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IDENTITIES

Enlightening Interdisciplinarity in STEM for Teaching

Guidelines to design and implement modules on curricular interdisciplinarity and STEM emerging interdisciplinarity in pre-service teacher education

- Intellectual Output O4 -

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IDENTITIES approach to interdisciplinarity

In this document, we present the general guidelines for the design and implementation of the modules on *curricular interdisciplinarity topics* and on *emerging STEM interdisciplinarity* in pre-service teacher education. In spite of their differences, all the modules have common features that incorporate the key principles of the IDENTITIES project and its common approach to interdisciplinarity that we focus on introducing in this document.

IDENTITIES taxonomy for Interdisciplinarity

In IDENTITIES we talk about interdisciplinarity and STEM integration, where STEM is used to refer to "a teaching approach that integrates content and skills specific to Science, Technology, Engineering & Mathematics". In the project, we refer to STEM Integration as: "working in the context of complex phenomena or situations on tasks that require students to use knowledge and skills from multiple disciplines" with particular attention to the dialogue between Computer Science, Physics, and Mathematics. Questions, such as: *What is the role of disciplines in this new paradigm? What are their relationships? What does it change in their traditional teaching if we adopt this perspective?* lead partners to adopt and define some terms helping designers, educators, and pre-service teachers progress to talking about and analyzing (among other aspects) interdisciplinarity.
The video "**IDENTITIES approach to interdisciplinarity**" (https://youtu.be/YElNsxehnxI) presents some keywords about interdisciplinarity (recontextualizing some terms introduced by Alvargonzález, 2011; Thomson Klein, 2017) helping to grasp the distinction between approaches to interdisciplinarity, all involving more than one discipline, in the context of IDENTITIES.

O2 and O3 modules for Interdisciplinarity: intrinsic distinction to approach ID

In the IDENTITIES Project two types of interdisciplinary topics have been addressed and implemented in two modules: *STEM advanced interdisciplinary topics* (in Chapter 3) and *Curricular interdisciplinary topics* (in Chapter 4). IDENTITIES project focuses on the tension(s) between the traditional organization of school knowledge in disciplines and the STEM interdisciplinary skills required by society and the labour market. Thus, we decided to explore, in the project, two kinds of interdisciplinarity, named curricular and STEM advanced interdisciplinarity topics.
In the *Curricular interdisciplinary topics* educational disciplines-interdisciplinarity, the pole "disciplines" prevail. The challenge is to avoid the collapse from the epistemic, social and cultural dimensions of a discipline to lists of data, methods, and information.
In the *STEM advanced interdisciplinary topics*, didactic narratives have to be still built and disciplines play the role of grounding new exploration on a solid basis and protecting from the insecurity given by the uncertainty and ambiguity of a new experience.

The video "**Curricular and STEM interdisciplinarity**" (https://youtu.be/CSBwQixd-vI) introduces the intrinsic distinction between both (non-independent) ways to approach interdisciplinarity.

Crossing the boundaries and activating interdisciplinarity

Interdisciplinarity is operationally explored in the design and analysis of the modules, through the theoretical perspective of Akkerman and Bakker (2011) of the "boundary objects and boundary learning mechanisms". The video "**The metaphor of boundary for interdisciplinarity**" (https://youtu.be/fK0XvtFeO_U) presents how researchers and designers in the IDENTITIES project proposed to recontextualise some of the core concepts, such as: "boundary", "boundary zones", "boundary crossing mechanisms" with respect to the focus of the project: detecting possible interdisciplinary zones to make disciplines interact and cross the traditional (or artificial) boundaries limiting their borders.

In order to manage the tensions that can emerge in navigating interdisciplinary boundary zone in search of both exploiting the disciplinary identities and transgressing them, the modules include and value "epistemological and linguistic activators". *Epistemological* activators refer to special boundaries or disciplinary objects that can support the activation of boundary learning mechanisms, that is, reflective comparisons within, between, and across disciplines (Ravaioli, 2020). Additional support for managing the tension can be provided by *linguistic* activators i.e., concepts, categories, or forms of linguistic representations that are used differently in different disciplines. The explicit comparisons of the entailed meanings ascribed in each discipline, help students unpack additional layers of the interaction between disciplines. In particular, the video **"The linguistic approach to interdisciplinarity"** (https://youtu.be/vQV7Id3JIVA) introduces the basic features of scientific language that make it different from natural ones and provides some examples to argue how linguistic choices made to present scientific topics affect the representations of the disciplines and their comprehension.

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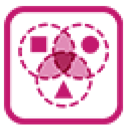
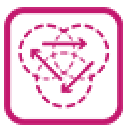
Icons and description of the IDENTITIES approach to ID

In the modules' description, three different groups of icons are used to orient reading. The first group corresponds to the keywords related to the IDENTITIES framework on interdisciplinarity. Table 1 describes the meaning assigned to each icon used to express the IDENTITIES approach to interdisciplinarity.

Tab. 1. Keywords for the IDENTITIES approach to interdisciplinarity

Icon	Meaning
	Identities of the discipline ● mathematics ■ physics ▲ computer science
	Interdisciplinary zone
	Boundary objects
	Boundary-crossing mechanisms
	Epistemological activators
	Linguistic activators

The didactic model for the design of the Teacher Education for Interdisciplinarity

The final structure of the IDENTITIES modules has been built according to a general didactic model to design and plan teacher education proposals. It corresponds to an adaptation of the proposal of the *Study and Research Paths for Teacher Education* (SRP-TE), as proposed in the framework of the Anthropological Theory of the Didactic (ATD). Within the ATD, a change in school paradigm, as the one proposed by STEM education and interdisciplinarity, is proposed to overcome some of the main didactic phenomena linked to the "monumentalization" of the knowledge to be taught (Chevallard, 2015). This change has been described in terms of a paradigm shift, from the "paradigm of visiting works" to the "paradigm of questioning the world". In the paradigm of questioning the world, the knowledge to be taught is associated with the study and inquiry into relevant questions. The step toward a change of paradigm in (pre-service and in-service) teacher education is approached using the didactic proposal of the *Study and Research Paths for Teacher education* (SRP-TE) (initially proposed in Ruiz-Olarría, 2015 and Barquero et al., 2018), which combines practical and theoretical questioning of outside- and inside-school scientific activities.

General overview of the design methodology for the design of the IDENTITIES modules

The adaptation considered of the SRP-TE in the context of the IDENTITIES project is mainly characterised by: i) the formulation of questions that are rich and relevant enough to be placed at the heart of pre-service teacher education programmes; ii) the facilitation, through the questions, of epistemological and didactic analysis tools of disciplinary and interdisciplinary knowledge at stake; iii) the detection of boundary objects and boundary-crossing mechanisms to switch on links between the disciplines and foster the analysis of interdisciplinary knowledge.

The adaptation of the structure of the particular SRPs-TE consists finally of four submodules (see Figure 1). Each of them asks participants to assume different roles, to facilitate facing complementary questioning about interdisciplinarity from different angles.

Submodule 1 starts with an initial question related to interdisciplinarity on the selected topic in scientific practices and about its possibilities to be transposed into secondary school education. These initial questions can be of different nature. They could be closer to the role of the disciplines and their interaction to address the topic at stake (*How have the different disciplines addressed a certain topic? How have these disciplines interacted to address certain problems or questions related to the topic?*); or, closer to the teaching profession (*How can interdisciplinarity practice can be transposed into Secondary school classrooms?*). Participants are asked to address these initial questions, under the role of "*interdisciplinary explorers*", to have a first look os what exists, and what emerges around them.

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In **Submodule 2**, participants are asked to experience an interdisciplinary project/activities, under the role of "*student*" or "*apprentice*", previously designed by the researchers-educators. The main goal of this module is to make participants carry out an unfamiliar interdisciplinary activity that could, to a certain extent, exist in an ordinary secondary classroom. This submodule is crucial to building together a shared *milieu* to "talk about" and to start "analyzing" (inter)disciplinary practice and the role of the disciplines.
Submodule 3 focuses on the collective analysis of the teaching experience that they come to be implemented, as students, but now changing and adopting the role of "*interdisciplinary analyst*". Participants are asked to analyse their previously developed activity on different levels or layers, e.g. epistemological, linguistic analysis of interdisciplinarity. At this stage, educators introduce some tools/instruments to progress collectively with participants on the analysis of interdisciplinarity.
Submodule 4 consists of sharing some secondary school experiences with participants (or, if there is the change, design and implement by themselves), linked to interdisciplinary topics and teaching projects iniciated in the previous submodules (1, 2 and 3). Through the presentation of a real case study in secondary school, participants are expected to use the tools for interdisciplinary analysis previously introduced to now question what and how can happen in a secondary classroom (under some specific conditions).

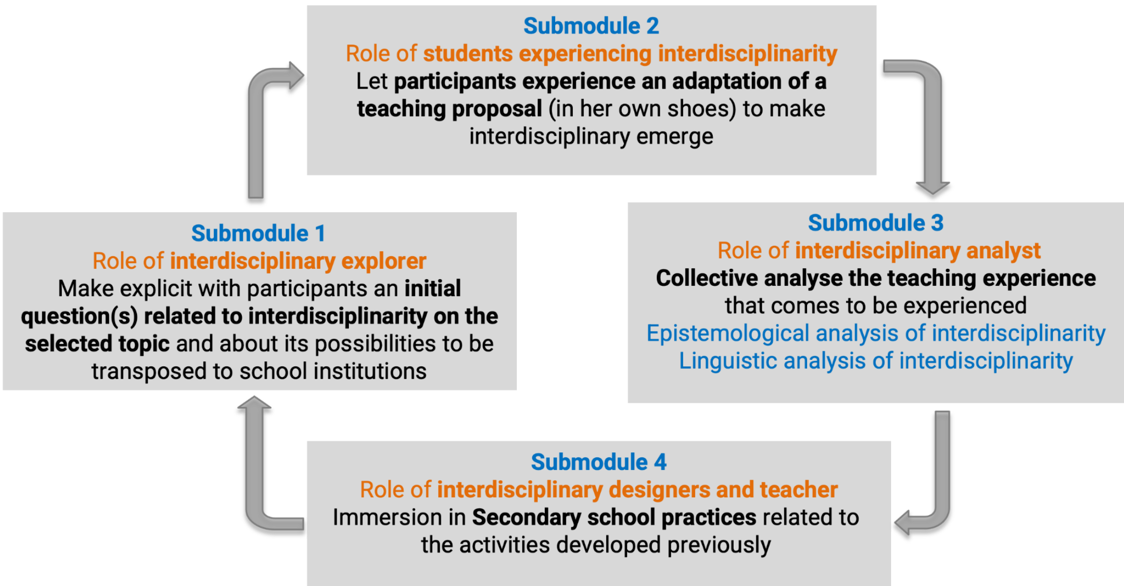

Fig. 1. Adaptation in submodules' structure of the SRP-TE
(adaptation from Barquero et al., 2018)

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Icons and description of the participants' roles in the module

As mentioned before, three different groups of icons are used to orient reading (the first group has been introduced in Table 1). The second set of icons (see Table 2) refers to the keywords related to the structure of the IDENTITIES modules, with respect to the role of participants.

Tab. 2. Keywords for the participants' roles in the module

Icon	Icons' definition
	Role of explorer
	Role of student
	Role of analyst
	Role of teacher-designer

In our guidelines for the design and implementation of the modules, we keep special attention to the rationale of the topics chosen; and, which are the aims and activities proposed in the different submodules (see Fig. 2) to help participants progress in making emerge, analyzing or designing interdisciplinarity.

