part to be exclusively due to violent motion as is the first straight part of the trajectory.“ (*)

(*) Renn, J., Damerow, P., Rieger, S., & Giulini, D. (2001). Hunting the white elephant: When and how did Galileo discover the law of fall?. Science in Context, 14(s1), 29.

The interpretative schemes were represented by the straight line and the circular line.

"Local motion, which is what we call 'translation', is always either straight, or circular, or a mixture of these two: because these two alone are simple. And the reason is that there are also only two simple quantities, the straight line and the circular one ".

(Aristotle)

Parabola was not
among the interpretative schemes!!

“Philosophy [nature] is written in that great book which ever is before our eyes -- I mean the universe -- but we cannot understand it if we do not first learn the language and grasp the symbols in which it is written. The book is written in mathematical language, and the symbols are **triangles, circles and other geometrical figures**, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth.”

(Galilei, The Assayer - Il Saggiatore,

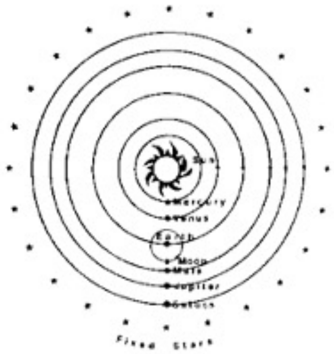
Radical changes

To “break down” the perfection of the celestial **shapes** – the spheres – an accept that other geometrical figures to describe the Cosmos (tringles, ellipses, hyperbola, parabola… conics…) are possible

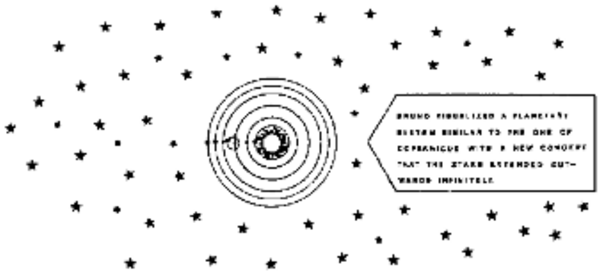
To “break down” the **material distinction** between the “incorruptible material of the celestial spheres” and the “corruptible matter of sublunar world”

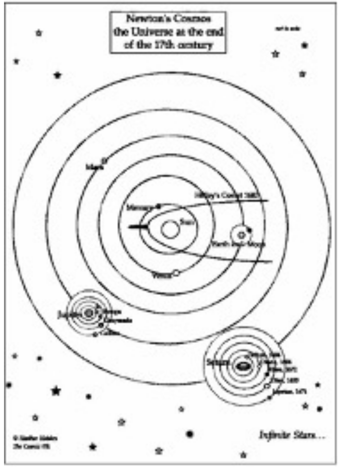
“Breaking down” the perfection of the **celestial shapes** – the spheres – an admit the other geometrical figures (tringles, ellipses, hyperbola, parabola… conics) to describe the Cosmos

Bruno



The Universe of Copernicus.





Copernicus

Kepler

Newton

“Breaking down” the **material distinction** between the “incorruptible material of the celestial spheres” and the “corruptible matter of sublunar world”

Pointing the telescope at the moon:

“It is not smooth, it is not a crystal ball”
(Galileo, by Liliana Cavani, 1968)

Now that the parabola is thinkable among the interpretative schemes and that the two worlds are made of the same substance, how can we prove that falling bodies have a parabolic trajectory?

The need of two further epistemological steps

The rigorous experimental method

The mathematical argumentative structure

In modern science, there is a particular intertwining between theory and practice, and for this to be possible simple respect for observed facts is not enough, a passive questioning of nature is not possible.

It is necessary to "write a script", prepare the phenomenon, ***purify it***, ***isolate it***, make it resemble an ideal situation that is not the phenomenon itself but what makes it intelligible.

Prigogine, I., & Stengers, I. (1977). La Nouvelle Alliance= La Nuova Alleanza= The New Alliance. Scientia Milano, 71(112), 287-332.

The dialogue takes place with:

- questions based on **theoretical assumptions**
- answers that must be decoded in the light of the answers expected from the theory.

