IDENTITIES

Enlightening Interdisciplinarity in STEM for Teaching

Student activity - O3

Parabola as a conic section: a historical dialogue between mathematics and physics

Co-funded by the Erasmus+ Programme of the European Union

Grant Agreement n°2019-1-IT02-KA203-063184

IDENTITIES
Enlightening Interdisciplinarity in STEM for Teaching

ACTIVITY 1

Conics in the history of mathematics: from Apollonius to Kepler

Primary sources

- Euclid's *Elements of Geometry (c. 300 BC)*
- Apollonius' *Conic sections (c. 200 BC)* (and some excerpts from Archimedes)
- Nicole d'Oresme work about "shapes of motion" (diagrams with latitudine-longitude and intermediate value theorem ante-litteram) (c. 1350)
- Descartes' *Geométrie* (1637)
- Kepler's *Parapolimena ad Vitellionem* (1604) about conics, starting from an analogy between reflection and refraction.

What are conic sections

Conic sections (or conics) are the curves obtained by intersecting a circular cone by a plane: hyperbolas, ellipses (including circles), and parabolas.

Proclus' *Commentary*: **Menaechmus**, pupil of Eudoxus and member of Plato's Academy, discovered these curves around 350 B.C.

Apollonius gave the conical sections their actual names:
- "hyperbola", from Greek "hyper", meaning "some added."
- "ellipse," from Greek for "something missing."
- "parabola," from Greek word "oaros" for "same."

https://math.uh.edu/~shanyuji/History/h-11.pdf

Conic sections as *loci of points*

Ancient mathematicians used the word “locus” for lines and surfaces.

Modern mathematicians regard lines and surfaces as sets of points, but this viewpoint was impossible for ancient scientists.

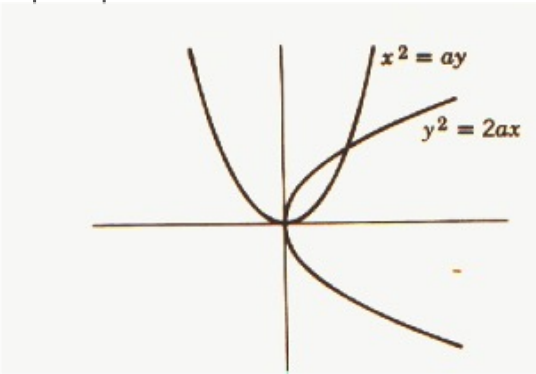
Aristotle wrote in his Physics: “**Nothing that is continuous can be composed of indivisible parts: e.g., a line cannot be composed of points, the line being continuous and the point indivisible** [Ar, p. 231].

Ancient mathematicians regarded **lines and surfaces only as “loci”** (τοποι), that is places for points.

Conic sections as *loci of points*

Menaechmus found that the duplication of a cube can be reduced to the finding two mean proportionals between a and b, that is:

a : x = x : y = y : b,
for b = 2a



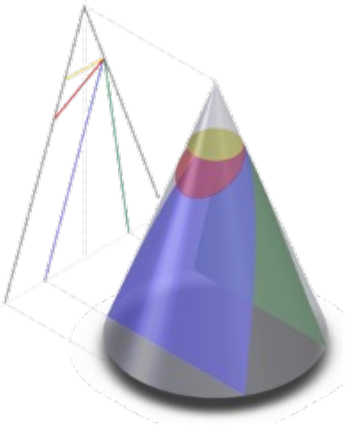
Menaechmus found that the solution x of equation is equal to the abscissa of the point of intersection of two parabolas:

$$x^2 = ay \text{ and } y^2 = 2ax$$

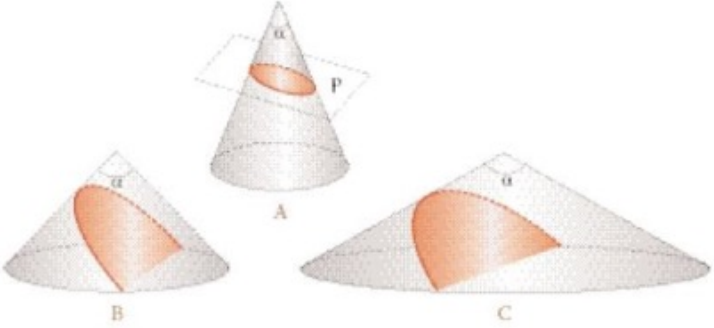
Euclid, *Elements* (IV-III a.C.).

Elements of Conics (Κωνικων στοιξεια)

Cone: figure obtained by rotating a right triangle around a cathetus.
Conic sections (which will later be called ellipse, parabola and hyperbola): sections with a plane perpendicular to the side of the cone, respectively:



acute angle (oxytoma)
rectangle (orthotome)
obtuse (amblystoma)

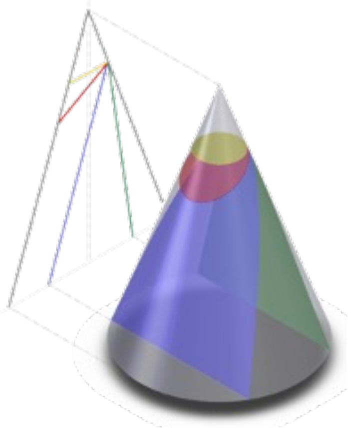


From the Greek terms: acute angle straight and obtuse

https://web.math.unifi.it/archimede/note_storia/Belle-Napolitani-Coniche.pdf

Apollonius, *Conic sections* (III a.C.).

If a line, extending to infinity and always passing through a fixed point, is made to rotate along the circumference of a circle that is not in the same plane as the point so that it passes successively through every point of that circumference, the rotating line will trace the surface of a double cone.



- *Conics as sections of the same cone*
- *Conics as loci, with focal properties (only ellipse and hyperbola..)*
- *Conics characterized by geometric properties relating to the areas (parabolic, elliptical, hyperbolic "application")*

https://web.math.unifi.it/archimede/note_storia/Belle-Napolitani-Coniche.pdf

Apollonius, *Conic sections* (III a.C.).

Apollonius proved that when any cone is sliced by a plane which is parallel to one of its tangent planes (but not containing a generating line), then the resulting section is a ***parabola****, in the sense that we know it as a curve:* ***one of its coordinates is proportional to the square of the other.***

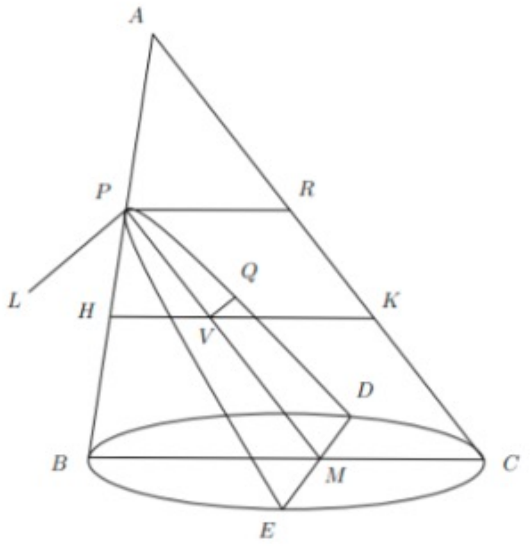
Apollonius (I, 11): known much earlier, generalized (does not depen right cone and on the tangent to be perpendicular to the axis).