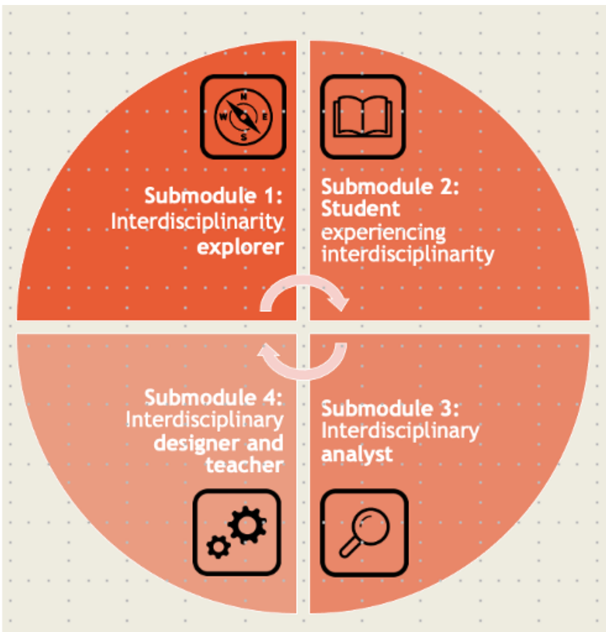

Fig. 2. Submodules' structure of the SRP-TE with the icons of the roles to be assumed

Generic description of the modules

In this document, we present an introduction and general description of the modules, with special attention to (1) the rationale of the selection of the interdisciplinary topic(s); (2) how the module uses the IDENTITIES approach to interdisciplinarity; (3) brief description of the aims and activities in each of the submodules (becoming explorers, students, analysts or teachers-designers). With this purpose, in the following chapters 3 and 4, each module is described according to the elements synthesized in Table 3.

Tab. 3. Tables used for the general description of the modules

Module	Title of the module
Rationale of the topic selected	Strengths of interdisciplinary contexts and topics chosen Selection of the ID topic, intrinsic O2 or O3 nature, and its disciplinary and ID nature
IDENTITIES' framework and aims pursued	ID questions and aims and professional teaching questions (justifying the design of the module)
Brief description of the submodules: aims and activities	
Becoming explorers	Description of the aims and activities to 'explore' ID in the topic proposed in the module and the role of the disciplines involved.
Becoming students	Description of the aims and activities planned to be experienced under the role of 'students' to approach ID
Becoming analyst	Description of the aims and tools offered for the analysis of Disciplinary Identities, Interdisciplinarity (ID), and of the role of the disciplines
Becoming teachers-designers	Description of the aims and activities planned in the module to experience as teachers and/or designers the interdisciplinarity
Link to the material in the IDENTITIES website	Link to the IDENTITIES link with the material to implement the module and the description of the possible Lesson Plan for its implementation.

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Overview of the designed modules

In the IDENTITIES project, the partners have worked on the design of eight modules: Four modules of the first type on *STEM advanced interdisciplinary topics* (O2); and, four more modules of the second type *Curricular interdisciplinary topics* (O3). Figure 3 summarizes the correspondence of each module in each kind of approach to interdisciplinarity, corresponding to O2 –**Intellectual Output O2**– or O3 –**Intellectual Output O3**–.

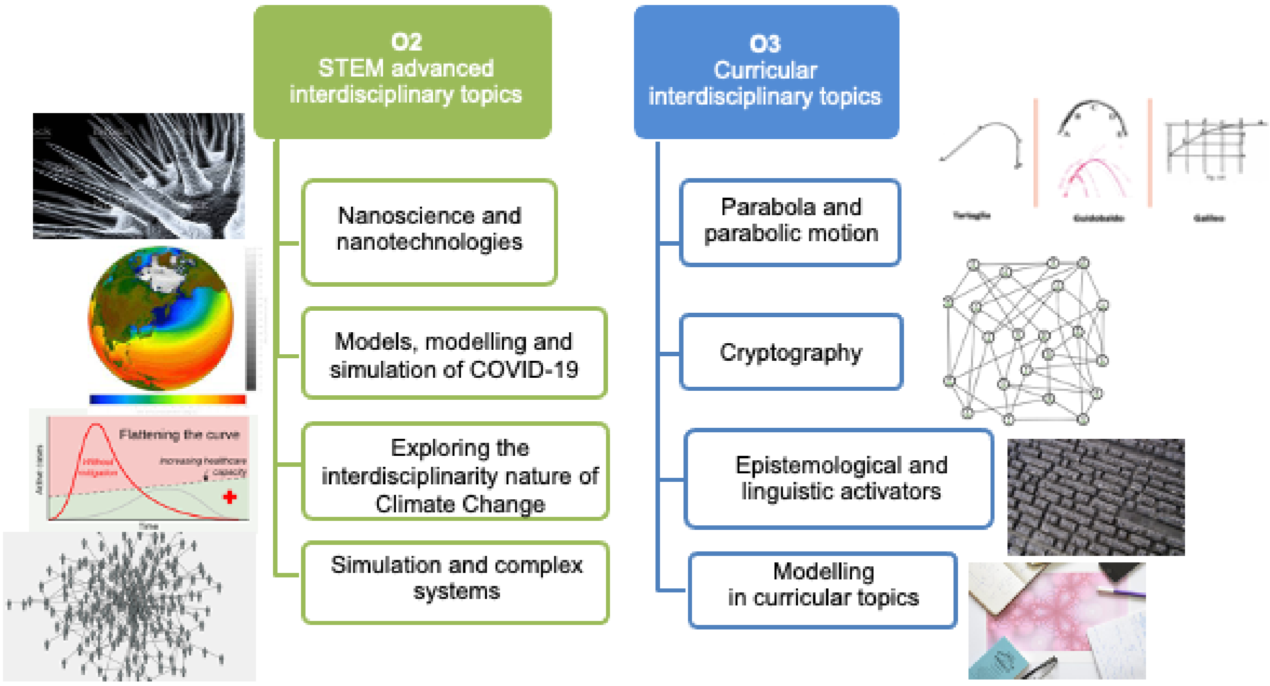

Fig. 3. Designed modules in IDENTITIES; corresponding to O2 or O3

Chapters 3 and 4 present the description of each IDENTITIES module. We start the O2 modules on **STEM interdisciplinary advanced topics** ([Chapter 3](#)), followed by the modules on **Curricular interdisciplinary topics** ([Chapter 4](#)).

More details about the description of the module, the material produced and the lesson plans to guide their implementation are accessible in the following link, as well as in the reports of O2 – Intellectual Output O2 – or O3 – Intellectual Output O3 -. Table 4 and 5 includes the title of each module and the links to the material and lesson plans available on the IDENTITIES website https://identitiesproject.eu/modules/. In the Annex we report graphical visualization of five modules in the form of posters.

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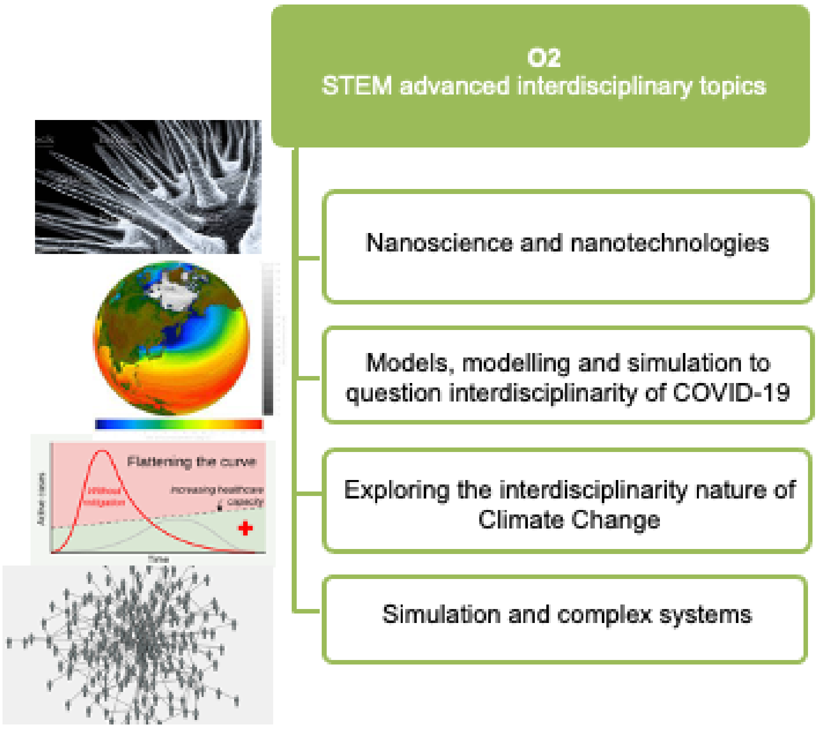
O2 modules on STEM interdisciplinary advanced topics

In this chapter, the four modules about STEM interdisciplinary advanced topics are presented.



Tab. 4. Titles of the modules produced in O2 and links to materials and lesson plans

O2: Teaching modules on emergent interdisciplinarity in advanced STEM topics		
	Title of the module	Links to description, material and lesson plan
1	Nanoscience and nanotechnologies	https://identitiesproject.eu/nanoscience-and-nanotechnology/
2	Models, modelling and simulation to question interdisciplinarity of COVID-19	https://identitiesproject.eu/models-modelling-and-simulation-to-highlight-the-interdisciplinarity-of-covid-19/
3	Exploring the interdisciplinarity nature of Climate Change	https://identitiesproject.eu/climate-change/
4	Simulation and complex systems	https://identitiesproject.eu/simulations-of-complex-systems/

The Module on Nanoscience and nanotechnologies

Rationale of the topic selected	The rationale for choosing Nanoscience-Nanotechnology (NST) for STEM teaching relies on the fact that: a) NST is by nature an interdisciplinary field, in which many disciplines interact. According to Kähkönen et al. (2016) "interdisciplinarity is the only thing that gives a name to nanoscience". In specific, physics, chemistry, biology, material science, medicine, computer science, and engineering are some indicative disciplines that are related to NST phenomena; b) NST is related to many contemporary real-world applications, and breakthroughs; c) being an ongoing field of research, it gives the opportunity to students to discover new methods and new ways of thinking as well as to cultivate views of Nature of Science and Nature of Technology; and d) it can engage students in relevant socio-scientific issues and issues of responsible citizenship (Kähkönen et al., 2016; Stavrou et al., 2018). Moreover, it is a topic that, although it is considered a contemporary advanced STEM topic, several connections with the current curricula can be used as 'entrance' points to NST concepts/phenomena/applications, e.g. atomic structure, orbitals, materials' properties (optical, electrical, etc.), polymers, etc.
IDENTITIES' framework and aims pursued	The NST module makes use of theoretical frameworks implemented and elaborated during the IDENTITIES project, as follows: a) The module uses informed views on Interdisciplinarity (Klein, 2017) both during the design of the module and the activities for the students, contrasting multi-, trans- and a-disciplinarity views. b) The module implements the boundary object framework (Akkerman & Bakker, 2011) as a means to explore the interconnections among disciplines. Particularly, the module designers implemented boundary objects such as modelling, instrumentation, and biomimicry as 'lenses' through which students would be facilitated to identify and analyse the relative disciplines and the interactions among them. Furthermore, it enacts students' epistemic agency by inviting participating students to reflect and identify additional boundary objects themselves, and also justifying their views. c) The project also implements epistemological activators (Ravaioli, 2020) as boundary objects. In specific, epistemological activators are themes that activate a meta-level of analysis in which the disciplines can be characterised, compared and intertwined. Hence, epistemological activators can assist in the cultivation of interdisciplinary consciousness. Additionally,

	linguistic activators (Polverini, 2022) are terms, syntaxes, and lexicons that can be used as another layer of identifying, comparing, and negotiating disciplinary knowledge and skills through the commonalities and differences that they get among the languages of different communities. d) The module implements ‘professional’ questions as a tool to orient students to the educational perspective and applicability of their teaching ideas. In specific, questions are been posed to students regarding the justification of introducing NST to schools, the core NST concepts to be taught, and the affordances and constraints of the transposition of NST knowledge and skills to schools.
Brief description of the submodules: aims and activities	
Becoming explorers	Aims: Contextualisation of NST to real-world problems and challenges that is related to. Significant NST applications and innovations are been introduced to students, whereas space is given to students to mention additional ones from their knowledge and experiences. The students are subsequently called upon to reflect and discuss the S-T-E-M disciplines engaged in the topics presented, and to preliminary identify boundary objects in the module in terms of concepts/methods/artefacts/questions or linguistic terms. Activities: After a round of exploration of real-world problems that NST contributes to, the instructor(s) present some main application areas of NST, such as environmental issues, energy crisis, overpopulation and increasing need for food, healthcare, and technological development. Subsequently, students work in groups and analyse the topics in the light of the disciplines involved and the boundary objects in terms of epistemological and linguistic activators in which they can identify interactions between disciplines.
Becoming students	Aims: The aim of this submodule is to offer students the opportunity to get informed and to practically experience NST-related activities in some indicative main application areas. These activities act as a reference space in which students will reflect on and perform an interdisciplinary analysis in the analyst submodule. Activities: Initially, students are introduced to core NST concepts/phenomena/applications (the 9 ‘big ideas’ of NST). Thereafter, students work in groups in 4 sets of activities concerning: a) NST & smart housing, in which students engage with NST applications such as thermochromic glass, waterproof surfaces and self-cleaning dyes. b) NST & alternative energy sources, in which students engage with third-generation solar cells (organic & dye-sensitised dollar cells) and compare them with conventional solar cells. c) NST & Medicine, in which students explore how nanoparticles of different sizes have different energy band gaps and hence

