IDENTITIES

Enlightening
Interdisciplinarity
in STEM
for Teaching

Proof in mathematics

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The MU puzzle

LETTERS: M, I, U
OUR SOUL POSSESSION: MI
RULE I: If you possess a string who last letter s I, you can add on a U at the end.
RULE II: Suppose you have Mx. then you may add Mxx to your collection.
RULE III: If III occurs in one of the strings in your collection, you may make a new string with U in place of III.
RULE IV: If UU occurs inside one of your strings, you can drop it.

Can you derive MU?

A theorem consists of (Mariotti et al., 1997)

- a statement
- the proof
- the theoretical framework of reference

+

metatheory (i.e., the set of formal rules that allow to derive theorems from the starting group of axioms and definitions)

Functions of proof and proving:

Function of proof in mathematics (Hanna, 2000)

- *verification* (concerned with the truth of a statement)
- *explanation* (providing insight into why it is true)
- *systematisation* (the organisation of various results into a deductive system of axioms, major concepts and theorems)
- *discovery* (the discovery or invention of new results)
- *communication* (the transmission of mathematical knowledge)
- *construction* of an empirical theory
- *exploration* of the meaning of a definition or the consequences of an assumption
- *incorporation* of a well-known fact into a new framework and thus viewing it from a fresh perspective

Approach to design and research in the IDENTITIES project

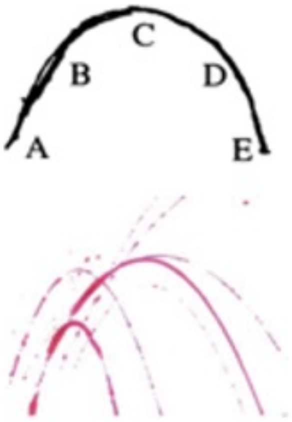
Cultural Analysis of the Content to be taught (CAC) (Boero & Guala; 2017), that encompasses **the understanding of how mathematics can be arranged in different ways according to different needs and historical or social circumstances, and how it enters human culture in interaction with other cultural domains.**

In the same perspective, we argue that teacher education should promote **CAC in mathematics and physics to promote interdisciplinarity.**

Argumentation & Proof are key topics

Parabolic motion in the history of physics (Renn et al., 2000)

Tartaglia



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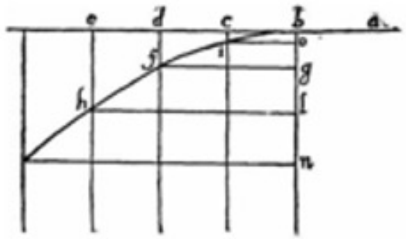
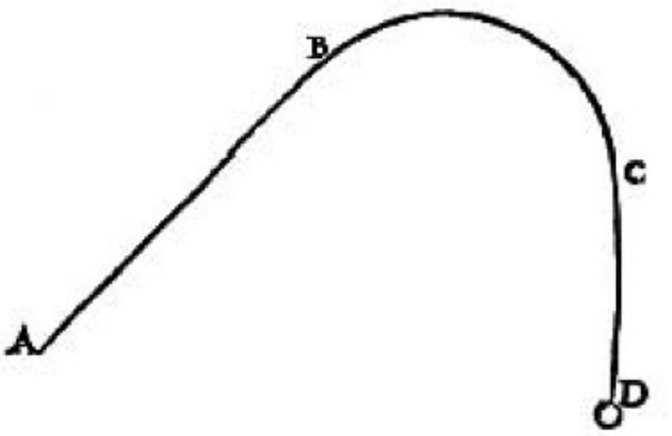


Fig. 108

Galileo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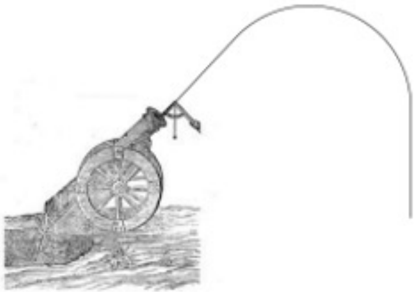