Apollonius determines the line PL, which he called ὀρθία πλεύρα ("right side"), PL is the perpendicular to the diameter PM at point P. The length of the line PL is given by the proportion:

$$PL : PA = BC^2 : BA \times AC$$

This segment allow to express this relation:

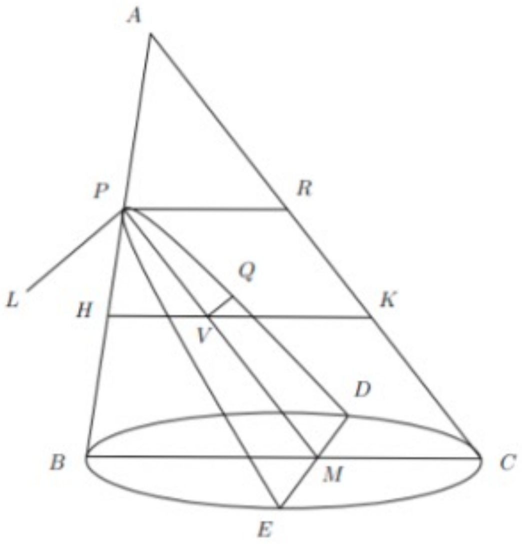
$$QV^2 = PL \times PV$$



Dimostrazione. Sia BC il diametro del cerchio di base del cono; DE l'intersezione fra il piano secante e cerchio di base. Sia HK una parallela a BC passante per il punto V. Poiché l'ordinata QV è anche parallela a DE, il piano passante per i tre punti H, Q, K sarà parallelo alla base del cono e lo taglierà in un cerchio di diametro HK.

Inoltre dato che QV è perpendicolare a HK (per l'osservazione 1), ne segue che

$$HV \times VK = QV^2 \tag{4}$$



Inoltre, per similitudine di triangoli:

$$\begin{aligned} BC : AC &= HV : PV \\ BC : AB &= VK : PA \end{aligned}$$

(per la seconda proporzione, si consideri il parallelogramma $PRKV$ costruito tracciando PR, parallela ad HK passante per P).

Di conseguenza,

$$BC^2 : AC \times AB = HV \times VK : PV \times PA. \tag{5}$$

Per (4), inoltre, avremo

$$HV \times VK : PA \times PV \ = \ QV^2 : PA \times PV$$

E quindi, per (5)

$$BC^2 : AC \times AB \ = \ QV^2 : PA \times PV.$$

Ma, per come abbiamo definito PL, si ha

$$PL : PA \ = \ BC^2 : BA \times AC$$

quindi

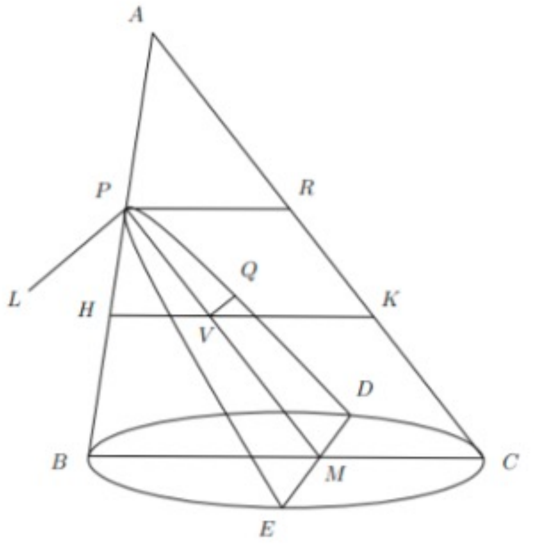
$$QV^2 : PA \times PV \ = \ PL : PA.$$

D'altra parte è ovvio che

$$PL : PA \ = \ PL \times PV : PA \times PV$$

e quindi si ottiene la tesi:

$$QV^2 \ = \ PL \times PV.$$



Apollonius, *Conic sections* (III a.C.).

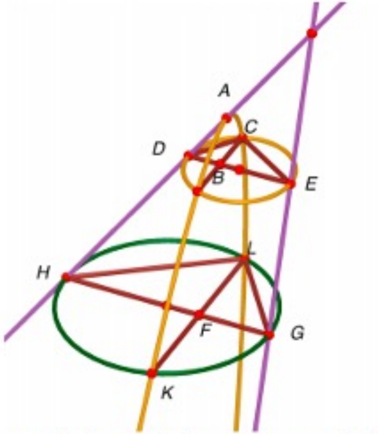


Figure 12. Lines, circles, and the parabola on the cone in Figure 11, for the proof.

In Figure 12 let the curve KAL be formed from slicing the cone with a plane parallel to the line $\overleftrightarrow{EG}$. Let GKH be a circular section and let D, C, and E lie on any parallel circular section (with C on the curve). Now LF is the geometric mean of HF and FG, i.e. $LF^2 = FH \times FG$. Likewise BC is the geometric mean of DB and BE, i.e. $BC^2 = BD \times BE$ (this is a fundamental property of the diameter of a circle and any chord perpendicular to it). Since the parabolic section is parallel to $\overleftrightarrow{EG}$, $BE = FG$. Since triangles ABD and AFH are similar to each other, we have the proportion $BD{:}BA = FH{:}FA$. If we think of sliding the parallel circular sections, and watching the two segments on the diameter, one of them is fixed ($BG = FE$), and the other (BD or FH) is proportional to the distance along the axis of the curve (BA or FA). Hence the distance along the axis of the parabola (BA or FA) is proportional to the square on the lateral distance out to the curve (BC or FL) by the geometric mean property of circles.

https://www.quadrivium.info/MathInt/Notes/Apollonius.pdf

Apollonius, *Conic sections* (III a.C.).

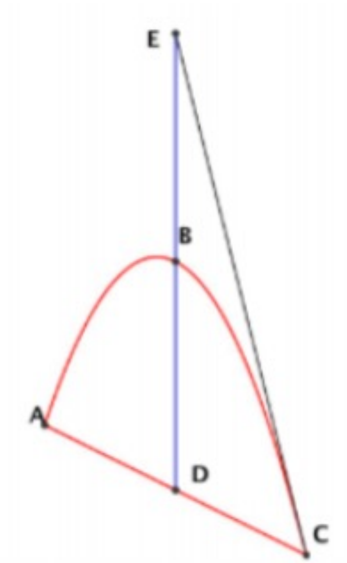
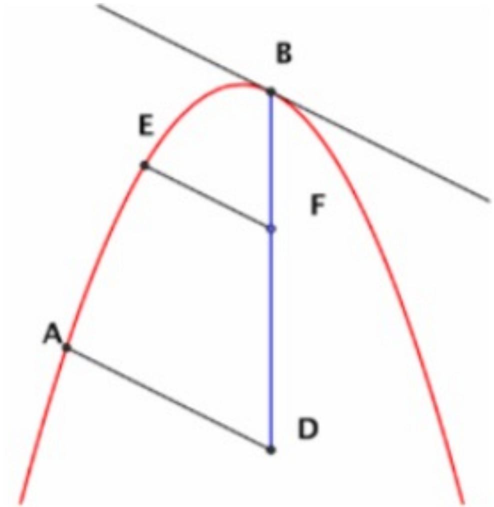


fig. 6

fig. 7

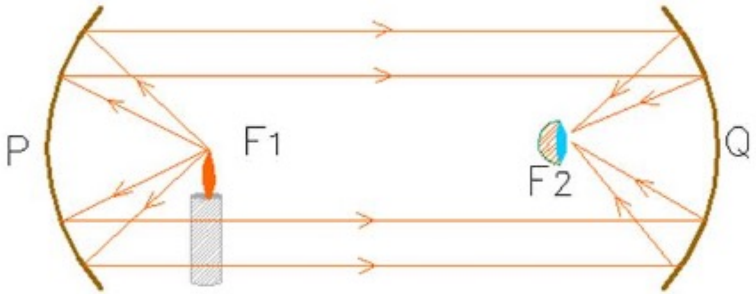


Apollonius, *Conic sections* (III a.C.).