	can be implemented for selective targeting of cancer cells in the human body. d) NST instrumentation, in which students engage with the main operation principles of instruments use in NST such as the Scanning Tunnelling Microscope (STM) and the Atomic Force Microscope (AFM). At the end of this submodule, time is given to student groups to recap and share their experiences with the other peer groups.
Becoming analysts	Aims: Introduction of theoretical frameworks of analysing interdisciplinarity, such as the epistemological and linguistic activators that act as boundary objects in this module. This submodule calls upon students to reflect on the boundary objects implemented by the researchers in the module, but also gives space to students to reflect on additional boundary objects they recognise. At the end of this submodule, students reflect on visual representations of STEM models (Ring et al., 2017) as a reflective epistemological activity on integrated STEM. Activities: The participants become “interdisciplinarity analysts” since, in groups, carry out a meta-reflection on the previous activity in terms of reflecting on boundary objects introduced by the researchers, which were deemed to be modelling (Develaki, 2020), instrumentation (Stevens et al., 2009), and biomimicry (Krohs, 2022). Students also reflect on linguistic terms that have different nuances among communities such as ‘properties’, ‘nanoscale’, ‘efficiency’ and ‘artificial/natural’. Furthermore, students are given the opportunity to identify additional boundary objects themselves and discuss about them. Finally, students are introduced to teacher-generated STEM models (Ring et al., 2017) and are called upon to comment on them, state their preferred STEM model representation, and optionally create and justify their own STEM model representation.

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The Module on COVID-19: Models, modelling and simulation to question interdisciplinarity of COVID-19

Rationale of the topic selected	The topic of modelling the evolution of COVID-19 was chosen in part due to its intrusion into our daily lives at the beginning of 2020. An authentic example of STEM advanced interdisciplinarity is recognised as one major issue for society that required the collective effort of putting different disciplines to react in front of unexpected questions. Indeed, the COVID-19 pandemic has shown more than ever that students and, more in general, citizens need to understand how mathematics and scientific advances contribute to the understanding of societal phenomena. In addition, "the pandemic illustrates perfectly how the operation of science changes when questions of urgency, stakes, values, and uncertainty collide" (Saltelli et al., 2020). Specifically, it has emerged the need to explore what kind of knowledge can models and modelling provide, how we may interpret their predictions, and more in general, what contribution they provide to the understanding of such a complex issue.
IDENTITIES' framework and aims pursued	The IDENTITIES approach for the design and implementation of the module is mainly evident in: (a) the formulation of questions that are rich and relevant enough to be placed at the heart of pre-service teacher education programmes (such as, the initial questions about "how have STEM disciplines contributed to the societal understanding of the evolution of COVID-19? How can this interdisciplinary practice be transposed to secondary schools?"; (b) the facilitation, through the questions, of epistemological and didactic analysis tools of disciplinary and interdisciplinary knowledge at stake; (c) the detection of boundary objects and boundary-crossing mechanisms to switch on links between the disciplines and foster the analysis of interdisciplinary knowledge.
Brief description of the submodules: aims and activities	
Becoming explorers	<u>Aims:</u> First exploration of the topic-specific questions for interdisciplinarity, identification of the disciplines involved, detection of new terminology diffused to society, etc. The submodule finishes with the delimitation of possible lines of inquiry involving models and modelling as well as the interaction among different disciplines. <u>Activities:</u> A selection of news collected during the months of pandemics is proposed to be analysed. Educators guide participants in a first analysis aiming to detect: e.g., relevant questions calling for disciplinary and interdisciplinary knowledge, answers provided by the scientific communities, and analysis of the terminology used.

Becoming students	Aims: The main goal of this submodule is to make participants carry out an unfamiliar interdisciplinary activity that could take place also in the classroom. The participants explore the issue of COVID-19 evolution from three different (but complementary) points of view: the real data processing, selection of variables, and their statistical analysis; the use of equation-based mathematical models for disease diffusion and the interpretation of the models' coefficients, accordingly to the data; and the implementation of an agent-based simulation using methods inspired by statistical physics to evaluate different types of social intervention. Activities: Educators ask participants to experience an interdisciplinary project, under the role of "students of interdisciplinarity", about: (1) The complexity of delimiting the system to model: analysing data; (2) The role of the equation-based models: what can we consider a 'good' model? what are models for?; (3) Agent-based models and simulations: Simulating scenarios to help to make decisions about societal restrictions.
Becoming analysts and designers	Aims: Introduction, transference, and adoption of tools for the epistemological, linguistic, and ecological analysis of interdisciplinarity and of discipline's identities Activities: The participants become "interdisciplinarity analysts" since, in groups, carry out a meta-reflection on the previous activity on three different levels. The first level, using the tool of questions-answers maps (Winsløw et al., 2013), aims to sketch the process followed through the dialectics between the specific questions that the group has faced, and the answers obtained. The second level requires recognizing in the lines of inquiry examples of boundary objects. Finally, participants analyse the kind of interaction among disciplines (i.e. boundary-crossing mechanisms) that happens when boundaries are at stake and are eventually overcome. In Submodule 4 some secondary school experiences linked to each line of inquiry are shared with participants. Then, they are expected to use the tools previously developed for interdisciplinary analysis to discuss the conditions to facilitate the implementation of interdisciplinary activities in real classrooms, as well as the constraints hindering the chances for interdisciplinarity to happen.

The Module on Climate change: Exploring the interdisciplinary nature of climate change

Rationale of the topic selected	The module concerns a theme, Climate Change, with the following features: a) The theme concerns a new STEM research field; b) Physics, Mathematics, Chemistry and Computer Science are core disciplines, although other disciplines can be involved (STE-A-M); c) The theme is educational, social, political and personally relevant and appropriate to be used within university courses for pre-service teacher education (in Physics, Mathematics and CS).
IDENTITIES' framework and aims pursued	The Climate change module makes use of IDENTITIES frameworks as follows: (a) *The authenticity of interdisciplinarity:* The module show that there are concepts or topics related to the STEM theme, that are intrinsically interdisciplinary and cannot be reduced to the sum of S+T+E+M: *How is the interdisciplinary - boundary zone established? When and how do you feel "inhabiting an exchange-boundary-space" and "facing the ambiguity and going out of the comfort zone"?* (b) *The S-T-E-M disciplinary identities:* The disciplines have an important role in the module, as sources of structured knowledge that can nurture the development of knowledge on the STEM theme. *What roles can be attached to the disciplines and the process of disciplinarization? Are there moments where you felt particularly at home?*; (c) *Epistemological and linguistic activators:* The module includes examples of "epistemological and linguistic activators": *What epistemological and linguistic activators do you find in the module? What makes them "epistemological and linguistic activators"?*
Brief description of the submodules: aims and activities	

Becoming explorers	Aims: The goal of the submodule is to allow students to explore the concept of circular causality within climate phenomena, allowing them to delve into the construction of causal maps. Within the submodule, the concept of feedback and causal circularity are explored, aiming to show the complex nature of climate and climate change in particular. The main objectives of the activities are i) to provide basic knowledge on causality, circularity and feedback; ii) to deliver the essential tools needed to explore the complexity of climate change; iii) to show the topic of biofuels as an example of a reality related to multiple aspects of the problem. Activities: The aforementioned concepts and goals are explored by pre-service teachers through 2 different activities. (Act. 1) Exploring feedback in climate systems. Pre-service teachers focus on the concept of feedback, looking at examples and videos aimed at explaining its importance, implications, and possible effects on daily life. (Act. 2) Reading a text on the use and production of biofuels. (Act. 3) Construction of a causal map on bio-fuels. The activity aims to make the pre-service teachers construct a causal map on biofuels, with particular concern for biodiesel. At the end of the activities, three claims are analysed, and each is followed by a question to start a discussion among all participants.
Becoming students	Aims: The main goal is to familiarise students with certain fundamental concepts of complexity to be able to recognise the interdisciplinary nature of complex systems such as climate change. The fundamental concepts of complexity that we explore are: i) Limited Predictability in Deterministic Systems; ii) Sensitivity in the initial conditions; iii) Critical States / Feedback. The objectives of exploring the aforementioned concepts are for pre-service teachers to observe: i) the inherent limitations in predicting the evolution of a deterministic chaotic system; ii) that some chaotic systems even though they appear random yield a form of order; iii) that as a system evolves it passes through critical states that dictates its final "form" (self-reinforcement). Activities: Pre-service teachers are introduced to 4 different activities. (Act. 1) Participants explore the possibility of long-term predictability through the comparison of weather forecasts (from 7 to 1 day before a given date) and the weather conditions for the given date. They discuss time sensitivity and the limited predictability of the weather as a complex system. (Act. 2) It concerns the limited predictability of deterministic chaotic systems and the forms of order that non-linear systems present. Pre-service teachers commented on the representations developed by a chaotic pendulum both in the case of harmonic oscillation and deterministic chaos. (Act. 3) Critical states in the overall "form" of the system are further explored with the activity about depicting the rotation of Benard Cells. (Act. 4) Participants reflect on the ideas of limited predictability and critical states in the context

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	of biodiesel. The activity is based on a concept map about choices in the procedure of biodiesel production and their effects.
Becoming analysts	<u>Aims:</u> The main objective is to have students analyze the topic of uncertainty in a disciplinary and interdisciplinary context, starting with the topic of climate change and then going on to generalize. The submodule focuses on the concepts of complexity and uncertainty, highlighting complexity as an epistemological activator and boundary object. Specific objectives of the submodule are: i) to grasp the epistemological consequences of complexity; ii) to differentiate among the three types of uncertainty present in climate modelling; iii) to highlight the three types of uncertainties within their own disciplines of belonging; iv) to be able to generalize the three types of uncertainty and confront them in an interdisciplinary perspective. <u>Activities</u>: Two activities are presented (Act. 1) Three questions are posed to think about the topic of complexity as it was seen in the previous sub-modules. (Act. 2) Discussion on the types of uncertainty in climate modelling. Uncertainty in climate projections arises from three sources: in the social and political conditions that determine future climate forcing; in our knowledge of how the climate system responds to that forcing; in the actual realization of climate for a particular time window. Pre-service teachers are divided into three groups, one for each type of uncertainty. Participants are asked to answer three questions, related to their discipline of expertise and fill out a scheme to meta-reflect on the perspectives opened by these issues.