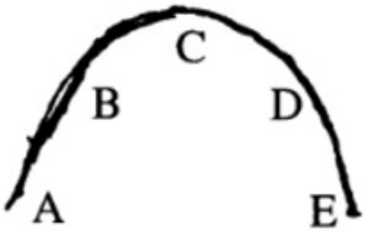
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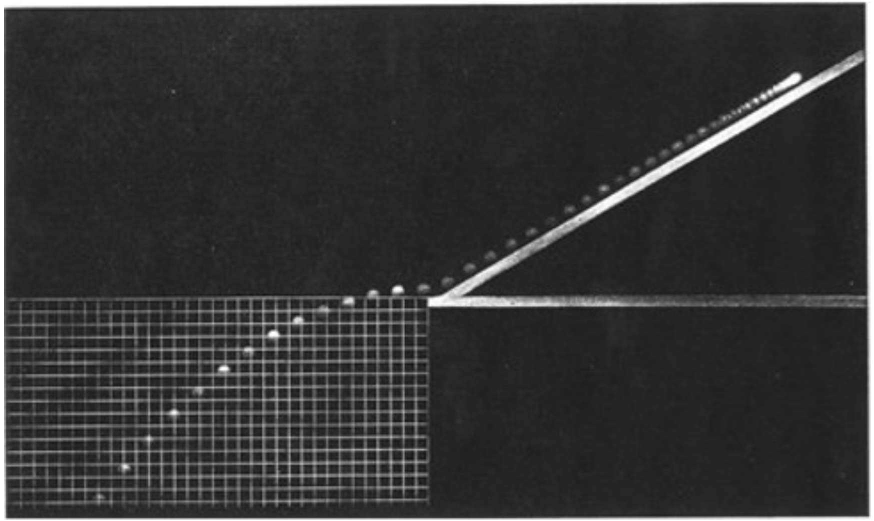
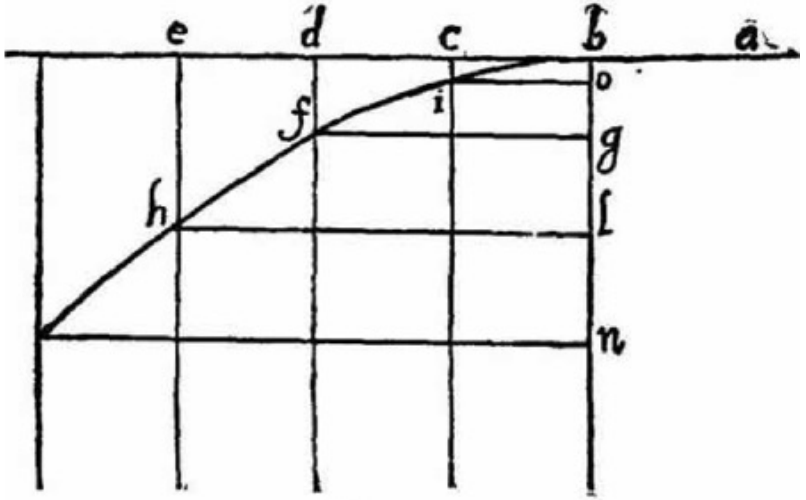
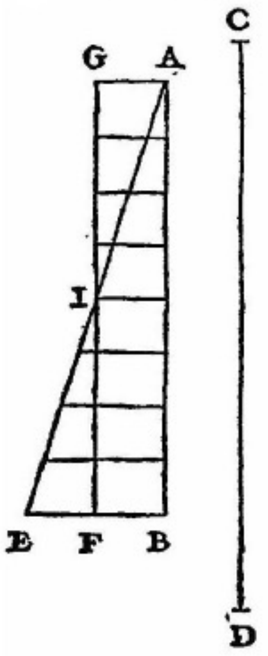


Galileo's proof





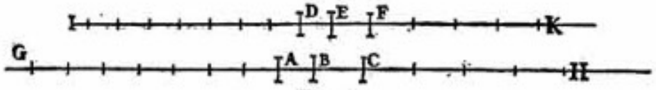


Fig. 40

From the above definition, four axioms follow, namely:

Axiom I ↩

In the case of one and the same uniform motion, the distance traversed during a longer interval of time is greater than the distance traversed during a shorter interval of time.

Axiom II ↩

In the case of one and the same uniform motion, the time required to traverse a greater distance is longer than the time required for a less distance.

Axiom III ↩

In one and the same interval of time, the distance traversed at a greater speed is larger than the distance traversed at a less speed.

Axiom IV[192] ↩

The speed required to traverse a longer distance is greater than that required to traverse a shorter distance during the same time-interval.

Theorem I, Proposition I ↩

If a moving particle, carried uniformly at a constant speed, traverses two distances the time-intervals required are to each other in the ratio of these distances.

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FOURTH DAY ↩

SALVIATI. Once more, Simplicio is here on time; so let us without delay take up the question of motion. The text of our Author is as follows:

THE MOTION OF PROJECTILES ↩

In the preceding pages we have discussed the properties of uniform motion and of motion naturally accelerated along planes of all inclinations. I now propose to set forth those properties which belong to a body whose motion is compounded of two other motions, namely, one uniform and one naturally accelerated; these properties, well worth knowing, I propose to demonstrate in a rigid manner. This is the kind of motion seen in a moving projectile; its origin I conceive to be as follows:

Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits. But if the plane is limited and elevated, then the moving particle, which we imagine to be a heavy one, will on passing over the edge of the plane acquire, in addition to its previous uniform and perpetual motion, a downward propensity due to its own weight; so that the resulting motion which I call projection [*projectio*], is compounded of one which is uniform and horizontal and of another which is vertical and naturally accelerated. We now proceed to [245] demonstrate some of its properties, the first of which is as follows:

THEOREM I, PROPOSITION I[269] ↩

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola.