The need of two further epistemological steps

The rigorous experimental method

The mathematical argumentative structure

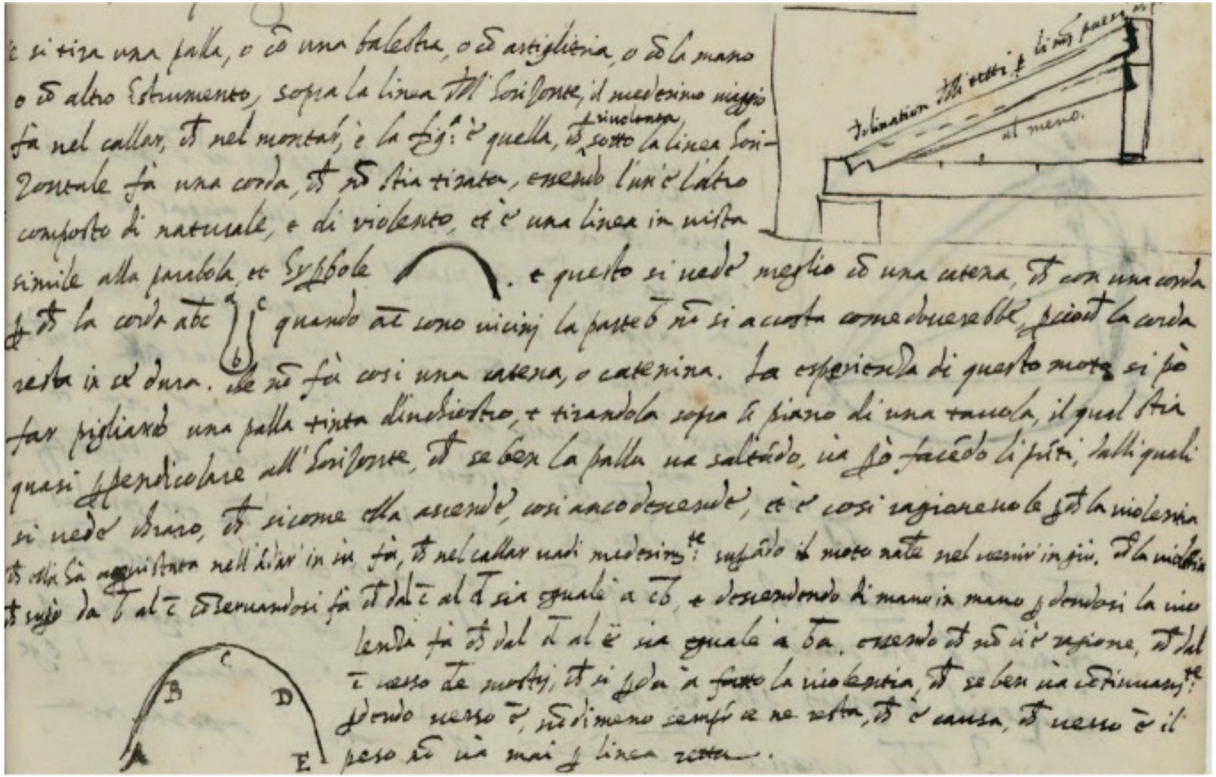
With the voices of Guidobaldo, Galileo (and Newton)

Guidobaldo del Monte

Pesaro, 1545 – Montebaroccio, 1607

He was one of the most prominent Italian mathematicians from the second half of the sixteenth century. He published influential texts on Archimedean mechanics and perspective that contributed substantially to a better understanding of the mathematical foundations of these sciences. He was also one of the most important patrons of the young Galileo. He had important dialogues and discussions with him and played a fundamental role in the discovery of the parabolic motion.

From Guidobaldo notes, 1592


Se si tira una palla, o con una balestra, o con artiglieria, o con la mano o con altro strumento, sopra la linea dell'orizzonte, il medesimo viaggio fa nel calar, che nel montar, e la figura è quella, che sotto la linea orizzontale fa una corda, che non stia tirata, essendo l'uno e l'altro composto di naturale, e di violento, et è una linea in vista simile alla parabola et hyperbole. E questo si vede meglio con una catena, che con una corda, perché con la corda abc quando ac sono vicini la parte b non si accosta come dovrebbe, perché la corda resta in sé dura, il che non fa così una catena, o catenina. La esperienza di questo moto si può far pigliando una palla tinta d'inchiostro, e tirandola sopra il piano di una tavola, il qual stia quasi perpendicolare all'orizzonte, che se ben la palla va saltando, va però facendo li punti, dalli quali si vede chiaro, che sì come ella ascende, così ancora descende, et è così ragionevole perché la violenza che ella ha acquistata nell'andar in su fa, che nel calar vadi medesimamente superando il moto naturale nel venir in giù, che la violenza che supera da b al c conservandosi fa che dal c al d sia eguale a cb, e descendendo di mano in mano perdendosi la violenza fa che dal d al e sia eguale a ba, essendo che non vi è ragione, che dal c verso de mostri, che si perda a fatto la violenza, che se ben va continuamente perdendo verso e, nondimeno sempre ne resta, che è causa, che verso e il peso non va mai per linea retta.