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The Module on Simulations of complex systems

Rationale of the topic selected	The topic of computational simulations of complex systems was chosen because, since the 50s, they have been an authentic part of the current methodological practices in research and professional endeavours. Indeed, nowadays, in almost all disciplines, computational methods in general - and simulations in particular - are intensively used to generate knowledge and advance inquiry. In particular, computational simulations were selected because they have an interdisciplinary nature at the intersection of traditional STEM disciplines (applied physics, mathematics, and computer science of simulations) but also connect science with humanities and societal studies.
IDENTITIES' framework and aims pursued	The IDENTITIES approach for the design and implementation of the module is mainly evident in (a) the search for the connections between the physical modelling of the simulated systems, their mathematical formulation, and the computational implementation; (b) the enhancement of the identity of physical modelling as a lens to describe societal phenomena, closing in this way the gap between the dichotomy of natural sciences vs social sciences; (c) the emphasis on computational simulations as boundary objects i.e., artefacts that bridge different domains and trigger interdisciplinary collaboration and understanding.
Brief description of the submodules: aims and activities	
Becoming explorers	<u>Aims:</u> The first submodule aims at a preliminary exploration of the properties of physical complex systems (non-linearity, high sensitivity to initial conditions, circular causality, emergent properties) and their educational potential. <u>Activities:</u> Two dialogical lectures are proposed to participants. In the first, the instructor presents examples of complex systems (double pendulum, arctic feedback, shivering and sweating in the human body, Rayleigh-Benard convection) to show how they embed properties that are significantly different from those of classical systems. The second lecture, instead, is focused on the research in science education about teaching and learning complexity. Specific attention is devoted to the types of explanations for emergent phenomena that novices elaborate. To make participants experience first-hand the issue, the instructor proposes them a group activity to explain the process of movement of sand dunes in the desert. After the teamwork and a collective

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	discussion, the instructor analyses the explanation formulated by a prospective teacher engaging with the same problem.
Becoming students	<u>Aims:</u> The main goal of this submodule is to introduce the notion of computational simulation and then to make participants experience equation-, agent- and network-based modelling and simulations by addressing specific case studies. <u>Activities:</u> The submodule comprises two main interactive lectures. In the first, several definitions of simulation are discussed and two main approaches are delimited: the equation-based approach (providing the example of Lotka-Volterra model) and the agent-based one (providing the example of Schelling model). In the second lecture, the participants are led to see in action the previously delimited distinction between equation- and agent-based approaches by analysing the epidemiological model of Susceptible-Infectious-Recovered by Kermack and McKendrick (1927); in the final part of the lecture, to these two approaches is flanked a third approach provided by network-based models to address epidemiological modelling. During the second lecture, several group activities are proposed to the participants.
Becoming analysts	<u>Aims:</u> The third submodule aims at making participants analyse from an epistemological and interdisciplinary perspective the difference between equation-, agent-, and network-based models. <u>Activities:</u> Two team-work activities are proposed to the participants to analyse the work carried out in the module so far. In the first one, the participants reflect on the comparison of the three types of models and simulations experienced; to do that, they are guided by a table with some questions (e.g. What kind of phenomena does the model allow us to deal with? What spatial and temporal structures are incorporated? What is the connection between the model and its computational implementation (in a simulation)? What allows us to visualise the simulation of models, in relation to the system to be studied?). In the second activity, the participants are asked to reflect on the boundary objects detectable in the module and the disciplines at stake. The module ends with a wrap-up that discusses the relation between physics and the interdisciplinary field of complex systems, toward the advent of the new discipline of computational social science that overcomes the traditional dichotomy between natural and social.

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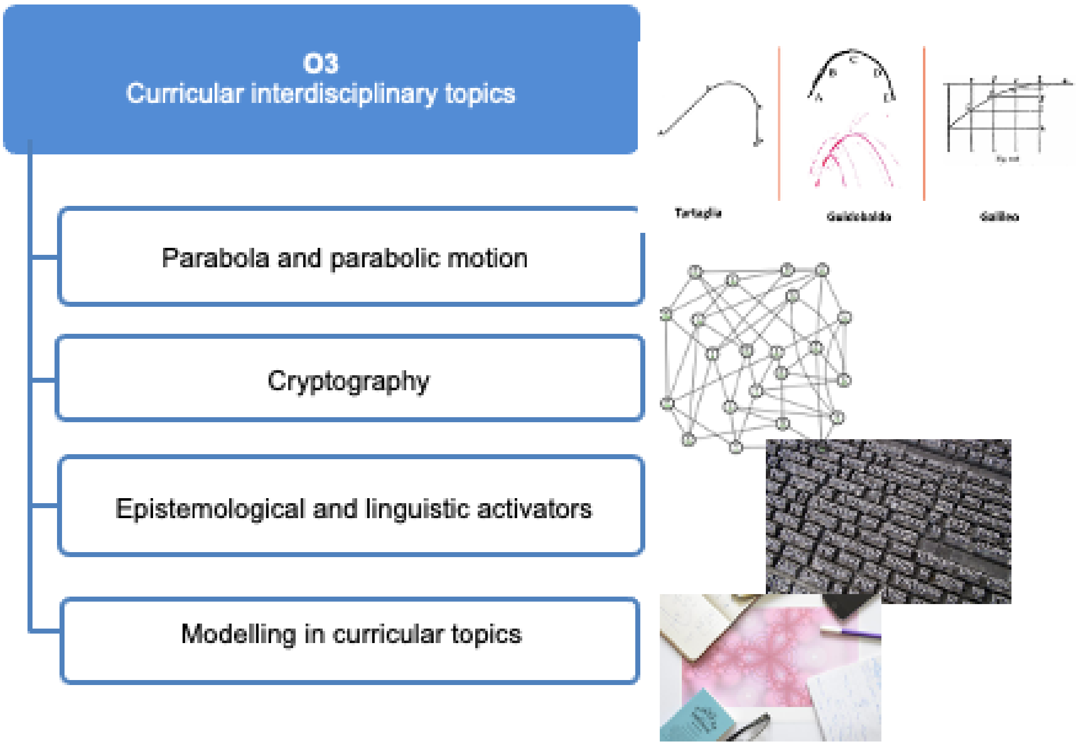
O3 modules on Curricular interdisciplinary topics

In this chapter, the four modules on Curricular interdisciplinary topics are presented.



Tab. 5. Titles of the modules produced in O3 and links to materials and lesson plans

O3: Teaching modules on curricular interdisciplinary topics		
5	Parabola and parabolic motion	https://identitiesproject.eu/parabola-and-parabolic-motion/
6	Cryptography	https://identitiesproject.eu/cryptography/
7	Linguistic and epistemological activators of interdisciplinarity	https://identitiesproject.eu/linguistics-and-epistemology/
8	Modelling in curricular topics	https://identitiesproject.eu/modeling/

The Module on Parabola and parabolic motion

Rationale of the topic selected	The module concerns the curricular themes of parabola and parabolic motion. The themes have been chosen since, even if they are obviously interrelated, habits, textbooks, and school practices have progressively led to the establishment of school narratives that have short-circuited the deep interdisciplinary connections that characterized the historical development of the two disciplines. The historical episodes are reconstructed as two emblematic cases to highlight the structural role of mathematics in physics and *vice versa*. The module has been designed to emphasize the connections so as to both make the epistemological structures of the disciplines visible and create a space where the student-teachers were encouraged to make deep interdisciplinary experiences.
IDENTITIES' framework and aims pursued	The IDENTITIES approach for the design and implementation of the module is mainly evident in the choice of the following design principles: DP1) Keep disciplinary technicalities as much simpler as possible so as to emphasize the interdisciplinary potential of the topic that emerges by analysing the theme from a historical, representational, and epistemological perspective (Renn et al., 2001). DP2) Unpack the epistemic core of disciplinary knowledge through the Family Resemblance Approach (Erduran & Dagher, 2014). DP3) Establish the boundary zone by introducing boundary objects ("curve-trajectories" and "proof"), and activating a variety of boundary-crossing mechanisms (Akkerman & Bakker, 2011). DP4) Create a safe learning environment, where student-teachers feel free to play with their ideas and are supported to find an equilibrium and inhabit the tension between feeling at their disciplinary home and accepting the risk of thinking out of the box and embracing the ambiguity of interdisciplinarity.
Brief description of the submodules: aims and activities	
Becoming explorers	Aims: The first submodule aims to introduce the tools to analyse the disciplines' identities and their integration. To this scope, the Family Resemblance Approach and the metaphor of the "boundary" are introduced. A further aim is to share their ideas on the knowledge that representations of curves embed and the criteria that can be used to establish if they are parabolas. Activities: Three teamwork activities are implemented. The first one guides the students to get acquainted with the vocabulary of Akkerman and Bakker

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	(2011) by sharing personal experiences of crossing boundaries and eventual difficulties they have met. The second one invites the students to reflect on aims & values of their disciplines, practices, methods, and forms of knowledge by sharing and commenting on examples of such epistemological elements. In the third activity, the student-teachers are invited, in group, to analyse a set of images that represents curves and point out criteria that can be used to establish if and why a curve can be recognised as a parabola.
Becoming students	<u>Aims:</u> The second submodule aims to unpack the interdisciplinary potential of the topic of parabola and parabolic motion, by analysing the theme from a historical, representational, and epistemological perspective. The analysis is carried out to show the co-evolution of the two disciplines and, specifically, to unveil boundary objects and boundary-crossing mechanisms that had a foundational role in the establishment or re-definition of the identities of physics and mathematics as disciplines. <u>Activities:</u> Through interactive lectures, the student-teachers, in the role of "students", are guided to experience two parallel narratives: the "physics" narrative where the structural role of mathematics is emphasised in the discovery of parabolic shape of projectile motion and the establishment of physics as discipline; the "mathematics" narrative, where the structural role of physics was stressed in the historical evolution that led to the re-classification of conics in mathematics. In the middle, the student-teachers are involved in an activity focused on reflecting on the construct of proof in mathematics and in physics.
Becoming analysts	<u>Aims:</u> The third submodule has two main related aims: 1) giving the participants the fundamental linguistic notion of the scientific language, the linguistic and the epistemological tools allowing them to analyse scientific texts; 2) recognising the elements of disciplinary's identities, boundary objects and boundary crossing mechanisms as they are represented in scientific texts (both historical texts and textbooks). <u>Activities:</u> To the participants, epistemological and linguistic grids are provided as a summary or enrichment of the second submodule. Through this activity, they are guided to apply the disciplinary and interdisciplinary knowledge and tools they have learned as lenses to break down stereotypes and unpack how textbooks convey specific images of mathematics and physics and of their possible relationships.

The Module on Cryptography

Rationale of the topic selected	Cryptography is nowadays an interdisciplinary domain in itself. Both mathematical elements (e.g., proofs, number theory) and Informatics elements (e.g., computational complexity, systems design, programming) are fundamental to solving the relevant social, technological, and scientific challenges it poses. Moreover, some cryptography elements encompass intertwined aspects of Informatics and Mathematics (e.g., one-way functions are both well-defined math functions and programs that satisfy specific security and efficiency criteria). The topic seems halfway through O2 and O3: while cryptography is crucial for today's societal challenges and its study involves different communities, the two most important disciplines involved (Informatics and Mathematics) are still very recognisable. While, historically, cryptography was in the realm of Mathematics, and while Informatics started as only a tool for faster computations, modern cryptography and its fields of application would not exist without Informatics. The current research in the field of cryptography stimulates a dialectic between the two disciplines, from Informatics to Mathematics (e.g., requiring new research in elliptic curves) and in reverse (e.g., using theorem provers to validate cryptographic properties). As said, cryptography still allows for "disciplinary projections": elements of the two disciplines are recognisable inside the field (analogously to what happens in bicultural societies[1]). We choose didactic activities where these projections can shed light on important disciplinary concepts, like computational complexity for Informatics and linear systems for Mathematics. The projections can stimulate exploration and learning of some relevant topics and ideas of each discipline, not only in the specific scope of cryptography but also in other Informatics and Mathematics areas. Moreover, this can foster the exploration of general interactions between the two disciplines. Consequently, developing learning in both disciplines and their interactions is relevant for secondary teachers who should teach cryptography either in Mathematics, Informatics courses, or interdisciplinary courses and projects..
IDENTITIES' framework and aims pursued	We consider the need to introduce cryptography as a social issue of contemporary society[2] (following a long-term presence in human history from ancient Greece to nowadays) because Cryptography is at the core of many of the economic, social, political interactions that happen in our interconnected, globalised digital society. That is why our module starts