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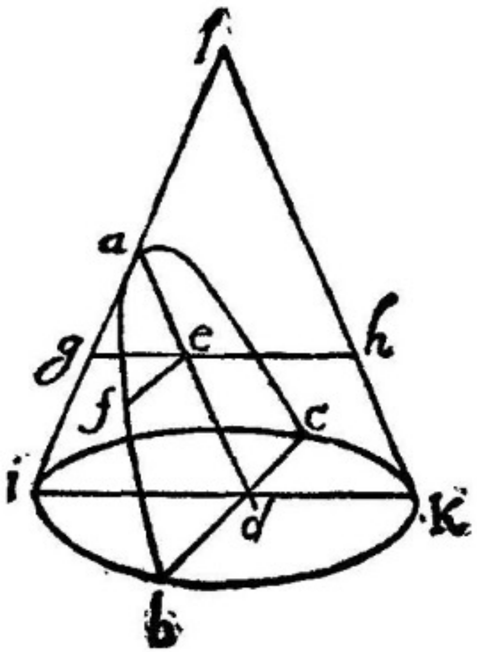

Fig. 106

SAGR. Here, Salviati, it will be necessary to stop a little while for my sake and, I believe, also for the benefit of Simplicio; for it so happens that I have not gone very far in my study of Apollonius and am merely aware of the fact that he treats of the parabola and other conic sections, without an understanding of which I hardly think one will be able to follow the proof of other propositions depending upon them. Since even in this first beautiful theorem the author finds it necessary to prove that the path of a projectile is a parabola, and since, as I imagine, we shall have to deal with only this kind of curves, it will be absolutely necessary to have a thorough acquaintance, if not with all the properties which Apollonius has demonstrated for these figures, at least with those which are needed for the present treatment.

SALV. Indeed, all real mathematicians assume on the part of the reader perfect familiarity with at least the elements of Euclid; and here it is necessary in your case only to recall a proposition of the Second Book in which he proves that when a line is cut into equal and also into two unequal parts, the rectangle formed on the unequal parts is less than that formed on the equal (i. e., less than the square on half the line), by an amount which is the square of the difference between the equal and unequal segments. From this it is clear that the square of the whole line which is equal to four times the square of the half is greater than four times the rectangle of the unequal parts. In order to understand the following portions of this treatise it will be necessary to keep in mind the two elemental theorems from conic sections which we have just demonstrated; and these two theorems are indeed the only ones which the Author uses. We can now resume the text and see how he demonstrates his first proposition in which he shows that a body falling with a motion compounded of a uniform horizontal and a naturally accelerated [*naturale descendente*] one describes a semi-parabola.

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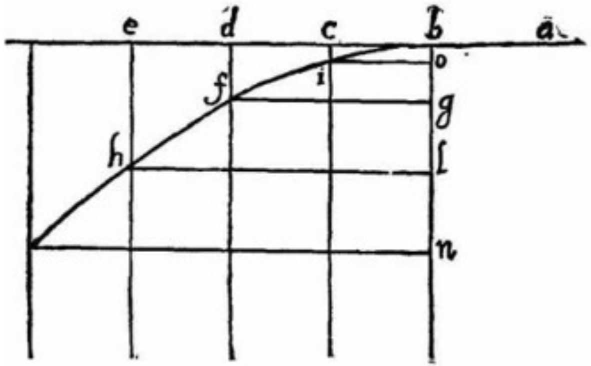


Fig. 108

Let us imagine an elevated horizontal line or plane *ab* along which a body moves with uniform speed from *a* to *b*. Suppose [249] this plane to end abruptly at *b;* then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular *bn*. Draw the line *be* along the plane *ba* to represent the flow, or measure, of time; divide this line into a number of segments, *bc, cd, de,* representing equal intervals of time; from the points *b, c, d, e,* let fall lines which are parallel to the perpendicular *bn*. On the first of these lay off any distance *ci*, on the second a distance four times as long, *df;* on [273] the third, one nine times as long, *eh;* and so on, in proportion to the squares of *cb, db, eb,* or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from *b* to *c* with uniform speed, it also falls perpendicularly through the distance *ci*, and at the end of the time-interval *bc* finds itself at the point *i*. In like manner at the end of the time-interval *bd*, which is the double of *bc*, the vertical fall will be four times the first distance *ci;* for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space *eh* traversed during the time *be* will be nine times *ci;* thus it is evident that the distances *eh, df, ci* will be to one another as the squares of the lines *be, bd, bc*. Now from the points *i, f, h* draw the straight lines *io, fg, hl* parallel to *be;* these lines *hl, fg, io* are equal to *eb, db* and *cb* respectively; so also are the lines *bo, bg, bl* respectively equal to *ci, df,* and *eh*. The square of *hl* is to that of *fg* as the line *lb* is to *bg;* and the square of *fg* is to that of *io* as *gb* is to *bo;* therefore the points *i, f, h,* lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, [250] the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola.