From Guidobaldo notes, 1592

If one throws a ball with a catapult or with artillery or by hand or by some other instrument above the horizontal line, it will take the same path in falling as in rising, and the shape is that which, when inverted under the horizon, a rope makes which is not pulled, both being composed of the natural and the forced, and it is a line which in appearance is similar to a parabola and hyperbola. And this can be seen better with a chain than with a rope, since [in the case of] the rope *abc*, when *ac* are close to each other, the part *b* does not approach as it should because the rope remains hard in itself, while a chain or a little chain does not behave in this way. The experiment of this movement can be made by taking a ball colored with ink, and throwing it over a plane of a table which is almost perpendicular to the horizontal.

Although the ball bounces along, yet it makes points as it goes, from which one can clearly see that as it rises so it descends, and it is reasonable this way, since the violence it has acquired in its ascent makes so that in falling it overcomes, in the same way, the natural movement in coming down so that the violence that overcame [the path] from *b* to *c*, conserving itself, operates so that from *c* to *d* [the path] is equal to *cb*, and the violence which is gradually lessening when descending operates so that from *d* to *e* [the path] is equal to *ba*, since there is no reason from *c* towards de that shows that the violence is lost at all, which, although it lessens continually towards *e*, yet there remains a sufficient amount of it, which is the cause that the weight never travels in a straight line towards *e*. (Guidobaldo, 1592, in Damerow et al., 1992, p.151-152)

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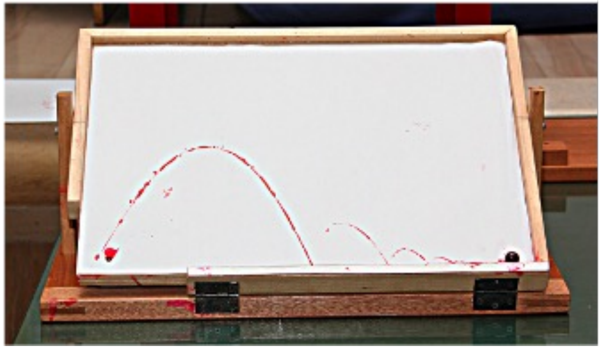

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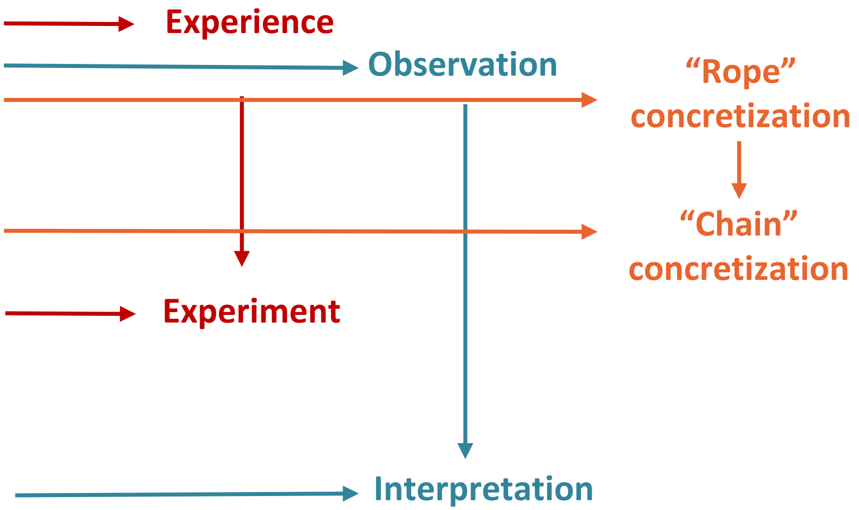