Conics as loci, focal properties (Book III)

The fires were not called fires but were "the points determined by the application" (Prop. 45)

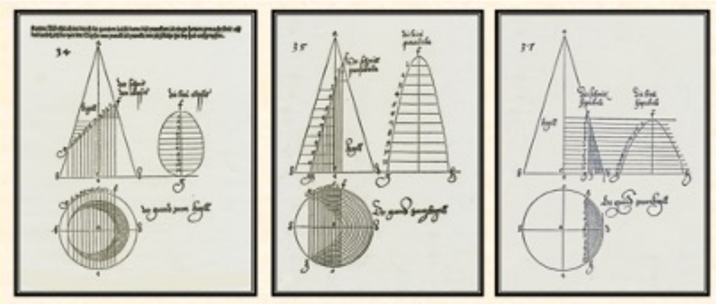
Apollonius does not speak of "focal point" for the parabola even if in ancient times the focal properties of the parabola were certainly known.



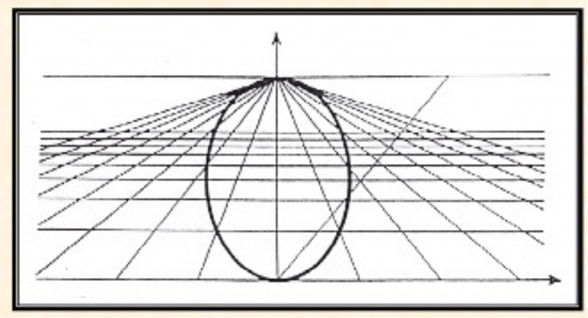
http://www.mathesisnazionale.it/mathesisbkp/archivio-storico-articoli-mathesis/68_83.pdf

Apollonius, *Conic sections* (III a.C.).

In book III important properties of the conics are introduced which are taken up by Pascal and Desargues and constitute, together with the innovations introduced by Guidobaldo Del Monte, a starting point for the study of perspective (*punctum concursus*).



Illustrazioni di A. DÜRER



The foundations of modern studies of Projective Geometry, but parabola had no focuses....

It will be Kepler who will significantly advance the study of the parabola in analogy to the other conics from this point of view, overcoming an epistemological obstacle.

At that time, there were no mathematical machines to draw a parabola… (Kepler)

https://people.dm.unipi.it/broglia/DIVULGAZIONE/Digitalpoint/itGP8.pdf

Bartolini Bussi, *The Meaning of Conics: Historical and Didactical Dimensions (2005)*

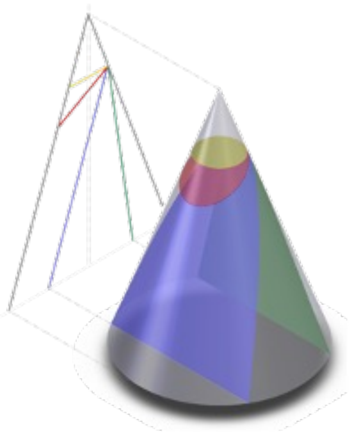
- Conics as sections of the same cone

- Conical as loci, with focal properties

- Conics characterized by geometric properties relating to the areas (parabolic, elliptic, hyperbolic "application")

- Conics as plane curves characterized with continuity by eccentricity and admits equation in polar coordinates

- Projective conics



Witelo, *Optics* (1270)

Witelo (13th century) takes up the study of conics for mirror applications

Predominantly physical interest for the study of optical phenomena

There are no mathematically significant theoretical innovations.

Witelo, 1270, Prop. 153, Book 9:

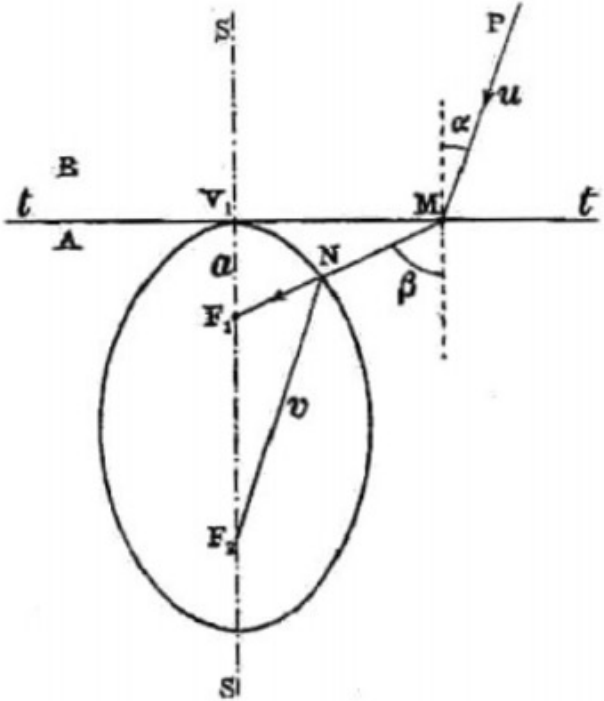
A mirror in the shape of a paraboloid of revolution concentrates the rays coming from a light source to infinity in the direction of the axis in the fire.

The ground is being prepared for a reconsideration of conics from a projective perspective

Johannes Kepler, *Astronomiae pars optica*, *Parapolimena* (1571-1630)

Refraction and reflection

Curve/Flat mirror



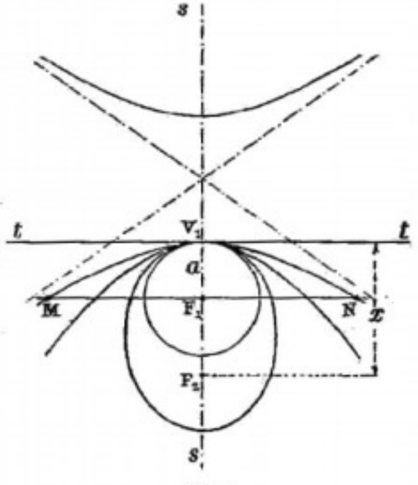
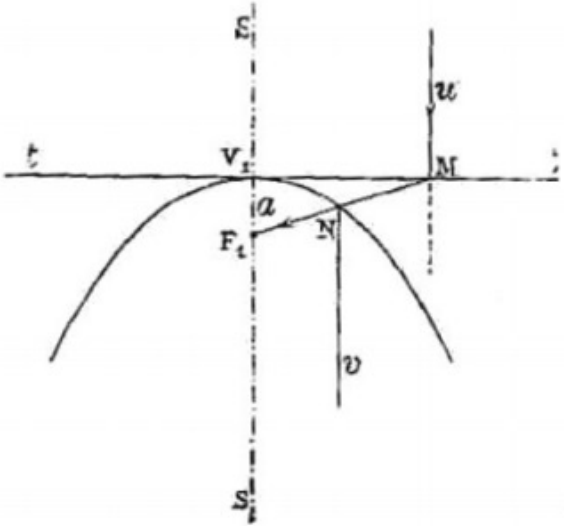


Fig. 1

http://www.mathesisnazionale.it/mathesisbkp/archivio-storico-articoli-mathesis/68_83.pdf

Johannes Kepler, *Astronomiae pars optica*, *Parapolimena* (1571-1630)