Parabolic Path

RWP Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = \frac{x}{v_0}$$

Next, substitute this result into the y equation to eliminate t:

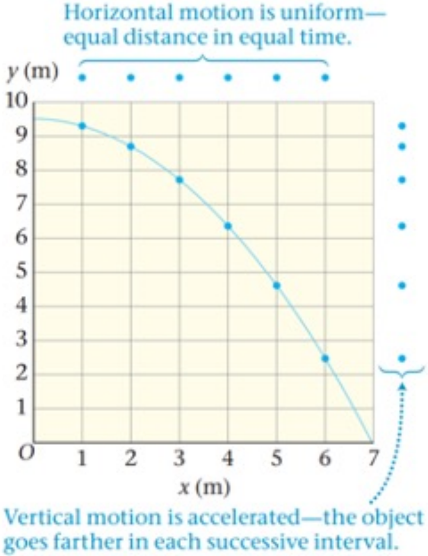
$$y = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2{v_0}^2}\right)x^2 \qquad \text{4-8}$$

It follows that y has the form

$$y = a + bx^2$$

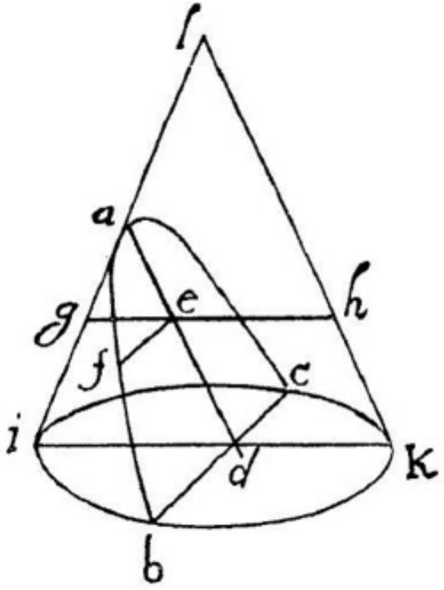
In this expression, $a = h =$ constant and $b = -g/2{v_0}^2 =$ constant. This is the equation of a *parabola* that curves downward, a characteristic shape in projectile motion. Examples of parabolas produced by real-world projectiles are shown in **FIGURE 4-7**.

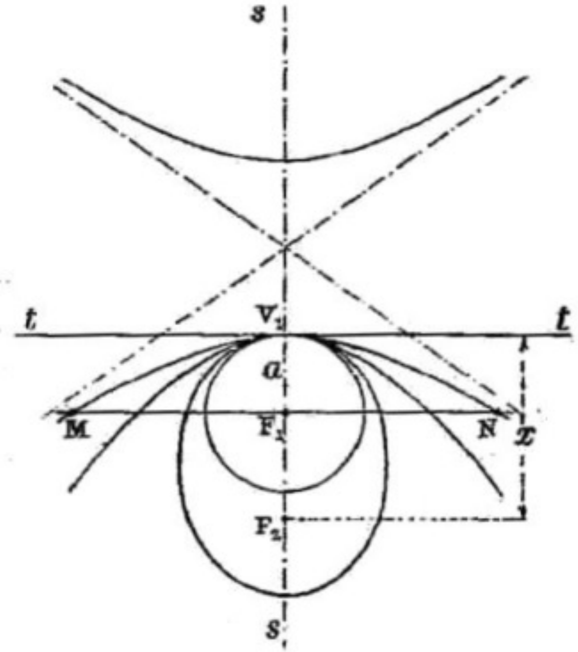
◂ **FIGURE 4-7 Visualizing Concepts**
Parabolic Trajectories Lava bombs (left) and fountain jets (right) trace out parabolic paths, as is typical in projectile motion. The trajectories are only slightly altered by air resistance.



Example from »Walker's textbook«

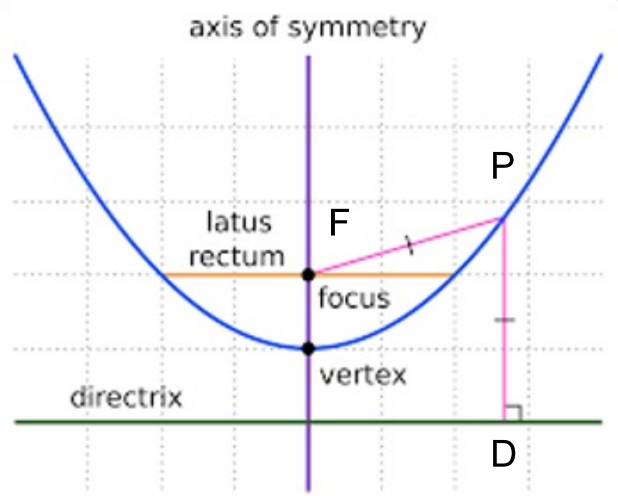
Parabola in the history of mathematics





We define a parabola as the locus of a point that moves such that its distance from a fixed straight line called the *directrix* is equal to its distance from a fixed point called the *focus*.

$$y = ax^2 + bx + c$$



Historical sources and textbooks: a comparison

Galileo: primary sources to consider in order to analyse the topic from the historical-epistemological point of view. Here parabolic motion is the key case.