The description

If one throws a ball with a catapult or with artillery or by hand or by some other instrument above the horizontal line

It refers to a concrete experience, which is not yet an experiment.

it will take the same path in falling as in rising

observation "by eye" apparently "neutral", but which highlights the symmetry between the ascent and descent of the ball

And the shape is that which, when inverted under the horizon, a rope makes which is not pulled

Guidobaldo gives substance to the observation

From the description to interpretation

We move on to the physical interpretation of the description of the previous observation (it will take the same path in falling as in rising)

both being composed of the natural and the forced

The two natural and violent motions are not separate but coexist and do not hinder each other (indifference of one movement to another). The curvilinear and symmetrical trajectory (the one reproducible by the rope) must lead to a revision of the concept of movement.

This is an important epistemological passage, a paradigm shift compared to the medieval one ...

is a line which in appearance is similar to a parabola and hyperbola

Guidobaldo dares to provide a mathematical description/interpretation of the trajectory, even these two curves could be compatible (in a similar view) but then the idea of the rope, replaced by a chain, returns.

From the description to interpretation

And this can be seen better with a chain than with a rope, since [in the case of] the rope *abc*, when *ac* are close to each other, the part *b* does not approach as it should because the rope remains hard in itself, while a chain or a little chain does not behave in this way.

He chose the chain because it does not present the same technical difficulties as the rope.

But there is more! Guidobaldo and Galileo had the idea that the chain, now excluding the rope, provided the best "analogy between the simultaneous actions of elongation and weight of the chain, on the one hand, and violent and natural motions on the other“ (Cerreta, 2019)

The question: what is the shape of the trajectory can find an answer. The trajectory is symmetrical and curvilinear describable, we would say today in modern terms, with a catenary arc.

From the description to interpretation

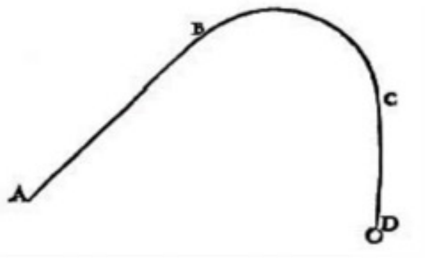
from which [the points that the ink leaves] one can clearly see that as it rises so it descends

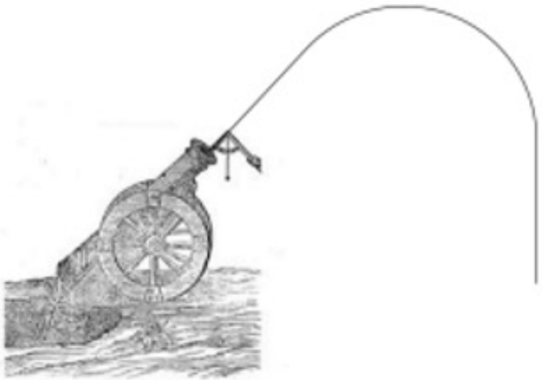
And the experiment confirms the initial observations.

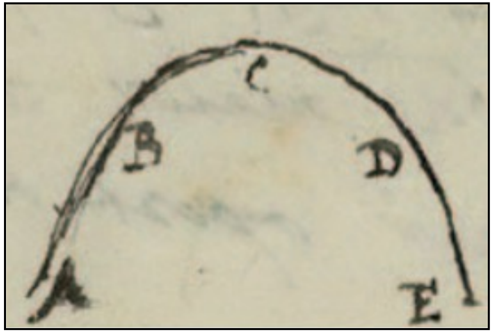
since the violence it has acquired in its ascent operates so that in falling it overcomes, in the same way, the natural movement in coming down so that the violence that overcame [the path] from *b* to *c*, conserving itself, operates so that from *c* to *d* [the path] is equal to *cb*, and the violence which is gradually lessening when descending operates so that from *d* to *e* [the path] is equal to *ba*, since there is no reason from *c* towards de that shows that the violence is lost at all, which, although it lessens continually towards *e*, yet there remains a sufficient amount of it, which is the cause that the weight never travels in a straight line towards *e*.