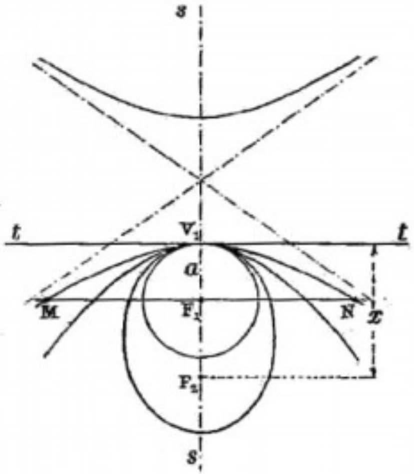
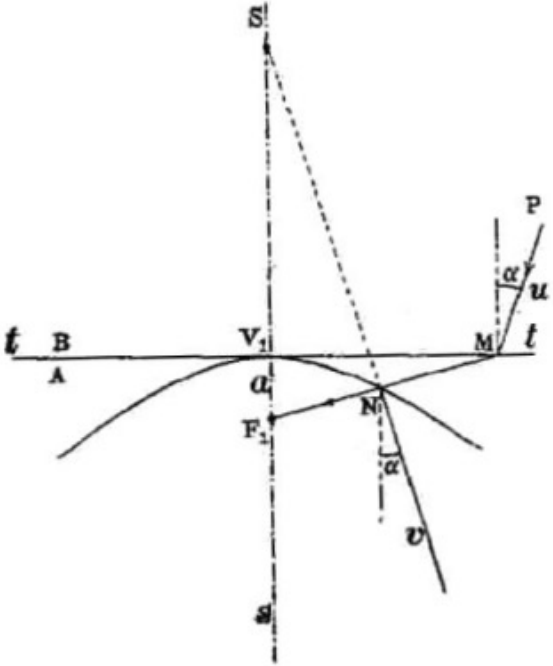


Fig. 1

Johannes Kepler, *Astronomiae pars optica*, *Parapolimena* (1571-1630)



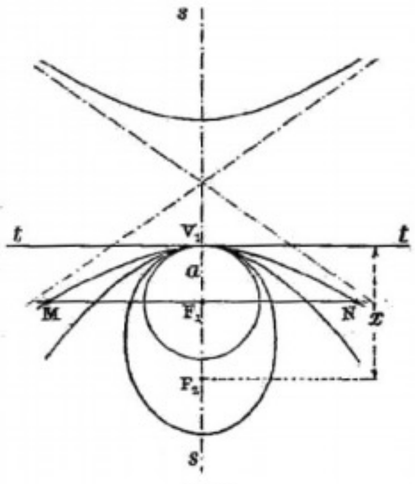


Fig. 1

Physics triggers innovation in Mathematics

1) Parallel lines as a special case of incident lines + infinity points (a revolution in Maths!)

1) Parabola included in a unifying plane classification of conics (parabola between ellipse and hyperbola)

infinity

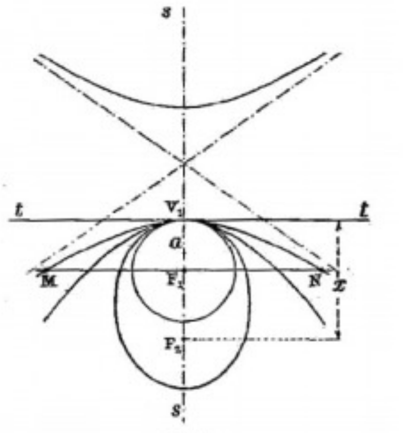
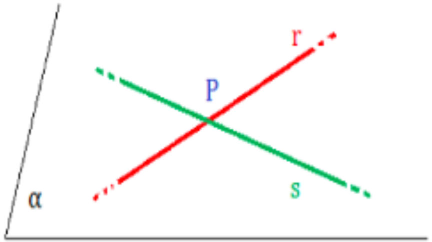
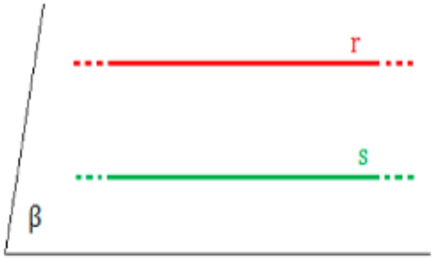


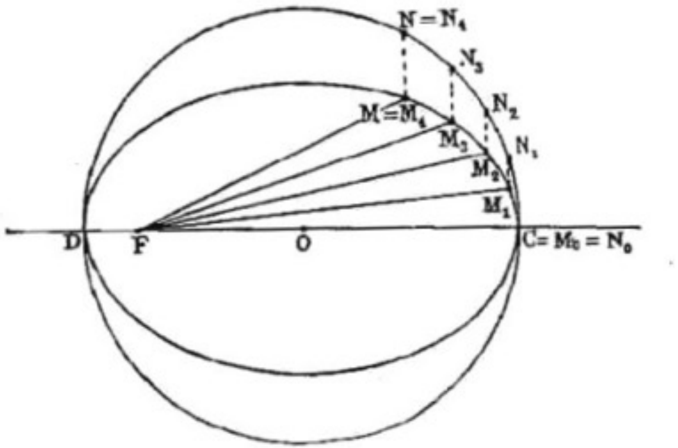
Fig. 1

infinity





Johannes Kepler: hypothesis of elliptic trajectory



History-pedagogy-mathematics/physics (HPM/Ph): an innermost relationship (Tzanakis, 2016)

Intertwined and bi-directional co-evolution, interdisciplinarity as the essence of the historical evolution of the two disciplines.

Historical cases can mirror both disciplinary authenticity and interdisciplinarity

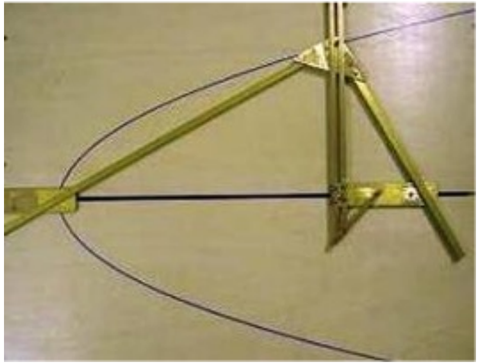
Maths → Physics

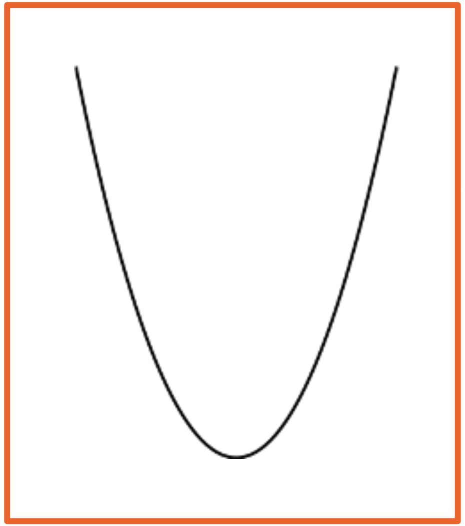
mathematics is the language of physics, not only as a **tool for expressing** ... but also as an indispensable, formative characteristic that shapes the physical concepts, by **deepening, sharpening, and extending their meaning**, or even endowing them with meaning.