Conceptions of disciplines and their relationship differs from today:

- foundative book, one of pillars of modern scientific method,
- an example of rich scientific text and scientific argumentation that intertwines explicitly many dimensions of knowledge that nowadays are codified in disciplines (mathematics, physics, engineering, philosophy).

Historical sources and textbooks: a comparison

Physics textbooks for secondary school: disciplinary didactical transposition that is consistent with the (implicit or explicit) didactical goals of the authors.

Parabolic motion is **a particular case of two-dimensional motion** and introduced deserving a lot of space to algebraic passages and formulas, also in the proof, with **a distance between Galileo's presentation increasing in Italian textbooks over time from the 80's to contemporary textbooks** (Bagaglini, Branchetti, Gombi, Levrini, Satanassi and Viale, 2021).

The results are very different narratives about parabolic motion in the History and at school (Satanassi et al., under review).

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Historical sources and textbooks: what can we learn from a comparison in terms of argumentation and proof (CAC in prospective teacher education)?

Parabolic Path

RWP Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

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Next, substitute this result into the y equation to eliminate t:

$$y = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2v_0^2}\right)x^2 \qquad \text{4-8}$$

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Theorem I, Proposition I

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola.

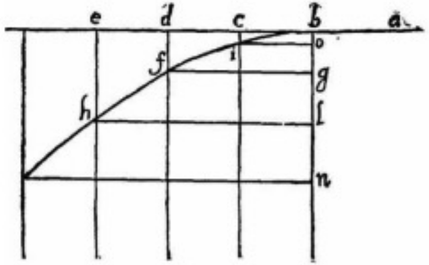


Fig. 108

Historical sources and textbooks: main differences

The way parabolic motion is presented **in physics textbooks for high school differs in terms of rationality from the historical books (epistemic, teleological and communicative dimensions of rationality,** Pollani, 2020).

The main differences can be due to:

- the **targets** (scientific community vs students),
- the **goals** (proposing a new theory vs teaching),
- the development of **disciplines and their epistemologies** (Euclidean geometry and study of motion vs M&P curriculum at school),
- **interdisciplinarity** (scientific discourse intertwining different dimensions vs combination of elements of knowledge taught with a disciplinary perspective).

A&P in historical sources and textbooks: a comparison

How are A&P are presented in two texts about parabolic motion?

- Galileo (1638; English translation)
- the chapter Two-Dimensional Kinematics in the physics textbook by Walker (2017; high school edition, translated also in Italian).

they deal with **proving/demonstrating that the trajectory of a projectile is an arc of parabola,** but….

the **meanings of the term "proving" seemed to change**, as well as **the way proof were presented and intertwined with other aspects** of the scientific argumentation.

What relevance for research about A&P in maths education?

Handbook about Argumentation and Proof (Durand-Guerrier et al., 2012)

To what extent should **mathematical proofs in the empirical sciences, such as physics, figure as a theme in mathematics teaching** so as to provide students with an adequate and authentic picture of the role of mathematics in the world?

Could a stronger emphasis on the process of **establishing hypotheses (in the empirical sciences) help students better understand the structure of a proof** that proceeds from assumptions to consequences and thus the meaning of axiomatics in general?

What relevance for research about A&P in maths education?

The **type of presentation of a proof** is also under investigation in mathematics education.

To what extent and how is the **presentation of a proof (verbal, visual, formal etc.) (in)dependent on the nature of the proof**?

Do students **perceive different types of proofs as more or less explanatory or convincing**?"

In Maths & Physics the verbal, the visual and the formal aspects of proof might play a different role in explaining and convincing students when "mathematizing" observation and reasonings about empirical phenomena or experiments

What relevance for research about A&P in maths education?

A&P in texts and the comparison with historical texts a key step to move from the historical-epistemological and cognitive analyses to the classroom practices, in particular considering teacher-students education.

This issue has been investigated by papers presented in CERME10 (Stylianides et al., 2018); among the themes discussed, we contribute to highlight **the role of language in teaching and writing proofs and to search for analytical frameworks for argumentation and proof in textbook expositions.**

Cognitive unity between argumentation and proof

The didactic value of inserting proof into an argumentative process and to move from a reproductive approach to demonstration to a productive one and to focus on proof as a process more than on proof as a product → construct of **cognitive unity** (Mariotti, Bartolini Bussi, Boero, Ferri & Garuti; 1997)

Need for didactical situations in which the **construction of a proof naturally follows from the exploration of a problematic situation by students.**

“during the construction of the conjecture, of the elements (“arguments”) that are used later during the construction of the proof” (p. 1).

Proving that the trajectory of a projectile motion is parabolic can be considered a **conjecture-proving problem**, according to the characterization of Mariotti et al. (2017).

Cognitive unity: continuity and discontinuity

We assume that **continuity should be pursued** also to guarantee a productive approach of students to proving in this field, in particular **when mathematics appears in the statements and semiotic representations of physical entities**.