[1] A good analogy can be bicultural identity or biculturalism, rather than multiculturalism (e.g. the coexistence of Europeans and Maori in New Zealand: the integration is clear, but is still very possible to recognize the two cultures).
[2] EU Agency for Network and Information Security (2016). *ENISA's Opinion Paper on Encryption.* https://www.enisa.europa.eu/publications/enisa-position-papers-and-opinions/enisas-opinion-paper-on-encryption

	with a discussion on end-to-end encryption, the debate around its societal advantages and risks, and stimulating the need to understand how it works to form an informed opinion. Moreover, Informatics education research agrees on the fact that communicating in secret and trying to decrypt messages without knowing the key is engaging and motivating for students[3]. We choose a public-key cryptography activity (based on a computationally hard problem on graphs) for epistemological and didactical reasons. Epistemologically, Informatics and Mathematics are deeply interconnected in the cryptography research field and discipline, and the activity, as we will see, can bring up many topics like algorithms, computational complexity, graphs, matrices, and linear systems. Educationally, informatics and cryptography have an inherent potential for *adidacticity*, that is, the potential for learning with a strong autonomy left to students' interactions with the problem. In fact, if one is trying to find the secret key of a cryptosystem (where encryption and decryption algorithms are public), the supposed key can be tested by decrypting the messages that have been encrypted with that key and see if the result is the original plaintext message. Analogously, in computer programming, students can autonomously test their program and check if it works by comparing the desired and actual result of the computation without waiting teacher's validation.
Brief description of the submodules: aims and activities	
Becoming explorers	<u>Aims</u>: Making students aware of the fact that cryptography is crucial today by contextualising it in a social debate about end-to-end encryption and backdoors. Students should realise what disciplines are involved and what knowledge they need to understand and form an opinion in a debate. Moreover, by analysing a historical piece on the birth of modern cryptography, students see how Informatics and Mathematics are intertwined in cryptography, beginning with *disciplinary projections*. During all the activities, students start encountering basic terminology about cryptography. <u>Activities</u>: Two individual activities. The first is watching an outreach video about how end-to-end encryption works (the fact that messages are fully encrypted from the sender to the receiver, so no one in the middle can read them) and the opportunity and risks of governments putting a backdoor (a way to decrypt those communications) for security reasons. The second one is reading an outreach text about the birth of asymmetric encryption, highlighting the interaction between mathematicians, computer scientists,

[3] Anke Lindmeier and Andreas Mühling. 2020. Keeping Secrets: K-12 Students' Understanding of Cryptography. In Proceedings of the 15th Workshop on Primary and Secondary Computing Education (Virtual Event, Germany) (WiPSCE '20). ACM, New York, NY, USA. https://doi.org/10.1145/3421590.3421630

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	and engineers who discovered it. After the two activities, students are given a reflection form about: skills needed to understand the debate; disciplines they recognise in the historical piece and their interactions; disciplinary-specific and crypto-specific terminology they found; initial thoughts about using the crypto interdisciplinary potential to teach crypto, Informatics and Mathematics in schools.
Becoming students	<u>Aims</u>: Students experience an example of a classroom activity designed according to the Theory of Didactical Situation (TDS). They will learn about public-key cryptography, explore the ideas of computational complexity and one-way functions, and manipulate interdisciplinary objects like functions, graphs, matrixes, and linear systems. From teachers' perspective, they will also experience firsthand how a didactical situation is organised. <u>Activities</u>: We start with a brief presentation on the fundamental elements of asymmetric cryptosystems. Then, we present a cryptosystem based on the perfect dominating set problem on graphs and show how it can be used to encrypt a message. After that, students are split into three groups: each group has to decrypt the same secret message but has different information on cryptographic and mathematical aspects of the system (i.e., in terms of the TDS, each group has a different organisation of the milieu) and an autonomous way to verify if they solved the problem (adidacticity). Afterwards, each group presents and discusses its strategies to decrypt the message. Finally, instructors lead an institutionalisation phase, based on the groups' presentations, to formalise the disciplinary aspects behind the activity. The different organisation of the three groups leads to different perspectives and approaches to the task and is intended to help student teachers think about the interdisciplinary objects encountered.

The Module on Epistemological and linguistic activators of interdisciplinarity

Rationale of the topic selected	The module has been designed as a general module to provide students with a more conscious approach to scientific texts, both as senders and receivers. It also intends to offer analytical tools that enable the recognition of disciplinary identities and curricular interdisciplinarity as they are presented in texts about scientific issues (historical texts, scientific papers, and textbooks). For this reason, the module deals with logic, linguistic, and epistemological issues with a particular focus on the impact of difficulties that an ambiguous or unclear scientific discourse can have on the comprehension of a message. The awareness of those aspects is important for understanding the texts adequately and deeply. Videos aimed to give the basics of the frameworks, terminology, and tools used in the module are available online.
IDENTITIES' framework and aims pursued	The IDENTITIES approach for the design and implementation of the module is mainly evident in the choice of the following design principles: DP1) keeping logical, linguistic, and epistemological technicalities as simple as possible in order to focus attention on the potential that the logic, linguistic and epistemological perspectives can add to the analysis of the interdisciplinarity; DP2) observing the scientific communication process, in its norms and needs, through the logic, linguistics, and epistemology approaches (Durand-Guerrier, 2008; Grice, 1975; Halliday & Martin, 1993; Habermas, 2003); DP3) recognizing and strengthening disciplinary identities in the curricular interdisciplinary discourse through recognizing differences and similarities among various disciplinary communities in the communication of scientific topics; DP4) reflecting on the boundary zone by introducing linguistic and epistemological activators (Bagaglini et al., 2021); DP5) building a collaborative learning environment in which teachers and students from different disciplinary communities are willing to

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	dialogue and share their disciplinary knowledge and perspectives in order to enrich their scientific identities.
Brief description of the submodules: aims and activities	
Becoming explorers	Aims: The first submodule focuses on exploring of processes involved in reading and understanding scientific texts, with particular attention to ambiguity and negation, also observed through translation issues. Activities: Two activities are implemented. In the first activity, students are required to read a textbook excerpt and answer a questionnaire about comprehension, disciplinarity, and interdisciplinarity issues, with an empirical teaching perspective. In the second activity, the relevance of logical analysis is stressed in facing the ambiguities of negation statements; the students are asked to translate them into their own languages in order to avoid ambiguity, showing them the importance of syntactic choices in structuring sentences.
Becoming students	Aims: The second submodule aims to show the differences between scientific language and common language, stressing how language's use varies among disciplinary communities and the possible criticalities that non-experts can have in understanding the scientific message. Further, it aims to consider epistemological issues about rational shared norms in building scientific communication. Activities: The second submodule includes two frontal lessons: one about linguistics and one about epistemology. The linguistic lecture is divided into two parts: the first part is dedicated to introducing the concept of scientific language, its historical development, its importance for the identity of the scientific community, and the specific characteristics which it assumes among disciplinary communities, in particular observing syntactical, morphological, lexical, and textual features; the second part is the explanation of what is implicit contents, how it structures the text and the negative consequences that they can have in activating misconceptions in students building their scientific knowledge. The epistemological lecture introduces Habermas' rationality construct in its three dimensions (epistemic, teleological, and communicative).
Becoming analysts	Aims: The third submodule aims to transform previous linguistic and epistemological knowledge into two analytical grids: the first looks at the scientific text's textual and syntactical structure, lexicon, and content; the second aims to bring out epistemological disciplinary issues. A second goal is to make the students aware of the complex organisation of the scientific discourse aimed to represent disciplinary images.

	Activities: The grids are used by students to point out the potentialities and criticalities of scientific texts in conveying disciplinary identities and interdisciplinarity, especially considering a didactic point of view. The students are requested to read a textbook excerpt, analyze it, and, subsequently, reformulate it using everyday language and different genres, such as the Galilean dialogue, making the implicit information of the source text explicit.

The Module on Modelling in mathematics and other disciplines

Rationale of the topic selected	The module concerns the curricular theme of modelling, which is transversal to mathematics, physics, and other sciences (with specificities in the different disciplines). The themes have been chosen since modelling appears in secondary school curricula in all countries, but its interpretation can be affected by habits of the disciplinary community that can vary a lot among different disciplines. In particular, it can be flattened or trivialized when dealing with mathematical modelling in physics, or it can be turned into an object (the 'model') rather than considering the process of modelling. The module has been designed to emphasize the contribution of different disciplines to an authentic modelling process suitable for secondary school students and exploiting only disciplinary contents and methods that can be considered curricular in all countries. The 'contamination' between different methods is alternated to the disciplinary analysis of the questions posed of their relevance to the discipline, to make the epistemological role of models and modelling within different disciplines visible where the student-teachers were encouraged to make deep interdisciplinary experiences.
IDENTITIES' framework and aims pursued	The IDENTITIES approach for the design and implementation of the module is mainly evident in the choice of the following design principles: DP1) Keep disciplinary technicalities as much simpler as possible so as to emphasize the interdisciplinary potential of the topic that emerges by modelling real problems and focus on the issues emerging in the process of modelling (Blum, 2015, Barquero, Bosch, & Gascón, 2019). DP2) Unpack the epistemic core of disciplinary knowledge through the Family Resemblance Approach (Erduran & Dagher, 2014).

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	DP3) Establish the boundary zone by introducing boundary objects (“model” and “approximation”), and activating a variety of boundary-crossing mechanisms (Akkerman & Bakker, 2011). DP4) Create a safe learning environment, where student-teachers feel free to play with their ideas and are supported to recognize the influence of disciplines in the formulation of questions to investigate a phenomenon. They will move from a paradigm of “modelling the real world” to the paradigm of modelling as elaboration of data collected and organized according to goals. Such goals could be related to issues posed by different agents (companies, societal agencies, research institutes, ..) or to the curiosity of the students themselves while exploring a situation.
Brief description of the submodules: aims and activities	
Becoming explorers	Aims: The first submodule aims to involve students in a mathematical modelling experience and to make them deal with two directions of mathematical modelling: vertical and horizontal modelization (Yvain-Prebiski & Modeste, 2019). The situations concern life sciences and industrial sciences. We use the results of this contemporary epistemological study to analyze and support the relevance of some problems designed to foster the devolution of the mathematical modelling process to the students. Moreover, in this submodule, students are asked to reflect on the novelties, choices and difficulties that emerged when dealing with a problem formulated by members of other fields. Activities: First, an adidactical situation is proposed to students concerning life science and industry; they are involved in a *milieu* where their exploration, guided by a question and supported by suitable media, should make emerge modelling strategies that the trainer could institutionalize as vertical and horizontal modelization in the “Becoming students” phase. We will use the strategy of “realistic fiction” in order to reduce the complexity of horizontal mathematization. The second activity guides the students to get acquainted with the vocabulary of Akkerman and Bakker by making explicit the assumptions that make this approach to modelling a mathematical one (identification) and what new aspects of mathematics they met in this process. Moreover, they are asked to reflect on eventual difficulties they have met during the activity due to the fact that the problem was formulated within other fields.
Becoming students	Aims: The second submodule aims to introduce the main epistemological features of mathematical modelling and modelling in sciences. After the explicitation of the notions of vertical and horizontal mathematization and their intertwining, students are introduced to the epistemological role of

	‘models and the game of modelling’ in science and particularly when dealing with the issue of radiation and absorbance in the green-house effect. The purpose of the submodule is to discuss with the students the epistemological dimension of modelling in different disciplines, showing the role of ‘modelling’ as a boundary object, triggering interesting identification and reflection at the boundary between maths and sciences. Activities: At the beginning, through interactive lectures, the student-teachers, in the role of “students”, are guided to recognize what are the invariant practices of horizontal mathematization they carried out in the explorer phase, and how they interplay with context-specific practices. In the second activity, the student-teachers are presented through a lecture about epistemological issues that can represent a demanding challenge for students, and more generally for citizens, related to climate science. While usually the attention is posed to the difficulties due to the use of non-linear models whose features are very different from the classic and mechanistic view of modelling (Pasini, 2005), puzzling epistemological issues also arise when very basic models are used in a curricular introduction to the greenhouse effect, which could easily leave students skeptical and detached (Tasquier, 2015; Tasquier et al., 2014). After being presented two different disciplinary epistemological aspects of modelling, students are asked to deal with mathematization in the organization of empirical data concerning the green-house effect into a mathematical model and coming back to the scientific issues.
Becoming analysts	Aims: The third submodule concerns the analysis of the mathematization in scientific modelling and interdisciplinary issues arising when different epistemologies of modelling are combined. Activities: To the participants, epistemological and linguistic grids are provided as a summary or enrichment of the second submodule. Through this activity, they are guided to apply the disciplinary and interdisciplinary knowledge and tools they have learned as lenses to break down stereotypes about modelling procedures in sciences and mathematics, highlighting their similarities, differences and interplays.

Concluding remarks

This document presents the methodological guidelines for the design and implementation of the modules on *curricular interdisciplinarity topics* and on *emerging STEM interdisciplinarity* in pre-service teacher education. All the modules have had common features that incorporate the central principles of the IDENTITIES project and a common approach to interdisciplinarity. In this sense, the Intellectual Output 4 (O4) has provided a methodological and analytical framework to support the work of the modules produced in O2 and O3.