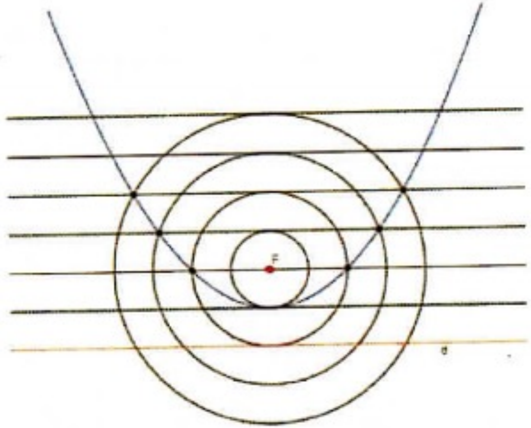
Physics→ Maths

physics constitutes a natural framework for **testing, applying and elaborating mathematical theories,** methods and concepts, or even motivating, stimulating, **instigating and creating all kinds of mathematical innovations.**

Back to the explorer activity….







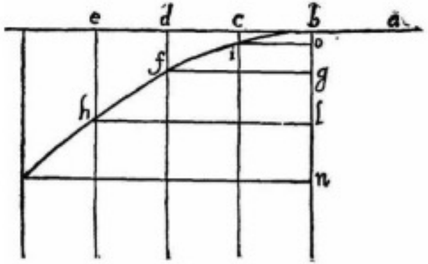
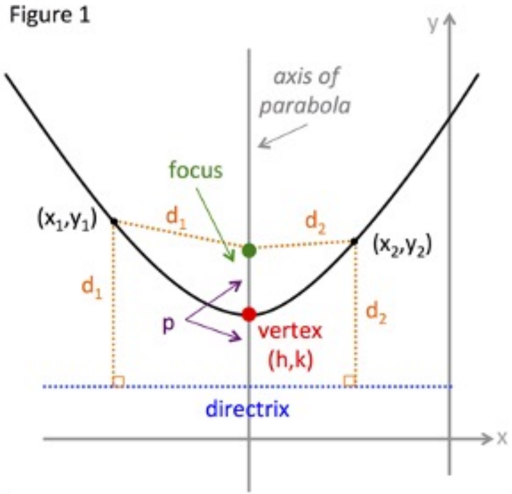
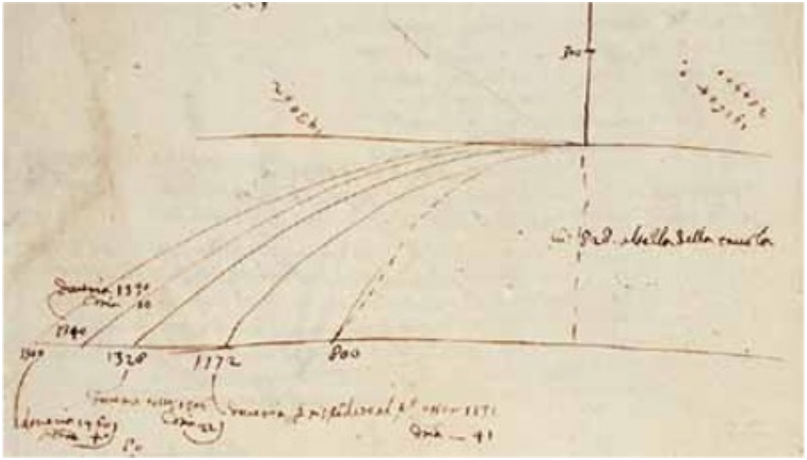


Fig. 108

Back to the explorer activity….





Whether and how can we prove that a “drawn curve” is a parabola?

Whether and how can we “draw” a parabola?

Whether and how can we prove that a motion is parabolic?

https://padlet.com/argyrisni/aje60yxrnc3g4zbc

Parabolic Path

RWP Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives

$$t = \frac{x}{v_0}$$

Next, substitute this result into the y equation to eliminate t:

$$y = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2{v_0}^2}\right)x^2 \qquad \text{4-8}$$

It follows that y has the form

$$y = a + bx^2$$

Theorem I, Proposition I

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola.

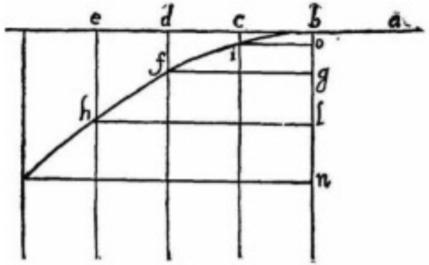


Fig. 108

Enlightening Interdisciplinarity in STEM for Teaching

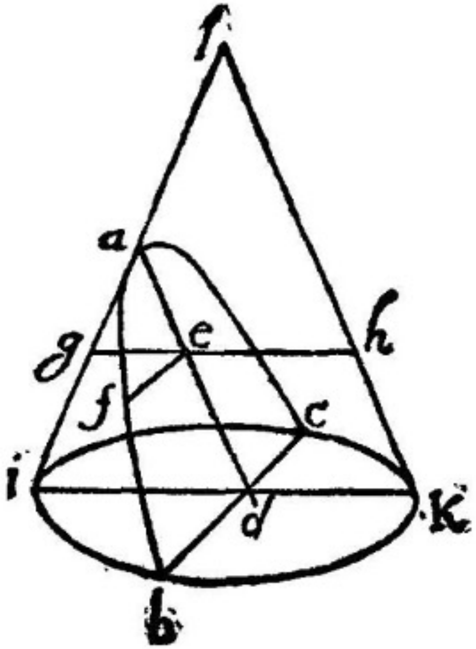
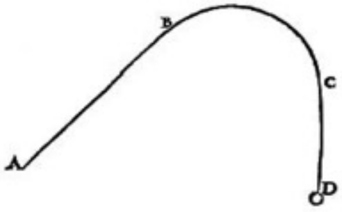


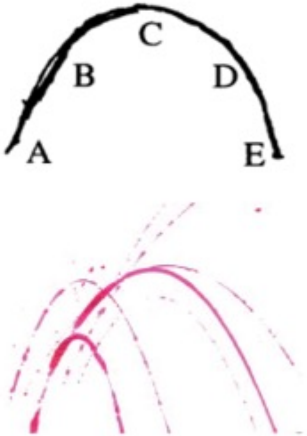
Fig. 106

SAGR. Here, Salviati, it will be necessary to stop a little while for my sake and, I believe, also for the benefit of Simplicio; for it so happens that I have not gone very far in my study of Apollonius and am merely aware of the fact that he treats of the parabola and other conic sections, without an understanding of which I hardly think one will be able to follow the proof of other propositions depending upon them. Since even in this first beautiful theorem the author finds it necessary to prove that the path of a projectile is a parabola, and since, as I imagine, we shall have to deal with only this kind of curves, it will be absolutely necessary to have a thorough acquaintance, if not with all the properties which Apollonius has demonstrated for these figures, at least with those which are needed for the present treatment.

SALV. Indeed, all real mathematicians assume on the part of the reader perfect familiarity with at least the elements of Euclid; and here it is necessary in your case only to recall a proposition of the Second Book in which he proves that when a line is cut into equal and also into two unequal parts, the rectangle formed on the unequal parts is less than that formed on the equal (i. e., less than the square on half the line), by an amount which is the square of the difference between the equal and unequal segments. From this it is clear that the square of the whole line which is equal to four times the square of the half is greater than four times the rectangle of the unequal parts. In order to understand the following portions of this treatise it will be necessary to keep in mind the two elemental theorems from conic sections which we have just demonstrated; and these two theorems are indeed the only ones which the Author uses. We can now resume the text and see how he demonstrates his first proposition in which he shows that a body falling with a motion compounded of a uniform horizontal and a naturally accelerated [*naturale descendente*] one describes a semi-parabola.