What happens to the flow of observation and conjectures about physical phenomena when mathematics enters the discourse? If teachers have to guide a classroom discussion to help the students to include these aspects, **is continuity between A&P pursued or do their interventions cause cognitive rupture**?

Teacher-students need examples and meta reflection to guide the students properly in such classroom discussions (textbooks analysis as prototypes of different ways the teachers scaffold students' approach to interdisciplinary A&P in the classroom)

Cognitive unity: continuity and discontinuity (Pedemonte, 2005)

– ***structural analysis***: refers to the link between the ***structures of statements used in argumentations and in proofs.*** There is structural cognitive unity when statements used in the argumentation are also used in the proof. Otherwise, there is structural cognitive rupture.

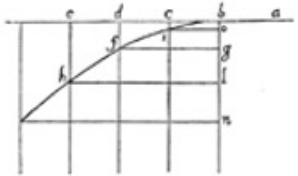
– ***referential analysis***: refers to the ***systems of reference used in argumentations and in proofs, that is, the systems of signs (drawings, calculations, algebraic expressions, etc.) and systems of knowledge (definitions, theorems, etc.) used.*** There is referential cognitive unity when some systems of signs or knowledge are used both in the argumentation and the proof. Otherwise, there is referential cognitive rupture.

Structural and referential analysis of relevant excerpts, modified according to our goal (interdisciplinary analysis of prototypes of A&P connections): **the study of local motions in Galileo (1638) and Walker' (2017).**

Statements in A&P related to parabolic motion and then systems of representation and knowledge belonging to both mathematics and physics (considered as disciplines taught at school in grades 9-10 in Italy in the textbook's analysis and as historical disciplines analyzing Galileo's excerpts).

By comparing the A&P steps, thanks to the structural and referential analysis, we detected **unity or rupture in both texts**.

By steady or uniform motion [1], I mean one in which the distances traversed by the moving particle [2] during any equal intervals of time [3], are themselves equal. [D1].	Definition of uniform motion using proportions (equal space in equal time)
A motion is said to be uniformly accelerated [4], when starting from rest, it acquires, during equal time-intervals [3], equal increments of speed.[...] the distances traversed [2] are proportional [D1] to the squares [5] of the times.	Definition of accelerated motion using proportions (equal increments of speed in equal time, space proportional to the square of time)
Imagine any particle projected along a horizontal plane without friction; if the plane is limited and elevated [6] the resulting motion which I call projection [7], is compounded of one which is uniform and horizontal [1] and of another which is vertical and naturally accelerated [4].	Definition of projectile, that incorporates the assumption of composition of motions
Theorem 1 – Proposition 1: A projectile [7] which is carried by a uniform horizontal motion [1] compounded with a naturally accelerated [4] vertical motion describes a path which is a semi-parabola [8].	Theorem formulated using previous definitions
The section of this cone [..] which is called a parabola [8] [..] the square [5] of *bd* is to the square [5] of *fe* in the same ratio [9] as the axis ad is to the portion *ae*.	Definition of parabola

Let us imagine an elevated [6] horizontal line or plane *ab* along which a body moves with uniform [1] speed from a to b. Suppose this plane to end abruptly at *b* [6] [..]. Draw the line be along the plane ba to represent the flow, or measure, of time; divide this line into a number of segments, *bc, cd, de,* representing equal intervals of time [3] [..] in proportion [D1] to the squares [5] of *cb, db, eb,* or, [..] in the squared ratio [9] of these same lines. [...] at the end of the time interval [3] *bd*, which is the double [D1] of *bc*, the vertical fall will be four times [D1] the first distance *ci*; [..] the distance traversed [2] by a freely falling body varies as the square [5] of the time; in like manner the space *eh* traversed [2] during the time *be* will be nine times [D1] *ci*; thus it is evident that the distances *eh, df, ci* will be to one another as the squares [5] of the lines *be, bd, bc.* The square [5] of *hl* is to that of *fg* as the line *lb* is to *bg* [D1]; and the square [5] of *fg* is to that of *io* as *gb* is to *bo*; therefore the points *i, f, h,* lie on one and the same parabola [8]. 	Proof is presented, where: - the same terms introduced before are used, as well as the same spatial representation (segments/intervals of time) - it is stressed the use of proportional reasoning, that was used to define the kinds of motions that are combined - G. recalls the assumptions about the composition of motions - G. recalls the setting associated to the definition of projectile with the same words - G. intertwines the definition of parabola and the characterization of accelerated motion in order to exploit the linguistic analogies to stress that the points must lie on a parabola.