Guidobaldo hypothesizes the interpretation

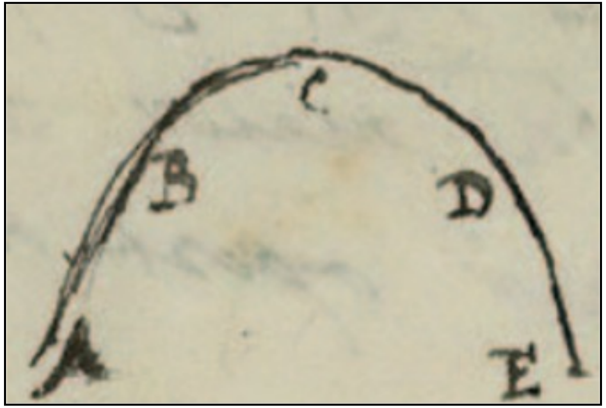
Where are we now? What did we learn ON physics from this episode?





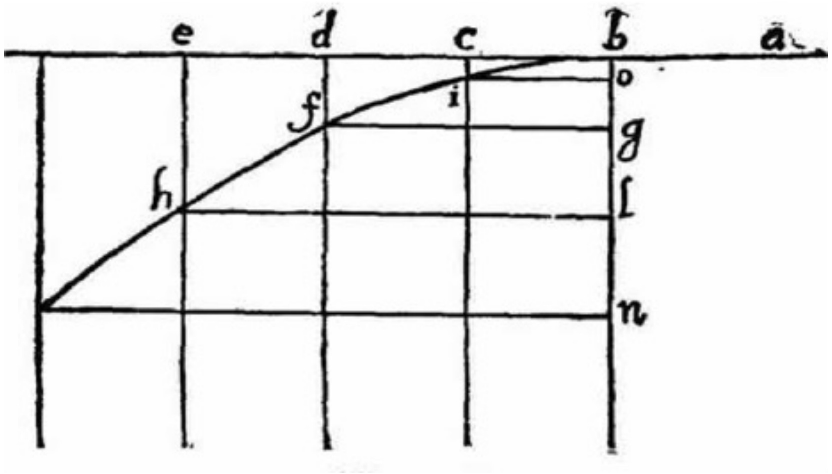


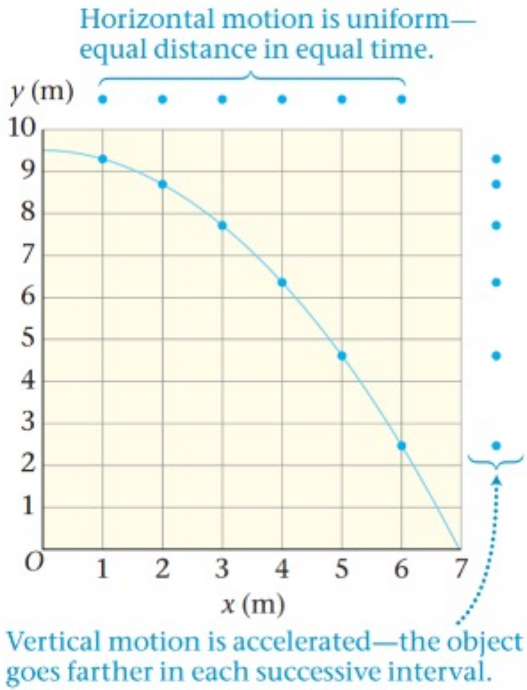
What curve is represented in the pictures? What "theoretical assumptions" stay at the basis of Guidobaldo's interpretation of the curve as a catenary?



What about these pictures?

Taken from the “Fourth day” of the *Discourses and Mathematical Demonstrations Relating to Two New Sciences* (1638), Galileo





Taken from the Walker’ textbook

What knowledge is embedded in the pictures?

Third day of the *Discourses and Mathematical Demonstrations Relating to Two New Sciences* (1638), definition of *uniform motion: "By steady or uniform motion, I mean one in which the distances traversed by the moving particle during any equal intervals of time, are themselves equal."*

This is the only definition that Galileo needs in order to deal with the topic of uniform motions. From the definition, four axioms are deduced:

> ***"Axiom I***
> *In the case of one and the same uniform motion, the distance traversed during a longer interval of time is greater than the distance traversed during a shorter interval of time.*
> ***Axiom II***
> *In the case of one and the same uniform motion, the time required to traverse a greater distance is longer than the time required for a less distance.*
> ***Axiom III***
> *In one and the same interval of time, the distance traversed at a greater speed is larger than the distance traversed at a less speed.*
> ***Axiom IV***
> *The speed required to traverse a longer distance is greater than that required to traverse a shorter distance during the same time-interval."*

What knowledge is embedded in the pictures?