Symmetry and proof





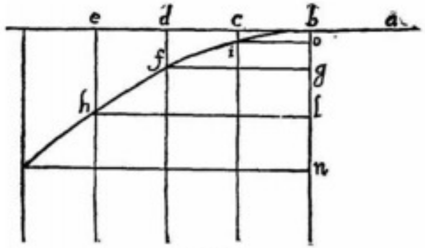


Fig. 108

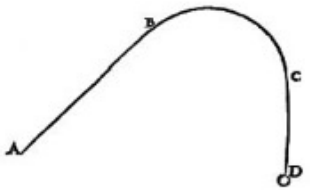
Tartaglia **Guidobaldo** **Galileo**

On the **left**, motion's representation of Tartaglia (1537); in the **middle**, the trajectory drawing in Guidobaldo's notebook and the reproduction of his ink experiment, done by Cerreta (2019); on the **right**, the figure supporting Galileo's demonstration of the parabolic trajectory.

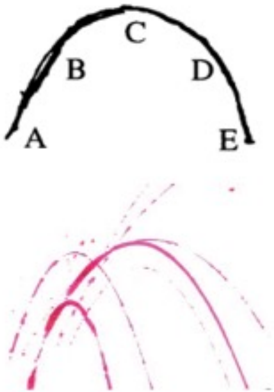
Symmetry and proof as epistemological activators

- The activity is titled ***Parabolic motion as foundational case to establish physics as discipline***. It is subdivided in four tasks.

- The activity has been designed to guide through the main **epistemological breakthroughs** that characterized the evolution of the physical thinking.



Tartaglia



Guidobaldo

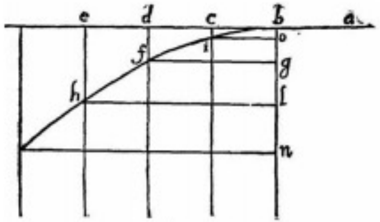
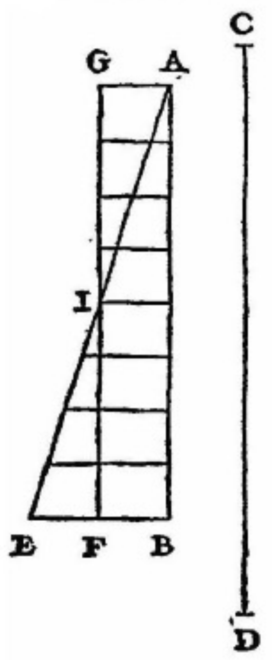


Fig. 108

Galileo

Proof: an activity to reflect about proof in mathematics and physics

Task 1: Write a proof of Pythagoras' theorem
Task 2: Proof in Euclid's *Elements*, exhaustion method and in analytical geometry
Task 3: The characterization of theorems, theory and metatheory
Task 4: Proof by Galileo about parabolic motion: an epistemological analysis



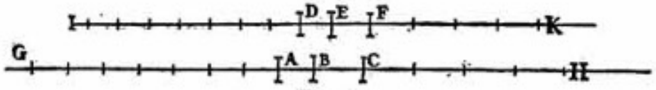


Fig. 40

From the above definition, four axioms follow, namely:

Axiom I

In the case of one and the same uniform motion, the distance traversed during a longer interval of time is greater than the distance traversed during a shorter interval of time.

Axiom II

In the case of one and the same uniform motion, the time required to traverse a greater distance is longer than the time required for a less distance.

Axiom III

In one and the same interval of time, the distance traversed at a greater speed is larger than the distance traversed at a less speed.

Axiom IV[192]

The speed required to traverse a longer distance is greater than that required to traverse a shorter distance during the same time-interval.

Theorem I, Proposition I

If a moving particle, carried uniformly at a constant speed, traverses two distances the time-intervals required are to each other in the ratio of these distances.

FOURTH DAY ↩

SALVIATI. Once more, Simplicio is here on time; so let us without delay take up the question of motion. The text of our Author is as follows:

THE MOTION OF PROJECTILES ↩

In the preceding pages we have discussed the properties of uniform motion and of motion naturally accelerated along planes of all inclinations. I now propose to set forth those properties which belong to a body whose motion is compounded of two other motions, namely, one uniform and one naturally accelerated; these properties, well worth knowing, I propose to demonstrate in a rigid manner. This is the kind of motion seen in a moving projectile; its origin I conceive to be as follows:

Imagine any particle projected along a horizontal plane without friction; then we know, from what has been more fully explained in the preceding pages, that this particle will move along this same plane with a motion which is uniform and perpetual, provided the plane has no limits. But if the plane is limited and elevated, then the moving particle, which we imagine to be a heavy one, will on passing over the edge of the plane acquire, in addition to its previous uniform and perpetual motion, a downward propensity due to its own weight; so that the resulting motion which I call projection [*projectio*], is compounded of one which is uniform and horizontal and of another which is vertical and naturally accelerated. We now proceed to [245] demonstrate some of its properties, the first of which is as follows:

THEOREM I, PROPOSITION I[269] ↩

A projectile which is carried by a uniform horizontal motion compounded with a naturally accelerated vertical motion describes a path which is a semi-parabola.

IDENTITIES
Enlightening Interdisciplinarity in STEM for Teaching

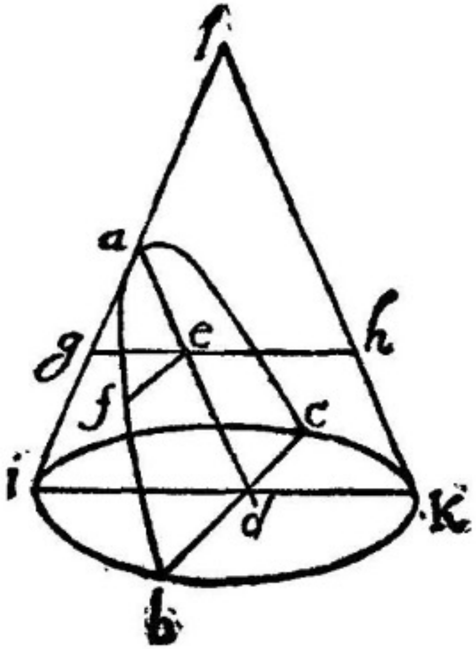


Fig. 106

SAGR. Here, Salviati, it will be necessary to stop a little while for my sake and, I believe, also for the benefit of Simplicio; for it so happens that I have not gone very far in my study of Apollonius and am merely aware of the fact that he treats of the parabola and other conic sections, without an understanding of which I hardly think one will be able to follow the proof of other propositions depending upon them. Since even in this first beautiful theorem the author finds it necessary to prove that the path of a projectile is a parabola, and since, as I imagine, we shall have to deal with only this kind of curves, it will be absolutely necessary to have a thorough acquaintance, if not with all the properties which Apollonius has demonstrated for these figures, at least with those which are needed for the present treatment.

SALV. Indeed, all real mathematicians assume on the part of the reader perfect familiarity with at least the elements of Euclid; and here it is necessary in your case only to recall a proposition of the Second Book in which he proves that when a line is cut into equal and also into two unequal parts, the rectangle formed on the unequal parts is less than that formed on the equal (i. e., less than the square on half the line), by an amount which is the square of the difference between the equal and unequal segments. From this it is clear that the square of the whole line which is equal to four times the square of the half is greater than four times the rectangle of the unequal parts. In order to understand the following portions of this treatise it will be necessary to keep in mind the two elemental theorems from conic sections which we have just demonstrated; and these two theorems are indeed the only ones which the Author uses. We can now resume the text and see how he demonstrates his first proposition in which he shows that a body falling with a motion compounded of a uniform horizontal and a naturally accelerated [*naturale descendente*] one describes a semi-parabola.