Analysis of excerpts

Big Idea 1 Two-dimensional motion is a combination of horizontal and vertical motions. The key concept behind two-dimensional motion is that the horizontal and vertical motions are completely independent of one another; each can be considered separately as one-dimensional motion.	The combination and independence of horizontal and vertical motions are initially introduced in a lateral box as Big Idea. The status of the statement in terms of elements of A&P (axiom, theorem) is not expressed.
Projectile Motion: Basic Equations We now apply the independence of horizontal and vertical motions to projectiles. Just what do we mean by a projectile? Well, a projectile is an object that is thrown, kicked, batted, or otherwise launched into motion and then allowed to follow a path determined solely by the influence of gravity.	The Big Idea is applied to projectile motion to obtain its equations and a phenomenological description of the projectile is presented.
Demonstrating Independence of Motion A simple demonstration illustrates the independence of horizontal and vertical motions in projectile motion. [..] Notice that the ball goes straight down, lands near your feet, and returns almost to the level of your hand in about a second. [...] To you, its motion looks the same as before.[...]The fact that you were moving in the horizontal direction the whole time had no effect on the ball's vertical motion — the motions are independent.. 	A figure represents a moving person with a roller skate and a falling ball; the two combined motions are represented with a reference to real life. The motion is seen also by an external observer and the trajectory is linear and vertical in the system of person and curved in the external system, that is represented through cartesian axes put onto the real life figure. The relativity of motion in different systems is used to demonstrate independence of motions.

To an observer who sees you walking by, the ball follows a curved path, as shown. The precise shape of this curved path—a parabola—is verified in the next section.  **(a)** FIGURE 4-4 **Visualizing Concepts - Independence of Motion** (a) An athlete jumps upward from a moving skateboard.	A picture (photo with a camera to a real world phenomenon) is proposed. In the description of the figure, it is mentioned the visualization of concepts and presented as one among other“examples of principle” of independence of motions. It is anticipated that the shape is a parabola and that this will be verified later.

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Analysis of excerpts

▸ **FIGURE 4-6 Trajectory of a projectile launched horizontally** In this plot, the projectile was launched from a height of 9.5 m with an initial speed of 5.0 m/s. The positions shown in the plot correspond to the times $t = 0.20$ s, 0.40 s, 0.60 s, ... Notice the uniform motion in the x direction, and the accelerated motion in the y direction.

and

$$y_0 = h$$

This is illustrated in Figure 4-3.

The initial velocity is horizontal in this case, corresponding to $\theta = 0$ in FIGURE 4-5. As a result, the x component of the initial velocity is simply the initial speed:

$$v_{0x} = v_0 \cos 0^\circ = v_0$$

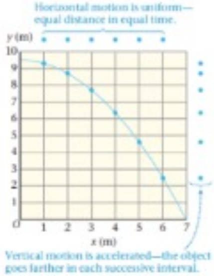
The y component of the initial velocity is zero:

$$v_{0y} = v_0 \sin 0^\circ = 0$$

Substituting these specific values into our fundamental equations for projectile motion (Equations 4-6) gives the following simplified results for zero launch angle ($\theta = 0$):

$$\begin{array}{lll} x = v_0 t & v_x = v_0 = \text{constant} & v_x^2 = v_0^2 = \text{constant} \\ y = h - \frac{1}{2}gt^2 & v_y = -gt & v_y^2 = -2g\Delta y \end{array} \qquad 4\text{-}7$$

Snapshots of this motion at equal time intervals are shown in FIGURE 4-6.



Dropped and Thrown Balls

PROBLEM-SOLVING NOTE

Identify Initial Conditions

The launch point of a projectile determines x_0 and y_0. The initial velocity of a projectile determines v_{0x} and v_{0y}.

A graph, resembling the one by Galileo but the use of x,y and units on the axes, is in a lateral box.
The horizontal uniform motion is presented using proportions (equal space in equal time) exploiting also signs on a figure (parenthesis, -) without expliciting the nature of this description as definition.The same happens to vertical accelerated motion, but proportions are not mentioned; there is a reference to intervals without mentioning time.
Symbolic expressions are used for the generic case and the Galileo case is obtained substituting a value into equations for projectile motion.

Parabolic Path

RWP Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = \frac{x}{v_0}$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2v_0^2}\right)x^2 \qquad 4\text{-}8$$

It follows that y has the form

$$y = a + bx^2$$

An algebraic version of the proof is presented (never named proof), with:
- reference to a curved path:
- the term“found” instead of verify
- use of symbolic expression of the two motions combined, as well as the parabolic generic equation
- no reference to assumptions about the combination of motions
- the use of terms algebraic “substitution” and “eliminate”
- no mention of the previous graph and the exemplification of principles of independence.

Conclusions and further developments

Galileo’s text is characterized by **structural and referential unity:**

he **mathematized the relationship between space and time with magnitudes and proportions** and **used always the same objects and properties** to merge the observation of phenomena, empirical laws and geometrical properties of conic sections.

the **mathematization of the experimental setting allowed him to prove, deductively,** that the trajectory is a semi-parabola, **under the hypothesis that the motion of a projectile results from a composition** of independent uniform and accelerated motion.

the theory of magnitudes bridges the concrete action of measuring and the theoretical comparison between geometrical magnitudes.