The modules presented in O2 and O3 are the results of at least two rounds of selection, design, implementation, and revision. Each of the modules has been tested, fully or partially, in at least 2 different contexts. Each round has involved different "actors": researchers, designers, pre-service teachers, and in-service teachers to which the IDENTITIES project wants to refer.

The production of the modules has been methodologically guided within the methodology of *Didactical Engineering* (DE) (Artigue, 1994; 2014). This framework involves a 4-phase iterative process involving the *a priori* analysis, the design, the implementation and observation, and the *a posteriori* analysis, validation, and development of the module, according to back-and-forth dynamics between theoretical hypotheses and empirical results. This general methodology has been compatible with *Design-Based Research* (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). They both have an explicit theoretical orientation that enriches the goal to design and realize good practices with the purpose of explaining why a teaching practice is successful.

In this last chapter, we follow the different phases of the didactic engineering methodology to highlight the main contributions of the joint work on the (re)design, implementation, and analysis of the IDENTITIES modules.

1st phase with the *Preliminary analysis of the strengths of interdisciplinary topics* in which different teams of researchers and designers have worked on the design of the IDENTITIES modules. This preliminary analysis has led to considering some templates with the aim that each group of designers could discuss and synthesize the rationale of the selected topic (as included in the summary of the modules, presented in Chapters 3 and 4 of this document). From this initial phase, we may underline the significance in IDENTITIES of the following aspects:

- Importance of the constitution and collaboration of teams of designers with different backgrounds (from Physics, Mathematics, and Computer Science Education, among others) to make disciplinary knowledge and practice emerge along the design work. This is the first footstep to creating the necessary infrastructure to support working together on choosing and analyzing the interdisciplinary topics and the strengths (and possible limitations) to be then planned for teacher education.
- Relevance of the analysis of the strengths (and possible limitations) of the interdisciplinary selected topics. This preliminary analysis has facilitated the delimitation of the interdisciplinary topics and the discussion on how and why its

selection responds to emergent STEM topics (O2) or curricular interdisciplinary topics (O3).
- This previous discussion has led the team to characterize the intrinsic distinctions to approach interdisciplinarity in the IDENTITIES project (see Chapter 1 of this document). A summary is offered in the video "Curricular and STEM interdisciplinarity" accessible at: https://youtu.be/CSBwQixd-vI) that introduces the distinction between both (non-independent) ways to approach interdisciplinarity.

Beyond the preliminary analysis, thanks to the several implementations of the modules, the discussion about the strengths of the interdisciplinary topics of the modules has been extended and enriched thanks to the participation of pre-service secondary school teachers (and, in some cases, in-service secondary school teachers and secondary school students). The design of surveys to get these opinions about the potentialities of interdisciplinary topics has been extremely useful for this purpose.

2nd phase with the *Didactic design of IDENTITIES modules* in which different teams have been working on the design of the IDENTITIES modules. From this design stage, we may underline the significance of IDENTITIES in the following aspects:

- The final structure of the IDENTITIES modules has been built according to a general didactic model to design and plan teacher education proposals. This general didactic model corresponds to an adaptation of the *Study and Research Paths for Teacher Education* (SRP-TE) proposal, as proposed in the framework of the Anthropological Theory of the Didactic (see Chapter 2, see Figures 1 and 2).
  Although the different focus of the modules, this general didactic model has been useful to compare which aims, activities and tools/instruments were proposed in each submodule.
- Along with the design of the module (and the corresponding submodules), there have emerged important contributions in creating tools to refer to and analyse interdisciplinarity and the role of the disciplines. The IDENTITIES project has progressed on:
  - The adoption of the *boundary metaphor* to recontextualize some of the core concepts, such as "boundary", "boundary zones" or "boundary crossing mechanisms" (from Akkerman and Bakker (2011)) with respect to the focus of the IDENTITIES (see Chapter 1, and the video "The metaphor of boundary for interdisciplinarity" (https://youtu.be/fK0XvtFeO_U).
  - The use of the notions related to the *epistemological and linguistic activators* in the search of both exploiting the disciplinary identities and transgressing them (see Chapter 1, section 3) (https://youtu.be/vQV7Id3JIVA).

3rd phase with the *Implementation, observation, and data collection*. From this stage, it might be underlined the significance of IDENTITIES in the following aspects:

- The IDENTITIES modules have been the results of at least two rounds of selection, design, implementation, and revision. Each of the modules has been tested with prospective teachers, fully or partially, in at least 2 different contexts: in the IDENTITIES Summer School – Crete (online) 2021 or Barcelona 2022) or in local conditions (in different countries).
- With respect to the modules implemented in the IDENTITIES Summer Schools, several decisions were made:
  - agreeing on a common methodology for the observation, data collection, and data sharing,
  - researchers-designers (not directly involved in the design of an implemented module) would act as 'external' observers of the module.
- Data collection has also included other members (pre-service teachers, researchers, students) by collecting their reactions and opinions in surveys and debriefing sections at the end of the modules (with teachers, students, and researchers).

Our joint work and collaboration on the implementation and observation have facilitated the *a posteriori analysis, validation, and development of the IDENTITIES module* (4th phase). In particular, with respect to:

- Progressing together in making decisions about future developments or redesigns of the modules.
- Adopting a module from a different partner, to be adapted (or partially redesigned) to be locally implemented.
- Progressing together on the kind of tools that have been used as researchers, as designers, and, in particular, how they have been offered to pre-service teachers for the analysis of interdisciplinarity. Special attention might be paid to:
  - tools for the *epistemological analysis* of the disciplinary and interdisciplinary knowledge at stake: such as the construction of a "causal map" (used in the module about *Climate Change*); or of the questions-answers map (used in the module *Models, modelling and simulation to question interdisciplinarity of COVID-19*).
  - tools for the *linguistic analysis* (see module on *Linguistic and epistemological activators of interdisciplinarity*)
  - or, more in general, the "Guides for the analysis of interdisciplinary" proposed in different modules including elements of the "boundary" metaphor and "epistemological and linguistic activators".

The results of the analyses have been presented at international conferences and are the object of submitted or in-progress research papers. The list of research products (papers, theses, reports, conference presentations) is the following:

- Aguada, M.R, Branchetti, L., Giménez, J., Levrini, O., Pipitone, C., & Sala-Sebastià, G. (2021). Interdisciplinariedad en educación STEM. Reflexiones y retos. *UNO: Revista de didáctica de las matematicas*, *93*, 45-51.

- Bagaglini, V., Branchetti, L., Gombi, A., Levrini, O., Satanassi, S., & Viale, M. (2021). Il ruolo del testo nell'interdisciplinarità tra matematica, fisica ed educazione linguistica: Il tema del moto parabolico tra testi storici e manuali di fisica per la scuola secondaria di secondo grado. *Italiano a scuola*, *3*(1), 133–184. https://doi.org/10.6092/ISSN.2704-8128/13083
- Barelli, E., Barquero, B., Romero, O., Aguada, M.R., Giménez, J., Pipitone, C., Sala-Sebastià, G., Nipyrakis, A., Kokolaki, A., Metaxas, I., Michailidi, E., Stavrou, D., Bartzia, I., Lodi, M., Sbaraglia, M., Modeste, S., Martini, S., Durand-Guerrier, V., Bagaglini, V., Satanassi, S., Fantini, P., Kapon, S., Branchetti, L., & Levrini, O. (2022). Disciplinary identities in interdisciplinary topics: challenges and opportunities for teacher education. In G.S. Carvalho, A.S. Afonso & Z. Anastácio (Eds.), Fostering scientific citizenship in an uncertain world (Proceedings of ESERA 2021), Part 13 (co-ed. M. Evagorou & M.R. Jimenez Liso), (pp. 934-943). Braga: CIEC, University of Minho. ISBN 978-972-8952-82-2.
- Barelli, E., Aguada, M. R., Barquero, B., Pipitone, C. (in preparation). Investigating the interdisciplinarity of COVID-19 through the identification of boundary objects: a module for pre-service STEM teachers.
- Barelli, E., Barquero, B., & Branchetti, L. (under review). Questioning the evolution of the pandemic in an interdisciplinary way: the design of a Study and Research Path for pre-service Teacher Education. *Rivista Matematica dell'Università di Parma*.
- Barelli, E., Branchetti, L., & Barquero, B. (2021). Questioning interdisciplinarity within teacher education: A module on the evolution of COVID-19 pandemic. Presented at the 14th International Congress on Mathematical Education (ICME), July 12-19 2021, online.
- Barquero, B. (2021). Questioning interdisciplinarity from society to school: Design and analysis of interdisciplinarity teaching projects. *Invited plenary lecture at Edu-SIMAI 2020+2021* (August 2021).
- Branchetti, L. (2020). Interdisciplinarità tra fisica, matematica e informatica nella formazione iniziale degli insegnanti: il progetto IDENTITIES. [Interdisciplinarity between physics, mathematics and computer science in pre-service teacher education: the IDENTITIES project]. Presented at the SIF conference – 2020, September 14th-18th (online).
- Levrini, O., Branchetti, L., Stavrou, D., Barquero, B., & Durand-Guerrier, V. (2021, August 30-September 3). *IDENTITIES: Integrate Disciplines to Elaborate Novel Teaching approaches to InTerdisciplinarity and Innovate pre-service teacher Education for STEM challenges.* [Poster presentation]. European Science Education Research Association (ESERA) Conference, Braga.
- Nipyrakis, A., Kokolaki, A., & Stavrou, D. (2020, July 10). IDENTITIES project: Novel Teaching approaches to Interdisciplinarity for STEM Challenges. Presented at the 6th International Scientific Conference of the Institute of Humanities and Social Sciences – 2020, Heraklion.
- Nipyrakis, A., Stavrou, D., & Avraamidou, L. (2023, April 18-21, accepted). *Interdisciplinary Pre-service Teacher Training.* [poster presentation]. National Association for Research in Science Teaching (NARST), Chicago.

- Pollani, L., & Branchetti, L. (2022). An experience of exploring the boundary between mathematics and physics with preservice teachers. *Pre-Proceedings of the Fourth Conference of the International Network for Didactic Research in University Mathematics*, 330–339.
- Vásquez, S., Barquero, B., & Bosch, M. (under review). A Study and Research Path about the Evolution of Pandemics at Secondary School: Conditions for an Interdisciplinary Approach. *Rivista Matematica dell'Università di Parma*.

Because of the richness of the data collected, further analyses are ongoing and the results will be presented at future conferences and published in research papers.

The last point that we want to stress (as expressed in Barelli et al., 2022) is related to the protagonists of the IDENTITIES approach: the student teachers. Teachers are the most important *agents* of change in students' learning. Hence, if we want students to develop interdisciplinary skills, we should start by teaching this in our pre-service teacher education programs. That is why the design process not only includes the reconstruction of disciplinary and interdisciplinary issues in the module but also implementations in teacher education that require the preservice teachers to take different roles with respect to what is learned. Student teachers engage in exploring case studies of interdisciplinarity through explicit discussion of boundary crossing, epistemological activators, and linguistic activators, then in instructional design that highlights aspects of interdisciplinarity, in collective reflections on this design, followed by microteaching and further reflections. Further steps in this direction may be made to enhance collaboration, not only between pre-service teachers and designers, educators, and researchers. It might also involve in-service teachers and their regular teaching practice in secondary school institutions to analyse the viability and transferability of the interdisciplinary didactic proposals and the new tools to be offered to the whole educational community.