"*Theorem 1 – Proposition 1*:

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola."

Laura!!

Galileo from *Dialogues Concerning Two New sciences*

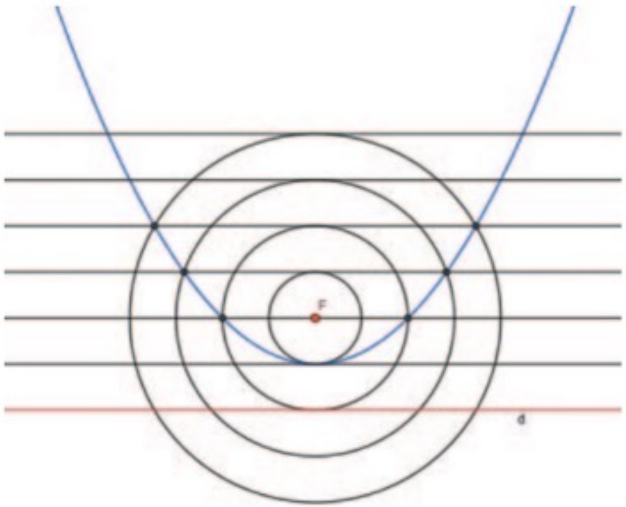
The remarkable way to draw the parabola

Sagredo asks as a preliminary "to have some easy and quick rules to be able above the plane [...] to mark this parabolic line“

In the text the protagonists are two “objects”: one mathematical (the parabola) and one physical the trajectory.

There are many ways of tracing these curves; I will mention merely the two which are the quickest of all. One of these is really remarkable; because by it I can trace thirty or forty parabolic curves with no less neatness and precision, and in a shorter time than another man can, by the aid of a compass, neatly draw four or six circles of different sizes upon paper. I take a perfectly round brass ball about the size of a walnut and project it along the surface of a metallic mirror held in a nearly upright position, so that the ball in its motion will press slightly upon the mirror and trace out a fine sharp parabolic line; this parabola will grow longer and narrower as the angle of elevation increases. The above experiment furnishes clear and tangible evidence that the path of a projectile is a parabola; a fact first observed by our friend and demonstrated by him in his book on motion which we shall take up at our next meeting. In the execution of this method, it is advisable to slightly heat and moisten the ball by rolling in the hand in order that its trace upon the mirror may be more distinct.