Conclusions and further developments

The **graphic representation plays a crucial role**, since the action itself to trace a line/curve with a motion of a point is a sort of ideal machine that draws a trajectory, **hybridizing the notion ofs trajectory and geometrical curve to treat the trajectory geometrically.**

In this case the structural role of mathematics clearly emerges: **“importing” the structure of Euclidean proof in the investigation of motion** allows to refine and strengthen argumentation.

Conclusions and further developments

Walker's chapter:

it is visible the effort **to consider the dimension of A&P: there are physical assumptions, a definition of projectile, examples that ground the assumptions** about the composition of independent motions on empirical facts, stressing that they are realistic.

Some referential choices are consistent: the motion of a projectile is a particular case of a more general motion, equations of evolution are used to derive new equations treating time and space as algebraic variables.

However, **many elements of rupture are present**.

Conclusions and further developments

the presentation of the argument concerning **physical principles and entities and the proof are presented with figures and pictures** related to real life, while in the **derivation of the equation they switch suddenly to algebraic language and analytical reasoning (substituting variables in functions)**.

definitions, principle, inference, proof are never mentioned.

the link between **empirical aspects and mathematical knowledge is hard to establish for a reader, because of the strong discontinuity in terms of use of signs** and semiotic registers for the expression of the statements (algebraic proof, Boero Morselli &Guala 2013).

Algebraic proof: a contribution from Mathematics education

At secondary schools, **in mathematics, an internal division** emerges that separates it into **the domains algebra, geometry, analysis, statistics** and so on (Boero, Guala & Morselli 2013) → difficulty at the didactic level, but also in building in students (and not only) a vision that, in addition to being **crystallized and sectoral, is also unrealistic.**

Morselli and Boero (2009): adaptation to mathematics teaching concerning the **construct of "rational behavior" for discursive practices,** proposed by Habermas, in particular regarding the **use of algebraic language in proofs.**

How might secondary students perceive it?

Algebraic language in proofs: mainly thought of in secondary school as the domain of synthetic geometry, leading to **a radical change in the forms of explanation when passing from geometry to algebra**.

This trend is found in the mathematics textbooks analyzed and represents the reference knowledge of students in mathematics; they will refer to this knowledge by thinking about the parabola and its equation in physics.

The transposition makes the many opportunities for interdisciplinary reflection disappear….

Can Galileo’s proof be considered a “boundary object” between mathematics and physics? Why and why not?

Parabolic Path

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Theorem I, Proposition I

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola.

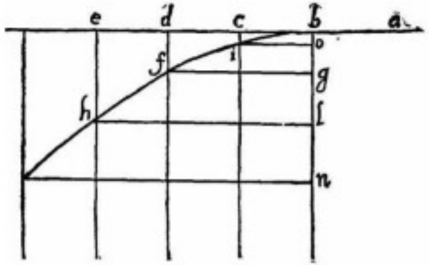
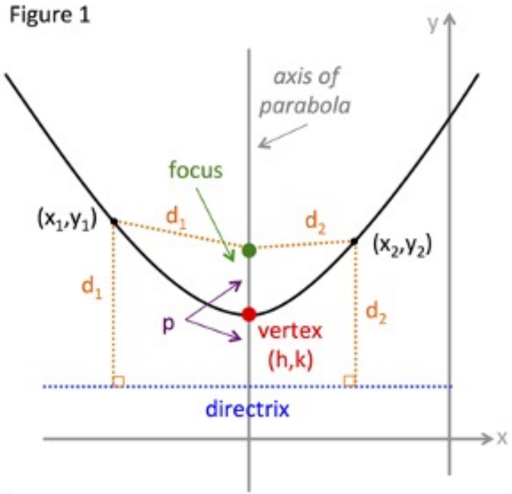
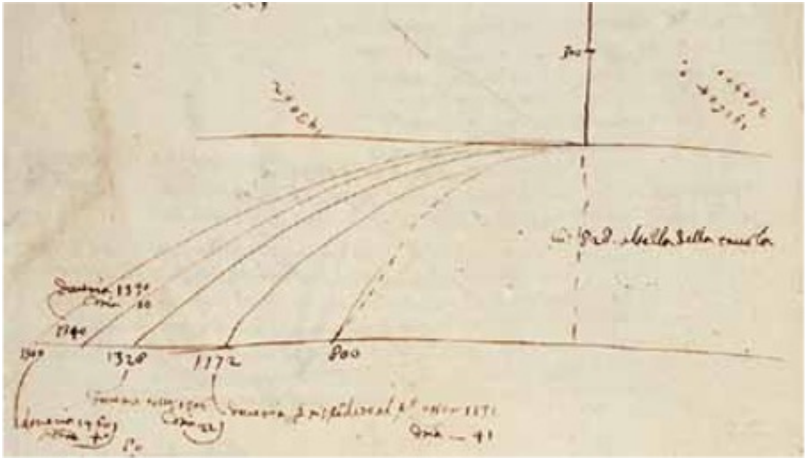


Fig. 108

Back to the explorer activity….





Whether and how can we prove that a “drawn curve” is a parabola?

Whether and how can we “draw” a parabola?

Whether and how can we prove that a motion is parabolic?

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