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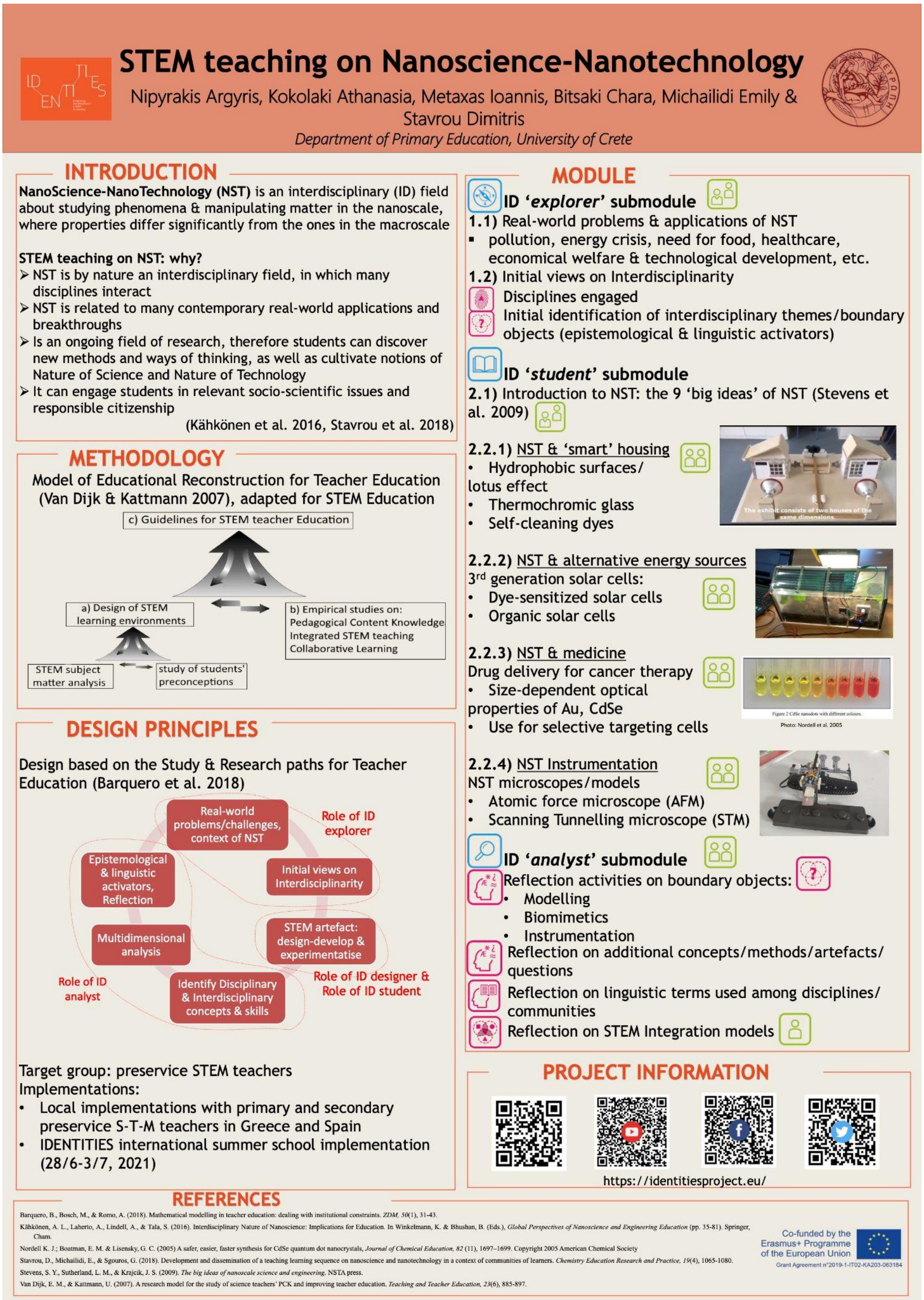
Annex: Graphical visualisation of the modules' structure



Models, modelling and simulations to question the interdisciplinarity of COVID-19

Barelli, E.[2], Barquero, B.[1], Pipitone, C.[1]
Aguada, M.R.[1], Bosch, M.[1], Giménez, J.[1] & Sala-Sebastià, G.[1]
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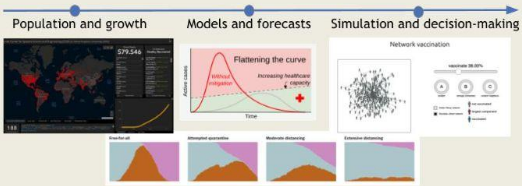
UNIVERSITAT DE BARCELONA
ALMA MATER STUDIORUM UNIVERSITÀ DI BOLOGNA

INTRODUCTION

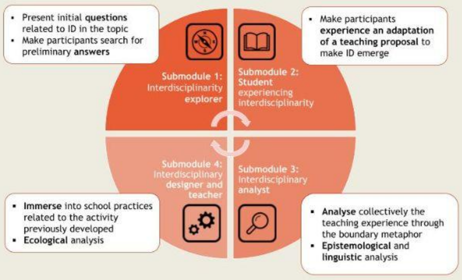
More than ever citizens, and students, need to understand how scientific advances contribute to the understanding of societal phenomena.



- How have STEM disciplines contributed to Society in understanding COVID-19 evolution?
- How can models, modelling and simulation contribute to understanding and decision-making?

METHODOLOGY

- An adaptation of a **Study and Research Path** for Teacher Education
- **Submodule structure** to question disciplinary identity and interdisciplinarity
- **Change of roles** explorers—students—analysts—designers/teachers to address interdisciplinarity



DESIGN PRINCIPLES

- Posing **professional teaching questions:** *How can scientific interdisciplinary practice be transposed to educational settings?*
- Transferring and using tools for the **epistemological** and **didactic analysis** of disciplines and interdisciplinarity
- Detecting **boundary zones** and **boundary-crossing** mechanisms to activate links between disciplines

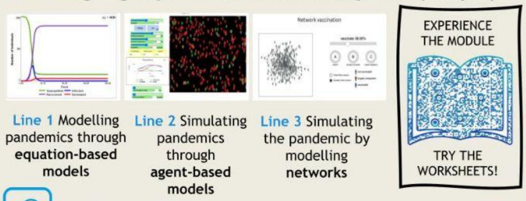
MODULE

Submodule 1 Exploring ID boundaries

- **Analysis of news** about scientific contributions to the understanding of COVID-19
- Detection of questions, interaction between disciplines, disseminated terminology

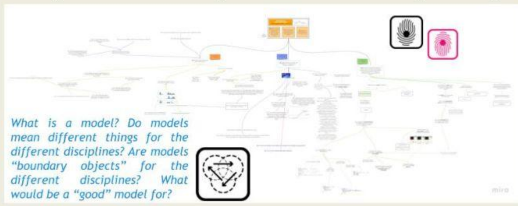
Submodule 2 Experiencing ID

- Working in groups in **lines of interdisciplinarity inquiry**



Submodule 3 Analysing disciplines and ID

- Collective group work on a **question-answer map**
- Analysis of the boundary ID zones and boundary-crossing



Submodule 4 Designing ID

Which questions could be transposed into secondary school education? Which opportunities or conditions would be offered that facilitate this transposition towards secondary school? Which difficulties could prevent the implementation in secondary schools?