The parabola constructed by points

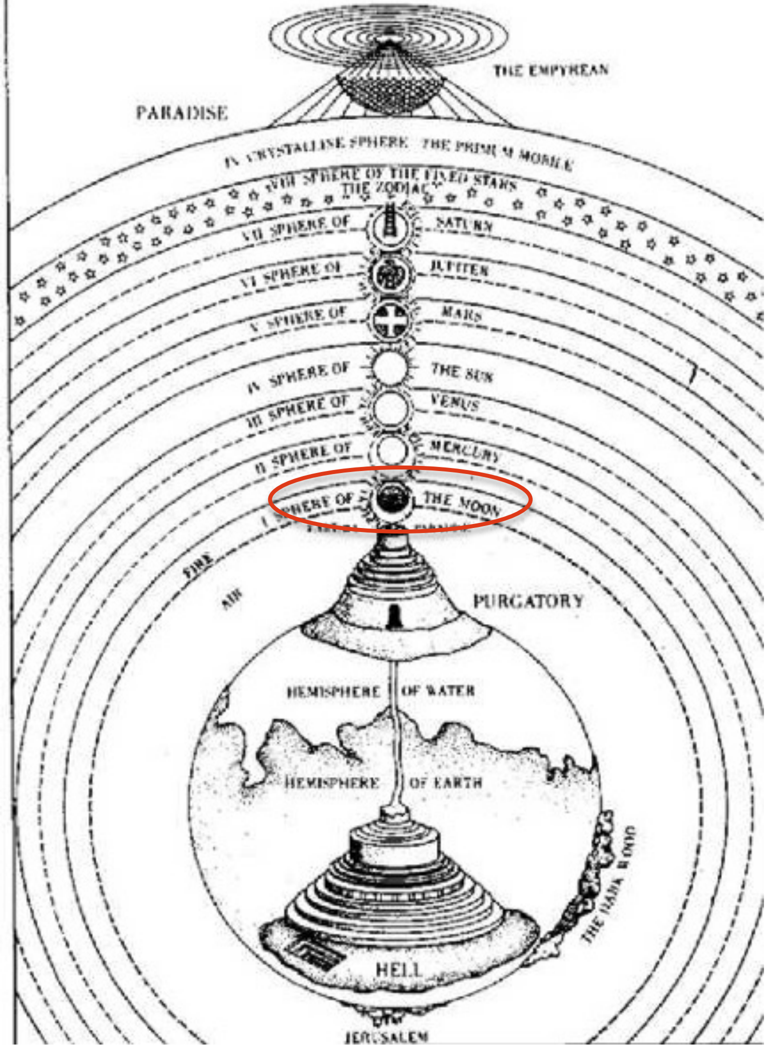


The points of the parabola are drawn as intersections between circumferences centered in the focus and radius *k* and lines parallel to the directrix and distant *k* from it. Given a length *k*, we trace the circumference with center *F* and radius *k* and the line parallel to the directrix and distant *k* from it. The two points of intersection between the circumference and the straight line have distance *k* both from the focus *F* and from the directrix line (*d*) and belong to the parabola. The two points are symmetrical with respect to the straight line passing through the focus and perpendicular to the directrix, that is to the axis of the parabola. Two points on the parabola can be plotted for any value of *k* that is grater than half the distance *s* between *F* and d. For *k=s/2* the only point of intersection is the vertex of the parabola.

There are many ways of tracing these curves; I will mention merely the two which are the quickest of all. One of these is really remarkable...

The opening sentence immediately clarifies the goal: to draw a parabola with some easy and quick rules, with a truly remarkable method.

But why is the method wonderful? For the simplicity of the drawing procedure? For the quick way in which the parabola is drawn? For the accuracy with which the drawing is obtained? Or other?



The medieval view of the universe in a (incredible semplified) nutshell

The medieval view of the universe and its legacy from ancient greek philosophy (Plato and Aristotle):

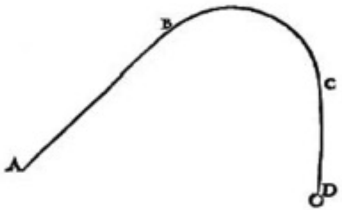
- The distinction between the celestian and the sub-lunar world;
- The different role of mathematics in the two worlds;
- Spheres and straight lines as the only «shapes» allowed to describe the world;
- The classification of the «basic motions» of sublunar world was based on the distinction between natural and violent.

The modern view of the universe in a (incredible simplified) nutshell

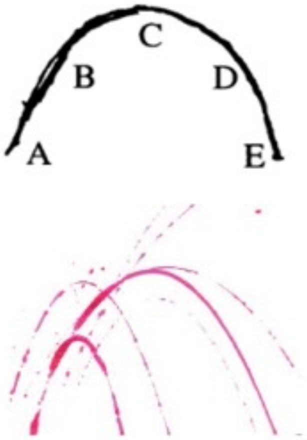
The modern view of the universe:

- The unification of the celestian and the sub-lunar world;
- The same structural role of mathematics in the two worlds;
- Spheres and straight lines as no longer the only «shapes» allowed to describe the world;
- The classification of the «fundamental motions» based on the distinction between uniform rectilinear motion and uniformly accelerated motion.

Let's go back to our images



Tartaglia



Guidobaldo

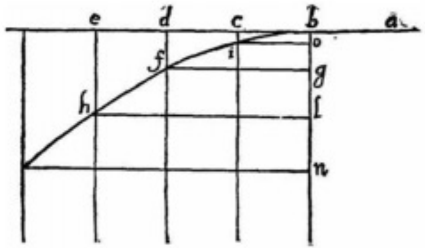


Fig. 108

Galileo

By observing the images and reading the passages, answer the following questions:

- What kind of knowledge do the three images embody?
- What epistemological elements distinguish figure *b* and *c* from figure *a*? To what extent do they characterize physics as a discipline?
- How would you describe the vertical orange bars? In other words, what concepts/elements/aspects activated the epistemological changes that moved knowledge from medieval to modern science?
- What role of mathematics in physics emerges from this case study?

IDENTITIES

Enlightening Interdisciplinarity in STEM for Teaching

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