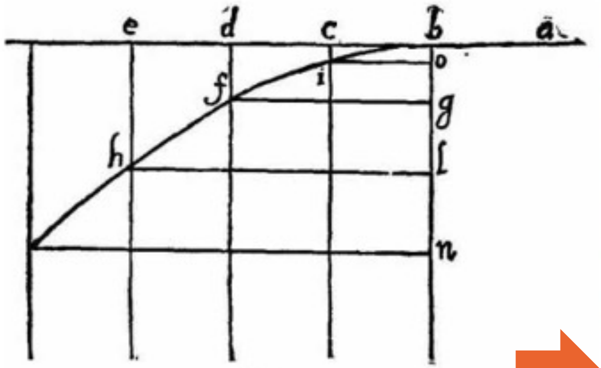


Fig. 108

Let us imagine an elevated horizontal line or plane *ab* along which a body moves with uniform speed from *a* to *b*. Suppose [249] this plane to end abruptly at *b;* then at this point the body will, on account of its weight, acquire also a natural motion downwards along the perpendicular *bn*. Draw the line *be* along the plane *ba* to represent the flow, or measure, of time; divide this line into a number of segments, *bc, cd, de,* representing equal intervals of time; from the points *b, c, d, e,* let fall lines which are parallel to the perpendicular *bn*. On the first of these lay off any distance *ci,* on the second a distance four times as long, *df;* on [273] the third, one nine times as long, *eh;* and so on, in proportion to the squares of *cb, db, eb,* or, we may say, in the squared ratio of these same lines. Accordingly we see that while the body moves from *b* to *c* with uniform speed, it also falls perpendicularly through the distance *ci,* and at the end of the time-interval *bc* finds itself at the point *i*. In like manner at the end of the time-interval *bd,* which is the double of *bc,* the vertical fall will be four times the first distance *ci;* for it has been shown in a previous discussion that the distance traversed by a freely falling body varies as the square of the time; in like manner the space *eh* traversed during the time *be* will be nine times *ci;* thus it is evident that the distances *eh, df, ci* will be to one another as the squares of the lines *be, bd, bc*. Now from the points *i, f, h* draw the straight lines *io, fg, hl* parallel to *be;* these lines *hl, fg, io* are equal to *eb, db* and *cb,* respectively; so also are the lines *bo, bg, bl* respectively equal to *ci, df,* and *eh*. The square of *hl* is to that of *fg* as the line *lb* is to *bg;* and the square of *fg* is to that of *io* as *gb* is to *bo;* therefore the points *i, f, h,* lie on one and the same parabola. In like manner it may be shown that, if we take equal time-intervals of any size whatever, and if we imagine the particle to be carried by a similar compound motion, [250] the positions of this particle, at the ends of these time-intervals, will lie on one and the same parabola.

Thomson Klein taxonomy to evaluate the kind of interdisciplinary output

Methodological Interdisciplinarity (MI) a method or a concept is taken from one discipline and applied in another to verify a hypothesis, formulate a theory or answer to a research question. The main goal is to improve the quality of results obtained in a single discipline. There is a contamination of epistemological knowledge, the <u>borrowing of some theoretical tools from another discipline can give us a new structure of the original discipline</u>.

Theoretical Interdisciplinarity (TI) is an evolution of the MI and it involves a more holistic, general view and a more coherent epistemology. The main results are the elaboration of conceptual frameworks during the analysis of problems, the integration of propositions across disciplines and the <u>new synthesis funded on the connection between models and analogies</u>.

Instrumental Interdisciplinarity (II) when MI serves some special needs of a single discipline. In the 80ies, instrumental interdisciplinarity gained visibility in informatics, biotechnology, or biomedicine (<u>development of new discipline</u>).

Critical Interdisciplinarity (CI) questions the dominant structure of knowledge and the educational system to transform it. It can destroy part of the system for reconstructing it. The deconstructing process and the seeking for disciplinary limits are the base for a <u>new epistemology</u>. <u>Asking critical questions and looking for a common answer</u> is part of the process of building new correspondences. The questions and the disciplines put in correspondence have changed, <u>the solidity of their borders crumbles and a common basis can raise</u>.

Enlightening Interdisciplinarity in STEM for Teaching

School science: parabolic motion in Italian textbooks

15. Il moto dei corpi lanciati in aria

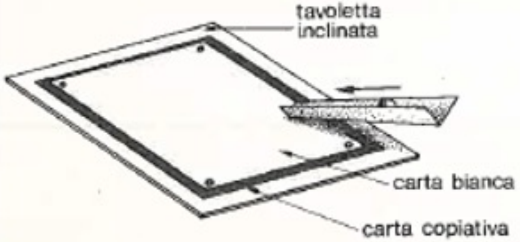
Un corpo lanciato in aria si muove, in generale, lungo una traiettoria curva: la figura 30 è stata copiata da una fotografia multiflash di una palla lanciata in aria in direzione obliqua. Cerchiamo di capire come si svolge il moto facendo alcune misurazioni sulla figura.

Innanzitutto: il moto della palla può essere considerato come se fosse composto dalla *sovrapposizione* di due moti: *un moto verticale* ed *un moto orizzontale*. Disegnando sulla figura con una matita a punta fine un reticolato

Un'ulteriore analisi della traiettoria della palla mostra che la sua forma è parabolica. Questo fatto può essere da te verificato riferendo la curva che rappresenta la traiettoria ad una coppia di coordinate spaziali cartesiane aventi l'origine nel suo punto più alto e l'asse verticale che punta verso il basso, coincidente con l'asse di simmetria della curva. La curva è una parabola se, chiamando x le ascisse (orizzontali) dei punti della traiettoria e y le loro ordinate (verticali), risulta che i valori di y sono direttamente proporzionali ai quadrati dei valori di x.

Traiettorie paraboliche come quella della figura 30 si ottengono ogni volta che un moto uniforme si combina con un moto uniformemente accelerato, ad angolo retto tra loro. Ciò avviene anche quando una biglia rotola obliquamente su un piano inclinato, se l'attrito è trascurabile. Un semplice esperimento ti permetterà di verificare questo fatto.

Registra le traiettorie paraboliche di una sferetta d'acciaio che farai rotolare obliquamente su una tavoletta di legno inclinata, su cui avrai fissato un foglio di carta copiativa con la parte inchiostrata verso l'alto, con sopra un foglio di carta bianca. Puoi lanciare la sferetta sul piano facendola rotolare giù da una guida ottenuta piegando una striscia di cartone nel senso della lunghezza (fig. 31).



Tracciando opportunamente degli assi cartesiani sulle traiettorie registrate, verifica che si tratta di parabole.

Nota. Forse sei curioso di sapere per quale ragione queste traiettorie sono proprio delle parabole. Eccola.

Se ci riferiamo alla palla della figura 30, si trova che essa si sposta orizzontalmente con velocità costante perché la resistenza del mezzo è trascurabile. Chiamiamo questa velocità v_x.

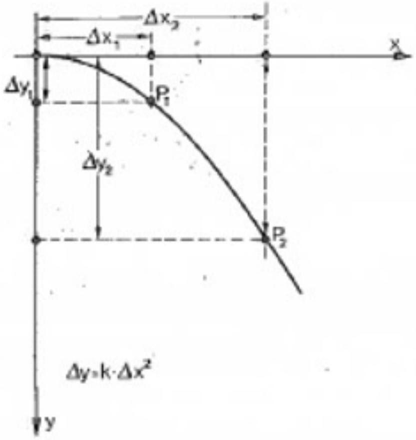
Lo spostamento verticale è invece uniformemente accelerato con accelerazione g.