PROJECT INFORMATION

https://identitiesproject.eu/

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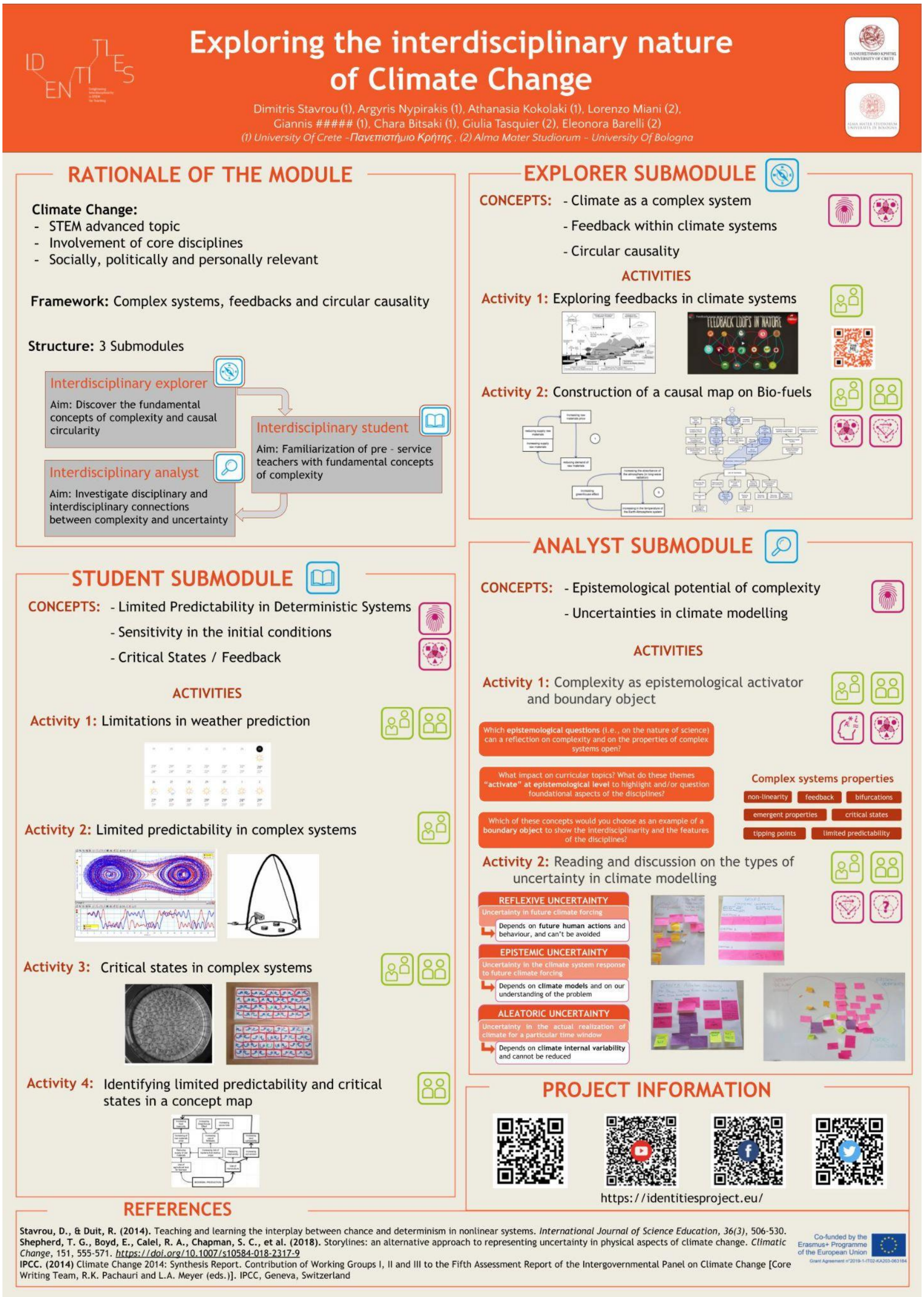
Exploring the interdisciplinary nature of Climate Change
Dimitris Stavrou (1), Argyris Nypirakis (1), Athanasia Kokolaki (1), Lorenzo Miani (2), Giannis ##### (1), Chara Bitsaki (1), Giulia Tasquier (2), Eleonora Barelli (2)
(1) University Of Crete -Πανεπιστήμιο Κρήτης , (2) Alma Mater Studiorum - University Of Bologna
RATIONALE OF THE MODULE
Climate Change:
- STEM advanced topic
- Involvement of core disciplines
- Socially, politically and personally relevant
Framework: Complex systems, feedbacks and circular causality
Structure: 3 Submodules
Interdisciplinary explorer
Aim: Discover the fundamental concepts of complexity and causal circularity
Interdisciplinary student
Aim: Familiarization of pre - service teachers with fundamental concepts of complexity
Interdisciplinary analyst
Aim: Investigate disciplinary and interdisciplinary connections between complexity and uncertainty
STUDENT SUBMODULE
CONCEPTS: - Limited Predictability in Deterministic Systems
- Sensitivity in the initial conditions
- Critical States / Feedback
ACTIVITIES
Activity 1: Limitations in weather prediction
Activity 2: Limited predictability in complex systems
Activity 3: Critical states in complex systems
Activity 4: Identifying limited predictability and critical states in a concept map
EXPLORER SUBMODULE
CONCEPTS: - Climate as a complex system
- Feedback within climate systems
- Circular causality
ACTIVITIES
Activity 1: Exploring feedbacks in climate systems
FEEDBACK LOOPS IN NATURE
Activity 2: Construction of a causal map on Bio-fuels
ANALYST SUBMODULE
CONCEPTS: - Epistemological potential of complexity
- Uncertainties in climate modelling
ACTIVITIES
Activity 1: Complexity as epistemological activator and boundary object
Which epistemological questions (i.e., on the nature of science) can a reflection on complexity and on the properties of complex systems open?
What impact on curricular topics? What do these themes "activate" at epistemological level to highlight and/or question foundational aspects of the disciplines?
Which of these concepts would you choose as an example of a boundary object to show the interdisciplinarity and the features of the disciplines?
Complex systems properties
non-linearity
feedback
bifurcations
emergent properties
critical states
tipping points
limited predictability
Activity 2: Reading and discussion on the types of uncertainty in climate modelling
REFLEXIVE UNCERTAINTY
Uncertainty in future climate forcing
Depends on future human actions and behaviour, and can't be avoided
EPISTEMIC UNCERTAINTY
Uncertainty in the climate system response to future climate forcing
Depends on climate models and on our understanding of the problem
ALEATORIC UNCERTAINTY
Uncertainty in the actual realization of climate for a particular time window
Depends on climate internal variability and cannot be reduced
PROJECT INFORMATION
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Stavrou, D., & Duit, R. (2014). Teaching and learning the interplay between chance and determinism in nonlinear systems. International Journal of Science Education, 36(3), 506-530.
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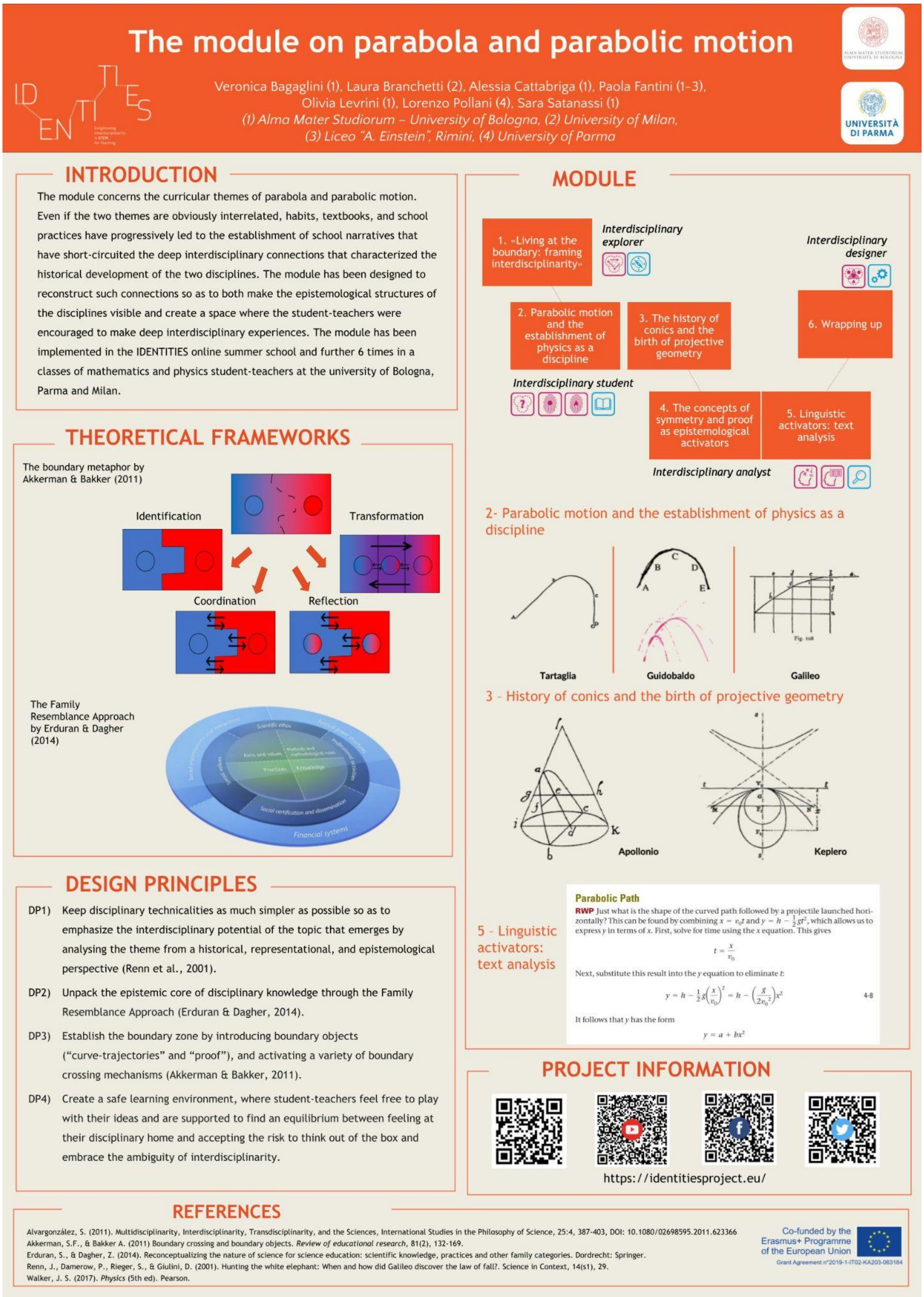
The module on parabola and parabolic motion
Veronica Bagaglini (1), Laura Branchetti (2), Alessia Cattabriga (1), Paola Fantini (1-3), Olivia Levrini (1), Lorenzo Pollani (4), Sara Satanassi (1)
(1) Alma Mater Studiorum – University of Bologna, (2) University of Milan, (3) Liceo "A. Einstein", Rimini, (4) University of Parma
UNIVERSITÀ DI PARMA
INTRODUCTION
The module concerns the curricular themes of parabola and parabolic motion. Even if the two themes are obviously interrelated, habits, textbooks, and school practices have progressively led to the establishment of school narratives that have short-circuited the deep interdisciplinary connections that characterized the historical development of the two disciplines. The module has been designed to reconstruct such connections so as to both make the epistemological structures of the disciplines visible and create a space where the student-teachers were encouraged to make deep interdisciplinary experiences. The module has been implemented in the IDENTITIES online summer school and further 6 times in a classes of mathematics and physics student-teachers at the university of Bologna, Parma and Milan.
THEORETICAL FRAMEWORKS
The boundary metaphor by Akkerman & Bakker (2011)
Identification
Transformation
Coordination
Reflection
The Family Resemblance Approach by Erduran & Dagher (2014)
Scientific ethos
Aims and values
Methods and methodological rules
Practices
Knowledge
Social certification and dissemination
Financial systems
DESIGN PRINCIPLES
DP1) Keep disciplinary technicalities as much simpler as possible so as to emphasize the interdisciplinary potential of the topic that emerges by analysing the theme from a historical, representational, and epistemological perspective (Renn et al., 2001).
DP2) Unpack the epistemic core of disciplinary knowledge through the Family Resemblance Approach (Erduran & Dagher, 2014).
DP3) Establish the boundary zone by introducing boundary objects ("curve-trajectories" and "proof"), and activating a variety of boundary crossing mechanisms (Akkerman & Bakker, 2011).
DP4) Create a safe learning environment, where student-teachers feel free to play with their ideas and are supported to find an equilibrium between feeling at their disciplinary home and accepting the risk to think out of the box and embrace the ambiguity of interdisciplinarity.
MODULE
1. «Living at the boundary: framing interdisciplinarity»
Interdisciplinary explorer
Interdisciplinary designer
2. Parabolic motion and the establishment of physics as a discipline
3. The history of conics and the birth of projective geometry
6. Wrapping up
Interdisciplinary student
4. The concepts of symmetry and proof as epistemological activators
5. Linguistic activators: text analysis
Interdisciplinary analyst
2- Parabolic motion and the establishment of physics as a discipline
Tartaglia
Guidobaldo
Galileo
3 - History of conics and the birth of projective geometry
Apollonio
Keplero
5 - Linguistic activators: text analysis
Parabolic Path
RWP Just what is the shape of the curved path followed by a projectile launched horizontally? This can be found by combining $x = v_0 t$ and $y = h - \frac{1}{2}gt^2$, which allows us to express y in terms of x. First, solve for time using the x equation. This gives
$t = \frac{x}{v_0}$
Next, substitute this result into the y equation to eliminate t:
$y = h - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = h - \left(\frac{g}{2{v_0}^2}\right)x^2$ 4-8
It follows that y has the form
$y = a + bx^2$
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Cryptography between Informatics and Mathematics

Iro Bartzia[1], Michael Lodi[2], Marco Sbaraglia[2], Simon Modeste[1], Viviane Durand-Guerrier[1]
[1]Université de Montpellier (France) [2]Alma Mater Studiorum · Università di Bologna (Italy)

THE MODULE

Cryptography as a… social issue in today's digital world
topic at the interface between Math and CS

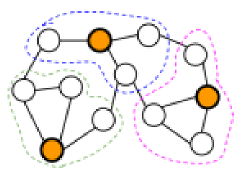
Submodule 1 Interdisciplinary explorer
Reflection on **real-world examples** (pros and cons of end-to-end encryption) and **historical reflection** (the birth of public-key cryptography)

S2 Interdisciplinary student
Participants experience a *Didactical Situation* as students to learn the main ideas of *asymmetric cryptography* through a cryptosystem based on graphs
- To experience interdisciplinarity
- As a starting point for discussing other CS and Math topics (e.g., computational complexity, graphs, linear systems, …)

S3 Interdisciplinary analyst
Collective analysis of the teaching activity they experienced
- Epistemological analysis of interdisciplinarity
- Linguistic analysis of interdisciplinarity

S4 Interdisciplinary designer and teacher
Design, implementation and analysis of other *Didactical Situations* on other fundamental concepts (e.g., computational complexity) in light of the previous analysis, focusing on the *interdisciplinary* and *learning potentials*

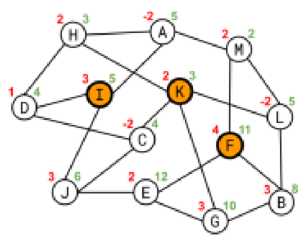
THE PDS CRYPTOSYSTEM



A vertex *v* of a graph dominates vertex *u* if there is an edge from *v* to *u*. The vertex *v* also dominates itself.

A **set of vertices** is a *perfect dominating set* (PDS) if each vertex of the graph is dominated by exactly one vertex in the set

Determining a PDS (if it exists) in a given graph is an **NP-hard problem**, i.e., we only know algorithms that take an **exponential time** w.r.t. the number of nodes



Secret message: 19
Decompose it as
a sum of random integers
19 = 3 + 2 + 4 + 3 + (-2) + 1 + (-2) + 2 + 3 + (-2) + 2 + 3 + 2
Put each summand on a node

For each node, compute the green value as the sum of all the red values on its neighbors and itself, e.g.:

```
sumC = C + D + K + J
   4 = -2 + 1 + 2 + 3
```

The **secret message** is **the graph with the green values only** (i.e., *without the red values and without the PDS marked*).

THE DIDACTICAL SITUATION

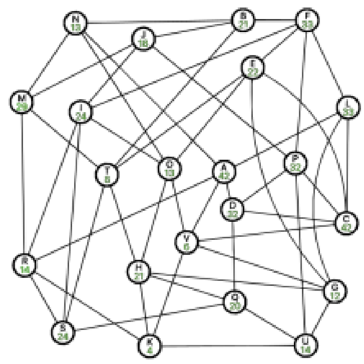
Encryption
The encryption technique (divide a number in red values, compute the green values) is presented to all students

Decryption
Students try to decrypt the secret message encrypted in the graph
➢ *Different groups have different information !*
➢ *All groups can independently check whether they have decrypted the message correctly*

- Group A is given the definition of PDS and the PDS for the given graph. They don't know the decryption algorithm
  - position of a *cryptographic engineer* with all the mathematical elements available; they need to combine them to design a public-key cryptosystem
- Group B is given the definition of PDS and the decryption algorithm (which uses the PDS). They do not know the specific PDS for the given graph
  - position of a *cryptanalyst* carrying out a person-in-the-middle attack (the attacker knows the public key, the encryption and decryption algorithms but not the private key)
- Group C has no information other than the encrypted message itself. They do not know the decryption algorithm, nor are they aware of the existence of a PDS
  - position of a *cryptanalyst* trying to find the plaintext message without necessarily finding the private key

A number has been encrypted with the method explained in the box on the left.

Can you decipher the secret message, i.e., find a number that opens the treasure chest?



PROJECT INFO

All teaching materials will be published from December 2022

https://identitiesproject.eu/

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