Dopo un tempo Δt dall'istante in cui il corpo è passato per il punto più alto della sua traiettoria, i suoi spostamenti orizzontale e verticale valgono:

$$\Delta x = v_x \cdot \Delta t \quad \text{e} \quad \Delta y = \frac{1}{2} g \, \Delta t^2$$

Se ricavi Δt dalla prima di queste espressioni e lo sostituisci nella seconda, trovi:

$$\Delta y = \frac{1}{2} g \, (\Delta x^2 / v_x^2) = \frac{g}{2 v_x^2} \Delta x^2 = \text{costante} \cdot \Delta x^2$$



Dunque lo spostamento verticale risulta proporzionale al quadrato dello spostamento orizzontale, e la traiettoria è proprio una parabola (fig. 32).

3.4.c Il moto dei proiettili

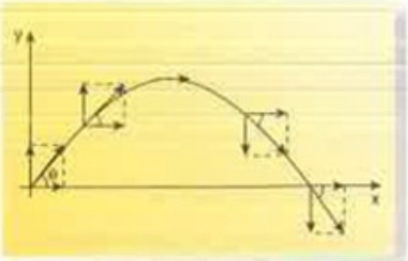


Fig. 3.33 Traiettoria di un proiettile sparato in una direzione formante un angolo θ con l'orizzontale. Si noti che la componente della velocità lungo x si mantiene costante.

Data la complessità della trattazione del moto bidimensionale nella sua espressione più generale possibile, ne proponiamo alcuni esempi iniziando con il moto dei proiettili.
Allo scopo si consideri un proiettile lanciato verso l'alto con velocità **v** in una direzione formante un angolo θ con l'orizzontale. Nell'analisi di questa situazione supporremo trascurabile la presenza dell'aria. Riferiamo il moto e la conseguente traiettoria al solito sistema cartesiano ortogonale Oxy con l'asse y rivolto verso l'alto come in figura 3.33.
In questo caso i valori delle grandezze cinematiche sono:
$a_y = -g$ (l'accelerazione verso il basso è dovuta alla gravità)
$a_x = 0$ (non vi è componente orizzontale dell'accelerazione)
$v_x = v \cdot \cos\theta$
$v_y = v \cdot \text{sen}\,\theta$
Dal momento che l'accelerazione non ha componente lungo l'asse x la componente orizzontale della velocità v_x rimane costante (in quella direzione il moto è rettilineo ed uniforme), mentre la componente verticale della velocità varia secondo le leggi del moto uniformemente accelerato ed il suo valore in un punto P qualsiasi sarà:

$$v_P = v_y - g \cdot t$$

Il moto risultante è dunque dato, istante per istante, dalla composizione (somma) di due moti: uno rettilineo ed uniforme lungo l'asse x, ed uno uniformemente accelerato lungo l'asse y. Le componenti dello spostamento del proiettile all'istante t sono:

$$x = v_x \cdot t \qquad\qquad y = v_y \cdot t - \frac{1}{2} \cdot g \cdot t^2$$

Ricavando t dalla prima equazione e sostituendolo nella seconda si ottiene:

$$y = \frac{v_y}{v_x} x - \frac{1}{2} g \frac{x^2}{v_x^2}$$

Essendo v_x, v_y e g valori fissati in maniera univoca una volta definite le condizioni iniziali (si noti tra l'altro che $v_y/v_x = \mathrm{tg}\theta$) l'equazione può essere scritta come:

$$y = b \cdot x - a \cdot x^2$$

È immediato riconoscere che questa è l'equazione di una parabola. Dunque la traiettoria del proiettile ha forma parabolica.

IDENTITIES Enlightening Interdisciplinarity in STEM for Teaching

Are these proofs?

Are they scientific explanations? If so, what kind?

What role does mathematics play?

How do students perceive it?

4 MOTI IN DUE DIMENSIONI: IL MOTO DEL PROIETTILE

Il moto di un punto materiale di massa m, lanciato con una certa velocità iniziale v_0 e soggetto alla sola azione della forza di gravità, è detto **moto del proiettile.**

Scegliamo un sistema di riferimento con l'asse x parallelo al suolo e l'asse y perpendicolare al suolo e diretto verso l'alto. Il principio di composizione dei moti consente di descrivere il moto del proiettile come la composizione di un moto rettilineo uniforme lungo l'asse x e di un moto uniformemente accelerato con accelerazione costante $-g = -9{,}8\ \mathrm{m/s^2}$ lungo l'asse y.

Indichiamo con x_0 e y_0 le componenti della posizione iniziale del proiettile e con v_{0x} e v_{0y} le componenti della sua velocità iniziale; le leggi orarie sono:

$$\vec{s}(t) \rightarrow \begin{cases} x(t) = x_0 + v_{0x}t \\ y(t) = y_0 + v_{0y}t - \frac{1}{2}gt^2 \end{cases}$$

$$\vec{v}(t) \rightarrow \begin{cases} v_x(t) = v_{0x} \\ v_y(t) = v_{0y} - gt \end{cases}$$

$$\vec{a}(t) \rightarrow \begin{cases} a_x(t) = 0 \\ a_y(t) = -g \end{cases}$$

Determiniamo l'equazione della traiettoria dalle leggi orarie della posizione, eliminando il parametro tempo; l'equazione cartesiana della traiettoria che si ottiene è quella di una parabola:

$$y = y_0 + \frac{v_{0y}}{v_{0x}}x - \frac{g}{2v_{0x}^2}x^2$$

Algebraic proof: a contribution from Mathematics education

At secondary schools, **in mathematics, an internal division** emerges that separates it into **the domains algebra, geometry, analysis, statistics** and so on (Boero, Guala & Morselli 2013) → difficulty at the didactic level, but also in building in students (and not only) a vision that, in addition to being **crystallized and sectoral, is also unrealistic.**

Morselli and Boero (2009): adaptation to mathematics teaching concerning the **construct of "rational behavior" for discursive practices,** proposed by Habermas, in particular regarding the **use of algebraic language in proofs.**

How do students perceive it?

Algebraic language in proofs: mainly thought of in secondary school as the domain of synthetic geometry, leading to **a radical change in the forms of explanation when passing from geometry to algebra**.

This trend is found in the mathematics textbooks analyzed and represents the reference knowledge of students in mathematics; they will refer to this knowledge by thinking about the parabola and its equation in physics.

The transposition makes the many opportunities for interdisciplinary reflection disappear….

Conclusions

Epistemological activators (of interdisciplinary learning potential):
objects meaningful within more than one discipline (like argumentation/proof, symmetry, line, …), so good candidates to be boundary objects, but also significant from an "internal" disciplinary epistemological point of view and likely to show the IDENTITIES of the disciplines through a learning mechanism at the boundary between disciplines.

Is this enough to activate the "learning potential" at the boundary?

Is this enough to trigger a fruitful discussion
about disciplinary IDENTITIES and interdisciplinarity?

Necessary but not sufficient…..

IDENTITIES

Enlightening
Interdisciplinarity
in **STEM**
for Teaching

ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA

UNIVERSITAT DE BARCELONA

ΠΑΝΕΠΙΣΤΗΜΙΟ ΚΡΗΤΗΣ UNIVERSITY OF CRETE

UNIVERSITÀ DI PARMA

Co-funded by the Erasmus+ Programme of the European Union

Grant Agreement n°2019-1-IT02-KA203-063184